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IFIC 10-16

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Abstract

We present an extension of the Standard Model (SM) based on the discrete flavor symmetry S_3 which gives a neutrino mass matrix with two-zero texture of Fritzsch-type and nearly diagonal charged lepton mass matrix. The model is compatible with the normal hierarchy only and predicts $\sin^2\theta_{13} \approx 0.01$ at the best fit values of solar and atmospheric parameters and maximal leptonic CP violation.

1 Introduction

Although there is a robust evidence that neutrinos are mixed, many aspects of the neutrino physics are not clearly understood yet. Among them, the comprehension of the values of the masses and mixing and the differences with respect to the quark sector are an open problem whose solution seems to be quite far from being found. Recent data from neutrino oscillations produced the following results:

$$0.36 \leq \sin^2\theta_{23} \leq 0.67 \qquad 0.27 \leq \sin^2\theta_{12} \leq 0.38 \qquad \sin^2\theta_{13} < 0.053\,, \tag{1}$$

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and

$$\begin{aligned} 2.07 \times 10^{-3}\ eV^2 \leq \Delta m^2_{atm} \leq 2.75 \times 10^{-3}\ eV^2, \\ 7.03 \times 10^{-5}\ eV^2 \leq \Delta m^2_{sol} \leq 8.27 \times 10^{-5}\ eV^2, \end{aligned} \tag{2}$$

at 99.73% confidence level [1] (see [2] for other recent interpretations of the neutrino data).

We have only hints coming from cosmological observations that the absolute values of the neutrino masses should be less than 1 eV [3]. In the quark sector the situation is quite different: not only the masses and the hierarchy in the up and down sectors are better known but also the mixing angles are well measured and strongly differ from the neutrino ones. A successful ansatz to reproduce these features in the quark sector is the Fritzsch-like texture [4], where both the up and down quark mass matrices have a simple form

$$M = \begin{pmatrix} 0 & A & 0 \\ A^* & C & B \\ 0 & B^* & D \end{pmatrix}. \tag{3}$$

Such a matrix (already described in, e.g., [5]) gives the well know relation

$$\tan\theta_{12} = \sqrt{\frac{m_1}{m_2}}, \tag{4}$$

which predicts the Cabibbo angle whose small value is a consequence of the strong hierarchy in the masses. A texture as in eq.(3) can also be employed for the Majorana neutrino mass matrix; this is a particular case of the class of two-zero texture [6] which, together with the two relations $\Delta m_{atm} = m_3^2 - m_1^2$ and $\Delta m_{sol} = m_2^2 - m_1^2$, fix the absolute neutrino mass scale as suggested in [7]. Unlike the quark sector, the solar and atmospheric angles can be large due to the fact that in the neutrino sector the hierarchy is not so strong.

Although a vast class of Fritzsch-like textures (and their phenomenological consequences) has been already studied in the literature, in this paper we propose a leptonic model based on the permutation symmetry S_3 which naturally gives rise to a Fritzsch-type neutrino Majorana mass matrix (and, in addition, to a nearly diagonal charged leptons). At tree level, the tau lepton acquires a mass via the spontaneous electroweak symmetry breaking (ESB) driven by one S_3 doublet and two S_3 singlets, whereas the electron and the muon remain massless. Higher order operators, mediated by just one Standard Model (and S_3) scalar singlet (called the flavon) are responsible for $m_e, m_\mu \neq 0$. In the neutrino sector, the Majorana mass matrix is generated by dimension five [8] and six operators.

The paper is organized as follows: in the next section we introduce the model; the scalar potential is studied in Sec.3; the lepton and neutrino mass matrices are introduced in Secs.4

and 5, respectively whereas their phenomenological consequences are discussed in Sec.6. Sec.7 is devoted to our conclusions.

2 The model

We propose a model based on S_3, the group of permutations of three objects, which is the smallest non-Abelian discrete group. S_3 contains one doublet irreducible representation and two singlets. This feature is useful to separate the third family of fermions from the other two and has been already used for model building [9]. For pioneers papers see [10] (and also references in [11]).

The group S_3 has two generators S and T satisfying the following relations:

$$S^2 = T^3 = (ST)^2 = 1 \ . \tag{5}$$

One possible realization is the so-called "T-diagonal" basis where

$$S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad T = \begin{pmatrix} \omega' & 0 \\ 0 & \omega'^2 \end{pmatrix}, \tag{6}$$

with $\omega' = e^{i2/3\pi}$. The tensor products involving pseudo-singlets are given by $1' \times 1' = 1$ and $1' \times 2 = 2$ while the product of two doublets is $2 \times 2 = 2 + 1 + 1'$ which, in terms of the components of the two doublets $A = (a_1, a_2)^{\rm T}$ and $B = (b_1, b_2)^{\rm T}$ in the T-diagonal basis, are as follows:

$$\begin{array}{ll} a_1 b_2 + a_2 b_1 \in 1 & \\ a_1 b_2 - a_2 b_1 \in 1' & \end{array} \qquad \begin{pmatrix} a_2 b_2 \\ a_1 b_1 \end{pmatrix} \in 2 \, . \tag{7}$$

The product $2^* \times 2$ is similar to 2×2 with the exchange of $a_1 \leftrightarrow a_2$.

Construction of the model

The Higgs sector is extended from one $SU(2)_L$-doublet to two $SU(2)_L$-doublets, $H_D = (H_1, H_2)$ belonging to a doublet irreducible representation of S_3 and other two $SU(2)_L$ doublets, H_S and H'_S, belonging to singlet representations of S_3. We also introduce an electroweak scalar singlet χ which turns out to be relevant to give a non-vanishing electron and muon masses. In order to have nearly diagonal charged lepton mass matrix we assume two further parity symmetries, so that the global discrete symmetry group of the model is $G = S_3 \otimes Z_5 \otimes Z_2$. The matter assignment under $G \otimes SM$ is summarized in Tab.1.

fields	$L_D = L_{1,2}$	L_3	$l_{R_D} = l_{R_{1,2}}$	l_{R_3}	H_D	H_S	H'_S	χ
$SU_L(2)$	2	2	1	1	2	2	2	1
Y	-1	-1	-2	-2	1	1	1	0
S_3	2	1	2	1	2	1	1	1
Z_2	$+$	$-$	$+$	$-$	$+$	$+$	$-$	$+$
Z_5	ω^2	ω	ω	ω^2	ω^3	ω^4	ω^4	ω^2

Table 1: *Matter assignment of the model. ω is the Z_5 charge $\omega = e^{i2/5\pi}$ and Y is the SM hypercharge in the convention $Y = 2(Q - T_3)$, where T_3 is the third component of the SM $SU(2)$ doublets.*

3 The scalar potential

The most general Higgs potential invariant under $G \times SM$ is as follows:

$$\begin{aligned}
V &= \mu_1 H_S'^\dagger H'_S + \mu_2 (H_D^\dagger H_D)_1 + \mu_3 H_S^\dagger H_S + \mu_4 |\chi|^2 + \lambda_1 |\chi|^4 \\
&+ (\lambda_2 H_D^\dagger H_D + \lambda_3 H_S^\dagger H_S + \lambda_4 H_S'^\dagger H'_S)|\chi|^2 + \lambda_5 [(H_D^\dagger H_D)]^2 + \lambda_6 [(H_D^\dagger H_D)_{1'}]^2 \\
&+ \lambda_7 [(H_D^\dagger H_D)_2]^2 + \lambda'_7 (H_D^\dagger H_D^\dagger)_1 (H_D H_D)_1 + \lambda_8 (H_S^\dagger H_S)^2 \\
&+ \lambda'_9 (H_D^\dagger H_D)_1 H_S'^\dagger H'_S + \lambda''_9 (H_D^\dagger H'_S)_2 (H_S'^\dagger H_D)_2 + \lambda'''_9 ((H_D^\dagger H'_S)_2^2 + h.c.) + \\
&+ \lambda'_{10} (H_D^\dagger H_D)_1 H_S^\dagger H_S + \lambda''_{10} (H_D^\dagger H'_S)_2 (H_S'^\dagger H_D)_2 + \lambda'''_{10} ((H_D^\dagger H_S)_2^2 + h.c.) + \\
&+ \lambda_{11} (H_D^\dagger H_D^\dagger)_2 (H_D H_D)_2 + \lambda_{12} (H_S'^\dagger H'_S)^2 + \\
&+ \lambda_{13}{}' H_S'^\dagger H'_S H_S^\dagger H_S + \lambda_{13}{}'' (H_S'^\dagger H_S'^\dagger H_S H_S + h.c.) + \lambda_{13}{}''' H_S'^\dagger H_S H_S^\dagger H'_S
\end{aligned} \tag{8}$$

where we used the subscripts $1, 1'$ and 2 to refer to the S_3 contractions when necessary and, for any Higgs fields, $\tilde{H} = -i\tau_2^T H^*$. In the case of real vev's, that is

$$\begin{aligned}
&\langle H_D \rangle = (v_1,\ v_2), \\
&\langle H_S \rangle = v_S, \\
&\langle H'_S \rangle = v'_S, \\
&\langle \chi \rangle = v_\chi,
\end{aligned} \tag{9}$$

the potential can be written as[4]

$$\begin{aligned} V &= (\lambda_{11}+\lambda_5+\lambda_6)(v_1^4+v_2^4)+v_S^4\lambda_8+v_S'^4\lambda_{12}+v_S^2v_S'^2\lambda_{13}+v_S'^2\mu_1+v_S^2\mu_3+ \\ &+ \left(\mu_4+v_S^2\lambda_3+v_S'^2\lambda_4\right)\chi^2+\lambda_1\chi^4+\left(v_S^2\lambda_{10}+v_S'^2\lambda_9+\mu_2+\lambda_2\chi^2\right)(v_2^2+v_1^2)+ \\ &+ \left(2v_2^2v_1^2(\lambda_5-\lambda_6+\lambda_7)\right). \end{aligned} \tag{10}$$

The minima of V are found solving the minimizing equations:

$$\begin{aligned} &\frac{\partial V}{\partial v_1}=2v_1\left[\lambda_2v_\chi^2+\lambda_{10}v_S^2+\lambda_9v_S'^2+2v_1^2(\lambda_{11}+\lambda_5+\lambda_6)+2v_2^2(\lambda_5-\lambda_6+\lambda_7)+\mu_2\right]=0, \\ &\frac{\partial V}{\partial v_2}=2v_2\left[\lambda_2v_\chi^2+\lambda_{10}v_S^2+\lambda_9v_S'^2+2v_2^2(\lambda_{11}+\lambda_5+\lambda_6)+2v_1^2(\lambda_5-\lambda_6+\lambda_7)+\mu_2\right]=0, \\ &\frac{\partial V}{\partial v_S}=2v_S\left[\lambda_3v_\chi^2+\lambda_{10}\left(v_1^2+v_2^2\right)+2v_S^2\lambda_8+v_S'^2\lambda_{13}+\mu_3\right]=0, \\ &\frac{\partial V}{\partial v_s'}=2v_s'\left[\lambda_4v_\chi^2+\lambda_9\left(v_1^2+v_2^2\right)+2v_S'^2\lambda_{12}+v_S^2\lambda_{13}+\mu_1\right]=0, \\ &\frac{\partial V}{\partial v_\chi}=2v_\chi\left[\mu_4+2\lambda_1v_\chi^2+\lambda_2\left(v_1^2+v_2^2\right)+\lambda_3v_S^2+\lambda_4v_S'^2\right]=0\,. \end{aligned} \tag{11}$$

The second equation is satisfied for $v_2=0$. From the remaining equations we can easily get the vevs of the other scalars in terms of the couplings of the Higgs potential; in particular, a solution with $v_1\neq 0$ can be found and the vev alignment of the S_3 Higgs doublet assumes the structure:

$$\langle H_D\rangle=(v,0)\,. \tag{12}$$

For this vev configuration, it is possible to find a huge region of the Higgs parameter space where the eigenvalues of the Hessian of the potential are all positive and therefore where the Higgs potential has a local minimum. Note that a solution of the form $\langle H_D\rangle=(0,v)$ is physically equivalent to eq.(12), producing the same phenomenology in the charged lepton and neutrino sectors. In fact it corresponds to the exchange of L_1 with L_2. We also verified numerically that, in the large parameter space where eq.(12) is a minimum, other solutions like $\langle H_D\rangle=v\,(1,1)$ do not produce positive definite Hessian. The mass spectra of the Higgs particles will be discussed elsewhere.

[4]Where $\lambda_i=\lambda_i'+\lambda_i''+\lambda_i'''$ for $i=9,10,13$.

4 Leptons

The most general Lagrangian invariant under $G \times SM$ is given by:

$$\mathcal{L} = \frac{y_1}{\Lambda}\overline{L_D} H_D l_{R_D} \chi^* + \frac{y_2}{\Lambda}\overline{L_D} H_S l_{R_D} \chi + y_3 \overline{L_3} H_S l_{R_3} \,, \tag{13}$$

where Λ is the cut-off scale. Higher order terms only appear at $\mathcal{O}(1/\Lambda^2)$ and will be considered negligible for our discussion. From eq.(12) the charged lepton mass matrix is:

$$M_l = \begin{pmatrix} \frac{y_2}{\Lambda} v_S v_\chi & 0 & 0 \\ \frac{y_1}{\Lambda} v v_\chi & \frac{y_2}{\Lambda} v_S v_\chi & 0 \\ 0 & 0 & y_3 v_S \end{pmatrix}. \tag{14}$$

When v_χ is equal to zero only the τ lepton is massive. The electron and muon masses are generated by the vev of the scalar χ and are then suppressed by the large scale Λ. The matrix $M_l M_l^\dagger$ has three distinct eigenvalues that can be identified with the squared charged fermion masses as:

$$\begin{aligned} m_e^2 &= \frac{\varepsilon^2}{2}\left(v^2 y_1^2 + 2 v_S^2 y_2^2 - v y_1 \sqrt{v^2 y_1^2 + 4 v_S^2 y_2^2}\right) \\ m_\mu^2 &= \frac{\varepsilon^2}{2}\left(v^2 y_1^2 + 2 v_S^2 y_2^2 + v y_1 \sqrt{v^2 y_1^2 + 4 v_S^2 y_2^2}\right) \\ m_\tau^2 &= v_S^2 y_3^2 \end{aligned} \tag{15}$$

where we introduced the short-hand notation $\varepsilon = v_\chi/\Lambda$. We see that for $\varepsilon \ll 1$, the hierarchy among the τ and the lightest charged leptons is easily reproduced although the latter, in absence of any fine-tuning among the Yukawas and/or the Higgs vevs, are expected to be of the same order of magnitude. We address this question in the next section. The mass matrix for the charged leptons can be written in terms of the physical lepton masses as:

$$M_l = \begin{pmatrix} \sqrt{m_e m_\mu} & 0 & 0 \\ -m_\mu(1 - \frac{m_e}{m_\mu}) & \sqrt{m_e m_\mu} & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \tag{16}$$

and the squared matrix $M_l M_l^\dagger$ is then diagonalized by:

$$U_L = \begin{pmatrix} \frac{1}{\sqrt{1+\frac{m_e}{m_\mu}}} & -\sqrt{\frac{m_e}{m_\mu}}\frac{1}{\sqrt{1+\frac{m_e}{m_\mu}}} & 0 \\ \sqrt{\frac{m_e}{m_\mu}}\frac{1}{\sqrt{1+\frac{m_e}{m_\mu}}} & \frac{1}{\sqrt{1+\frac{m_e}{m_\mu}}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -0.07 & 0 \\ 0.07 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{17}$$

5 Neutrino

The neutrino masses are generated by non-renormalizable operators of dimension 5 and 6 invariant under the group $G \times SM$[5]:

$$\begin{aligned}\Lambda \cdot \mathcal{L}_\nu \;=\;& y_1^\nu (\overline{L}_D \overline{L}_D)_1 (\tilde{H}_D \tilde{H}_D)_1 + y_2^\nu (\overline{L}_D \overline{L}_D)_2 (\tilde{H}_D \tilde{H}_D)_2 + y_3^\nu (\overline{L}_D \tilde{H}_D)_1 (\overline{L}_D \tilde{H}_D)_1 + \\ & y_4^\nu (\overline{L}_D \tilde{H}_D)_{1'} (\overline{L}_D \tilde{H}_D)_{1'} + y_5^\nu (\overline{L}_D \tilde{H}_D)_2 (\overline{L}_D \tilde{H}_D)_2 + \\ & y_6^{\nu\prime} (\overline{L}_D \overline{L}_D)_1 (\tilde{H}_S \tilde{H}_S)_1 \, \chi^*/\Lambda + y_6^{\nu\prime\prime} (\overline{L}_D \overline{L}_D)_1 (\tilde{H}'_S \tilde{H}'_S)_1 \, \chi^*/\Lambda + \\ & y_7^\nu \overline{L}_3 \overline{L}_3 (\tilde{H}_D \tilde{H}_D)_1 \, \chi/\Lambda + y_8^{\nu\prime} \overline{L}_3 \overline{L}_3 \tilde{H}_S \tilde{H}_S + y_8^{\nu\prime\prime} \overline{L}_3 \overline{L}_3 \tilde{H}'_S \tilde{H}'_S + \\ & y_9^\nu (\overline{L}_D \tilde{H}_D)_1 \overline{L}_3 \tilde{H}'_S \,,\end{aligned} \tag{18}$$

where we assumed that the large energy scale which suppresses these operators is of the same order of the cutoff scale Λ. The only operators of dimension six are those proportional to $y_6^\nu = y_6^{\nu\prime} + y_6^{\nu\prime\prime}$ and y_7^ν. From eq.(19) the neutrino mass matrix is as follows:

$$M_\nu = \begin{pmatrix} 0 & 2\, y_6^\nu (v_S^2 + v_S'^2) v_\chi/\Lambda & 0 \\ 2\, y_6^\nu (v_S^2 + v_S'^2) v_\chi/\Lambda & (y_2^\nu + y_3^\nu + y_4^\nu) v^2 & y_9^\nu v v'_S \\ 0 & y_9^\nu v v'_S & y_8^\nu (v_S^2 + v_S'^2) \end{pmatrix} \equiv \begin{pmatrix} 0 & b & 0 \\ b & a & c \\ 0 & c & d \end{pmatrix} , \tag{19}$$

where $y_8^\nu = y_8^{\nu\prime} + y_8^{\nu\prime\prime}$. Before discussing the phenomenological consequences of such a matrix, it is useful to get an estimate of the relevant Yukawa parameters and a relation among the vevs v and v_S. Comparing eqs.(14) and (16) and using the parameterization in eq.(19) we get:

$$\begin{aligned} \sqrt{m_e m_\mu} &= y_2 \, v_S \, \varepsilon \\ m_\mu \left(1 - \frac{m_e}{m_\mu}\right) &= y_1 \, v \, \varepsilon \\ m_\tau &= y_3 \, v_S \\ a &= y^\nu \, v^2 \\ b &= 2\, y_6^\nu (v_S^2 + v_S'^2) \, \varepsilon \,, \end{aligned}$$

where we assumed that $y_2^\nu + y_3^\nu + y_4^\nu = y^\nu$. We assume

$$v \gtrsim v_s \sim \mathcal{O}(100)\,\text{GeV}, \qquad \varepsilon \sim \mathcal{O}(10^{-2})$$

[5]Dimension 7 operators can be built, for instance, adding the singlet χ^2 or doublets $H^\dagger H$ to the previous $d = 5$ operators and will then be neglected.

then if

$$y_1 \sim \mathcal{O}(10^{-1}), \qquad y_2 \sim \mathcal{O}(10^{-2}), \qquad y_2 \sim \mathcal{O}(10^{-2}),$$

we have the correct charged lepton mass hyerarchies. We then consider the b/a ratio

$$\frac{b}{a} \;=\; \frac{2\, y_6^{\nu} \varepsilon (v_S^2 + v_S'^2)}{y^{\nu}\, v^2\, v_S}. \tag{20}$$

We numerically verified that $\left(\frac{a}{b}\right) \sim \mathcal{O}(1)$ so the Yukawa parameters must satisfy

$$y_6^{\nu} \sim \mathcal{O}(1), \quad y^{\nu} \sim \mathcal{O}(10^{-2}).$$

With these assumptions, the hierarchy in the charged leptons is recovered, higher order terms with more that one flavon insertions can be safely neglected and the largest vev is generated by the S_3 singlet Higgs H_S that can be identified with the Standard Model Higgs.

The mass matrix in eq.(19) depends on five real parameters, one of which is related to the Dirac phase. The other four parameters can be fixed using the experimental information from both solar and atmospheric sectors, namely the solar and atmospheric mixing angles and squared mass differences. The model allows for correlations among the angle θ_{13} and the CP phase δ that can be easily obtained using the zeros of the Fritzsch texture. The previous mass matrix is diagonalized by a unitary matrix U^{ν} as

$$U^{\nu T} M_{\nu} U^{\nu} = \mathrm{diag}(\mu_1, \mu_2, \mu_3) \tag{21}$$

where $\mu_i = m_i e^{i\phi_i}$ and ϕ_i are Majorana phases. Writing U^{ν} in the CKM-like form[6]

$$U^{\nu} = \begin{pmatrix} c_{12}^{\nu} c_{13}^{\nu} & c_{13}^{\nu} s_{12}^{\nu} & e^{-i\delta_{\nu}} s_{13}^{\nu} \\ -c_{23}^{\nu} s_{12}^{\nu} - c_{12}^{\nu} e^{i\delta_{\nu}} s_{13}^{\nu} s_{23}^{\nu} & c_{12}^{\nu} c_{23}^{\nu} - e^{i\delta_{\nu}} s_{12}^{\nu} s_{13}^{\nu} s_{23}^{\nu} & c_{13}^{\nu} s_{23}^{\nu} \\ -c_{12}^{\nu} c_{23}^{\nu} e^{i\delta_{\nu}} s_{13}^{\nu} + s_{12}^{\nu} s_{23}^{\nu} & -c_{23}^{\nu} e^{i\delta_{\nu}} s_{12}^{\nu} s_{13}^{\nu} - c_{12}^{\nu} s_{23}^{\nu} & c_{13}^{\nu} c_{23}^{\nu} \end{pmatrix}, \tag{22}$$

and using the fact that the elements $(M_{\nu})_{11}$ and $(M_{\nu})_{13}$ are zero (see eq. (19)), eq.(22) implies:

$$\begin{aligned} \mu_2 &= \mu_1 \frac{\cos\theta_{12}^{\nu}\left(-\cot\theta_{12}^{\nu}\cos\theta_{23}^{\nu} + \sin\theta_{12}^{\nu}\sin\theta_{13}^{\nu}\sin\theta_{23}^{\nu} e^{i\delta_{\nu}}\right)}{\cos\theta_{23}^{\nu}\sin\theta_{12}^{\nu} + \cos\theta_{12}^{\nu}\sin\theta_{13}^{\nu}\sin\theta_{23}^{\nu} e^{i\delta_{\nu}}}, \\ \mu_3 &= -\mu_1 \frac{\cos\theta_{12}^{\nu}\cos^2\theta_{13}^{\nu}\sin\theta_{23}^{\nu} e^{-i\delta_{\nu}}}{\sin\theta_{13}^{\nu}\left(\cos\theta_{23}^{\nu}\sin\theta_{12}^{\nu} + \cos\theta_{12}^{\nu}\sin\theta_{13}^{\nu}\sin\theta_{23}^{\nu} e^{i\delta_{\nu}}\right)}. \end{aligned} \tag{23}$$

Our model is compatible with the normal mass ordering only because the ratio $|\mu_2|^2/|\mu_3|^2$ is always less than 1; expanding it up to second order in $\sin\theta_{13}^{\nu}$ we get:

$$\frac{|\mu_2|^2}{|\mu_3|^2} = \cot^2\theta_{12}^{\nu}\, \cot^2\theta_{23}^{\nu}\, \sin^2\theta_{13}^{\nu} + \mathcal{O}(s_{13}^{\nu 3}), \tag{24}$$

[6]We have used the short-hand notation $s_{ij}^{\nu} = \sin\theta_{ij}^{\nu}$ and $c_{ij}^{\nu} = \cos\theta_{ij}^{\nu}$.

and we checked that higher order corrections do not modify our statement. The mass differences are written as:

$$\Delta m^2_{sol} = m_1^2 \left[\frac{\cos\theta^\nu_{23}(\cos\theta^\nu_{12}\cos\theta^\nu_{23}\csc^2\theta^\nu_{12} - 2\cos\delta^\nu\cot\theta^\nu_{12}\sin\theta^\nu_{13}\sin\theta^\nu_{23})}{D_\nu} \right], \quad (25)$$

and

$$\Delta m^2_{atm} = m_1^2 \left(-1 + \frac{\cos^2\theta^\nu_{12}\cos^2\theta^\nu_{13}\cot^2\theta^\nu_{13}\sin^2\theta^\nu_{23}}{D_\nu} \right), \quad (26)$$

where

$$D_\nu = \cos^2\theta^\nu_{23}\sin^2\theta^\nu_{12} + \sin 2\theta^\nu_{12}\sin 2\theta^\nu_{23}\sin\theta^\nu_{13}\cos\delta^\nu/2 + \cos^2\theta^\nu_{12}\sin^2\theta^\nu_{13}\sin^2\theta^\nu_{23}\,. \quad (27)$$

From the ratio $\alpha = \Delta m^2_{sol}/\Delta m^2_{atm}$ we find a relation between α and the mixing angles s^ν_{12}, s^ν_{13} and s^ν_{23}

$$\alpha = 2\sin^2\theta^\nu_{13}\cot\theta^\nu_{23}\csc\theta^\nu_{12}\sec\theta^\nu_{12}(-\sin\theta^\nu_{13}\cos\delta_\nu + \cot 2\theta^\nu_{12}\cot\theta^\nu_{23})\,. \quad (28)$$

which will be used below to constrain the physical θ_{13} and δ.

6 Phenomenology

To study the phenomenological implication of our model, it is necessary to relate the parameters in eq.(22) to the physical ones. This can be achieved introducing the rotations from the charged lepton sector described in Sec.4; the resulting mapping is a set of implicit relations that are quite cumbersome and will not be explicitly presented here. We limit ourselves to describe the procedure which allows us to extract the predictions of our model. The lepton mixing is defined by $V = U_L^\dagger U^\nu$ and we can write

$$U^\nu = U_L V\,, \quad (29)$$

where V is parametrized in the standard form as in eq. (22) replacing s^ν_{ij} and c^ν_{ij} with the physical (that is measurable) $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$, respectively. Taking the ratio of U^ν_{23} and U^ν_{33} from eq. (29) we find an expression for s^ν_{23} in terms of the physical parameters θ_{12}, θ_{23}, θ_{13} and the Dirac phase δ (and the corrections from the charged leptons). In the same way, always using eq. (29), we can express s^ν_{12} and s^ν_{13} as a function of θ_{12}, θ_{23}, θ_{13}, δ; finally, δ_ν is the argument of the element (13) of the matrix $U_L V$. In this way we have all the parameters θ^ν_{12}, θ^ν_{13}, θ^ν_{23} and δ_ν as a function of the neutrino mixing angles θ_{13}, θ_{12}, θ_{23}

and the phase δ. These relations can be inserted into eq. (28) to get an implicit connection among the mixing parameters and α, which is a characteristic of our model. Also the lightest mass eigenstate can be related to the same parameters and Δm^2_{atm} using eq. (26).

In the left panel of Fig.1 we show the dependence of $\sin^2\theta_{13}$ as a function of δ taking θ_{12}, θ_{23} and α inside their experimental ranges. In particular, the solid line represents the 1σ correlation when also the other parameters are left free to vary in their 1σ allowed ranges quoted in [1], whereas the 2σ correlation is represented by the dot-dashed line. Finally, dashed line is the relation obtained when θ_{12}, θ_{23} and α are fixed to their best fit values. We also included the upper limit on $\sin^2\theta_{13}$ at 3σ (upper horizontal dashed line) and the best fit value of ref. [1] (lower horizontal dashed line). We can see that, even considering the 2σ uncertainty, the predicted values for $\sin^2\theta_{13}$ are different from zero so that, to a very good accuracy, our model is compatible with deviation from $\theta_{13} = 0$ for any value of the CP violating phase. The precise value of θ_{13}, however, relies on the assumed magnitude for δ; in particular, the CP conserving case $\delta = 0$ is the most promising one to allow large θ_{13} (even above the current limits) whereas around $\delta \sim \pm\pi$ we get the smaller θ_{13} allowed in our model. It is interesting to observe that, in the case of maximal CP violation [7] and for the other oscillation parameters to their best fit values, the predicted $\sin^2\theta_{13}$ is fully compatible with the best fit value obtained in [1], $\sin^2\theta_{13} \sim 0.01$. Notice that, in the case of diagonal charged lepton mass matrix, the pattern of the $\theta_{13} - \delta$ correlation would have been quite similar, as it can be seen investigating the right panel of Fig.1. The fact that the corrections coming from U_L in eq.(17) are as large as the values of $\sin^2\theta_{13}$ is responsible for lowering the allowed θ_{13} for $\delta \sim \pm\pi$. For maximal CP violation at the best fit point, the Jarlskog invariant [14] is as follows:

$$J = c_{12}s_{23}c^2_{13}s_{12}s_{23}s_{13}\sin\delta = 0.023\,. \tag{30}$$

The next observable we want to discuss is the effective mass m_{ee} entering in the neutrinoless double beta decay. In the basis where the charged leptons are diagonal, m_{ee} is nothing but the (11) element of the neutrino mass matrix. According to eq.(19), this should vanishes as long as the rotation in the charged leptons is proportional to the identity matrix. Since this is not the case, a non-vanishing m_{ee} is generated by the rotation (17) and it is expected to be small because of the smallness of its off-diagonal entries. This is what we can observe in Fig.(2), where we plot the model predictions for m_{ee} as a function of the lightest neutrino mass m_1. For m_1 below $\mathcal{O}(10^{-2})$ eV we get $|m_{ee}| \sim 10^{-3}$ eV and then outside the range of

[7] Maximal CP violation can be observed in incoming experiments T2K and NOνA, see for instance [12,13].

future experimental sensitivities. We also see that the allowed range for the lightest neutrino mass is around $10^{-3} - 10^{-2}\, eV$; this is because, as already mentioned in the introduction, the Fritzsch texture gives a correlation between θ_{12}^{ν} [8] and the ratio m_1/m_2 that, together with the two measured square mass differences, fix the absolute neutrino scale in this range.

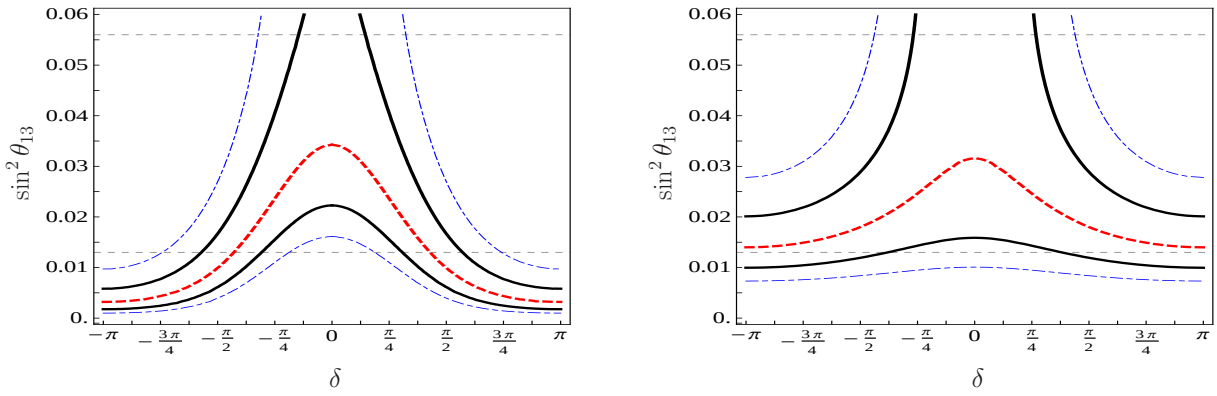

Figure 1: *Left panel: correlation among* δ *and* $\sin^2\theta_{13}$ *as obtained in our model. The* 1σ *result, obtained varying the other oscillation parameters also in their* 1σ *allowed ranges, is showed with solid lines, whereas the* 2σ *result is showed with the dot-dashed line. The dashed line is the relation obtained when* θ_{12}, θ_{23} *and* α *are fixed to their best fit values. Horizontal lines represent the upper limits on* $\sin^2\theta_{13}$ *(upper dashed line) and the best fit values (lower dashed line) from [1]. Right panel: the same as the left panel but assuming exactly diagonal charged lepton mass matrix.*

7 Conclusion

In this paper we have studied a leptonic model based on the discrete S_3 permutation flavor symmetry. We extended the scalar sector of the Standard Model by introducing three more Higgs doublets and one scalar singlet. We have carefully studied the problem of the minimization of the potential and found that a solution of the form $(v, 0)$ for the Higgsses in the

[8] Note that in the case of diagonal charged leptons the angle θ_{12} corresponds exactly to θ_{12}^{ν} and therefore from eqs. (4) and (23), the solar mixing angle does not depend on the absolute scale of neutrino mass m_1, while in our case this relation acquires a small correction proportional to $\sqrt{m_e/m_\mu}$.

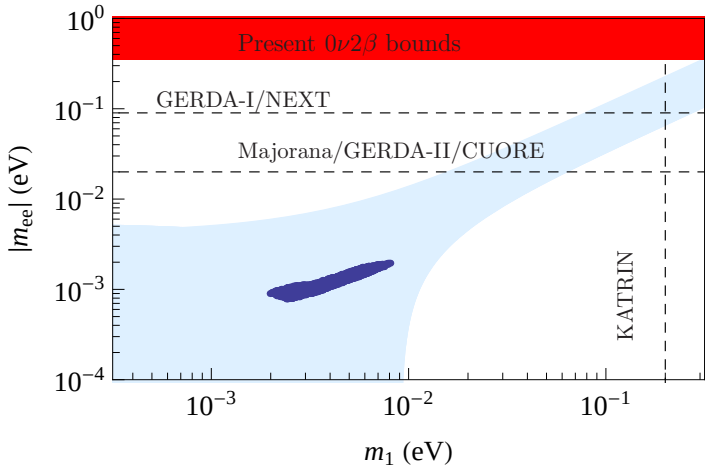

Figure 2: *Model predictions for $|m_{ee}|$ as a function of the lightest neutrino mass m_1. We also show the allowed regions for the normal hierarchy (gray band). The two dashed horizontal lines represent the experimental sensitivity of some of the forthcoming experiments while the dashed vertical line is the upper limit for the sum of the absolute neutrino masses from cosmological data. For references to experiments see [15–19].*

S_3 doublet representation is a viable minimum of the potential. With such a minimum, we obtain a two-zero Fritzsch-texture for the neutrino mass matrix and a nearly diagonal and hierarchical charged lepton mass matrix. As a consequence of the two zeros of the Fritzsch texture, we get a strong correlation between the reactor angle θ_{13} and the Dirac CP phase δ. In particular, for $\delta \sim \pm\pi/2$ we predict $\sin^2\theta_{13} \approx 0.01$, a value which is very close to the best fit value quoted in [1]. Beside the reactor angle, we also investigated the prediction for the effective mass m_{ee} governing the rate of the $0\nu\beta\beta$ decay, founding $m_{ee} \approx 10^{-3}\, eV$, one order of magnitude less than the sensitivities of the future experiments.

8 Acknowledgments

Work of SM and EP supported by the EC contract UNILHC PITN-GA-2009-237920, by the Spanish grants FPA2008-00319 and CDS2009-00064 (MICINN) and PROMETEO/2009/091 (Generalitat Valenciana) and by European Commission Contracts MRTN-CT-2004-503369 and ILIAS/N6 RII3-CT-2004-506222. D.M. was supported by the Deutsche Forschungsgemeinschaft, contract WI 2639/2-1. D.M. also acknowledges the AHEP Group of Valencia

for their hospitality during the earlier stages of this work.

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