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# Phase structure of strongly interacting matter and simulations of heavy-ion collisions using a quark-hadron model

S. Schramm<sup>1,2,3</sup>, J. Steinheimer<sup>2</sup>, V. Dexheimer<sup>1,2,4</sup>, H. Stöcker<sup>1,2,5</sup>

<sup>1</sup>FIAS, Johann Wolfgang Goethe University, Frankfurt am Main, Germany

<sup>2</sup>Institut für Theoretische Physik, Johann Wolfgang Goethe University, Frankfurt am Main, Germany

<sup>3</sup>CSC, Johann Wolfgang Goethe University, Frankfurt am Main, Germany

<sup>4</sup>Department of Physics, Gettysburg College, 300 North Washington Street  
Gettysburg, PA 17325, USA

<sup>5</sup>Gesellschaft für Schwerionenforschung, Darmstadt, Germany

**Abstract.** We consider the phase structure of hadronic and hadron-quark models at finite temperature and density. The basis for the hadronic part is an extension of a flavor-SU(3)  $\sigma - \omega$  model. We study the effect on the phase diagram by adding additional hadronic resonances to the model. With the resulting equation of state we investigate heavy-ion collisions using hydrodynamical simulations. In a combined approach we include quarks and the Polyakov loop field in the calculation and study chiral symmetry restoration and the deconfinement transition.

## 1. The Model Description

A key topic in the study of relativistic heavy-ion collisions is the investigation of the phase structure of highly excited strongly-interacting hadronic and quark matter. This includes the phase transition to a chirally symmetric and deconfined phase at high temperatures and/or densities. From the experimental side one approach to study the transition behavior is to use different beam energies to sample the transition region over a range of excitation energies and densities.

At zero chemical potential QCD lattice simulations show a smooth cross-over of the system to a phase with deconfinement and chiral symmetry restoration. Some lattice calculations at finite chemical potential suggest the existence of a critical endpoint of a line of first-order phase transition at  $T \approx 160$  MeV and  $\mu_q \approx 120$  MeV [1]. In the following we investigate the consequences of phase structures of that type by considering chiral hadronic and combined hadron-quark models. The model description of the hadronic system contains the lowest baryonic and mesonic SU(3) multiplets. The interaction of the scalar mesons and baryons generates the vacuum masses of the

baryons. The general interaction of baryons and the scalar (BM) and vector mesons (BV) reads (assuming static and isospin-symmetric systems)

$$\begin{aligned} \mathcal{L}_{\text{BM}} &= - \sum_i \bar{\psi}_i (g_{i\sigma}\sigma + g_{i\zeta}\zeta) \psi_i \\ , \quad \mathcal{L}_{\text{BV}} &= - \sum_i \bar{\psi}_i (g_{i\omega}\gamma_0\omega^0 + g_{i\phi}\gamma_0\phi^0) \psi_i . \end{aligned} \quad (1)$$

$\sigma$  is the scalar non-strange meson where as  $\zeta$  denotes the scalar ( $\bar{s}s$ ) state, whereas  $\omega$  and  $\phi$  are the corresponding vector mesons. The sums extend over the baryon octet states. Additional terms in the Lagrangian include mass terms and quartic self-interactions of the vector mesons whereas the self-interactions of the scalar meson multiplet induce the spontaneous breaking of chiral symmetry. The effect of non-zero current quark masses are included by introducing an explicit chiral-symmetry breaking term (for a detailed discussion of the Lagrangian see [2]). As a consequence of the coupling terms the effective masses of the baryons are generated through their couplings to the chiral condensates:

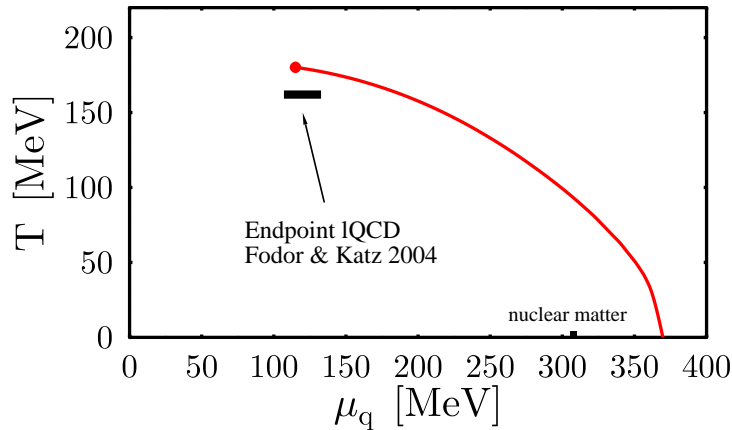
$$m_i^*(\sigma, \zeta) = g_{i\sigma}\sigma + g_{i\zeta}\zeta + \delta m_i, . \quad (2)$$

with a small explicit mass term  $\delta m_i$ . All parameters of the model are fixed by symmetry relations, hadronic vacuum and nuclear matter saturation properties ([2]). It has been shown that by fine-tuning parameters the model provides a good quantitative description of the properties of finite nuclei and the structure of neutron stars [2, 3].

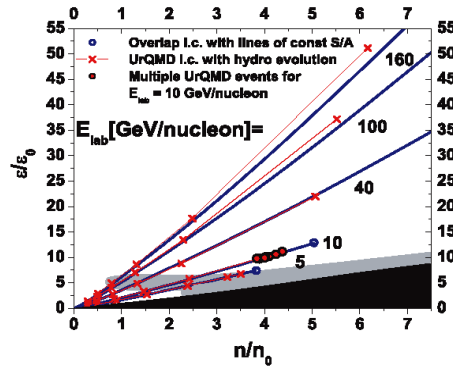
## 2. Phase Diagram in the Hadronic Model

Restricting the baryonic degrees of freedom to the lowest lying octet, the model exhibits cross-over phase transitions towards the chirally restored phase for high temperatures as well as for high densities [2]. Including higher resonances this behaviour can change. In order to avoid introducing many new unknown coupling constants for the baryon-meson interactions for higher multiplets we model the influence of such heavy baryonic states by adding a single resonance with mass  $m_R = m_0 + g_R\sigma$  and vector coupling  $g_{R\omega} = r_V g_{N\omega}$  and a degeneracy factor. The mass parameters,  $m_0$ ,  $g_R$ , the relative vector coupling  $r_V$  and the degeneracy represents free parameters, adjusted to reproduce a phase diagram with a critical end-point as suggested by lattice simulations. An extended discussion of the procedure can be found in [4, 5].

By maximizing the pressure one obtains self-consistent gap equations for the meson fields, which are solved numerically. As can be seen in Fig. 1, with appropriate choice of parameters it is possible to generate a phase diagram with a line of first-order phase transitions ending in a critical end-point at  $T \approx 180$  MeV and  $\mu_q \approx 110$  MeV, close to the lattice values [1]. We use the resulting equation of state in a hydrodynamical simulation of central Pb+Pb heavy-ion collisions. The evolution of the system in the energy density and density plane is shown in Fig. 5. The shaded area shows the region of the mixed phase where the system passes through a first order phase transition. The



**Figure 1.** Line of first order phase transition in the  $T$ - $\mu$  diagram.



**Figure 2.** Evolution of the averaged energy density and density of a central Pb-Pb collision system in a hydrodynamic simulation. The initial conditions were generated by a UrQMD simulation (thin lines) and by an overlap model (thick lines). The shaded area marks the region of the phase transition. The region marked in black is physically not accessible.

figure suggests that the critical end-point is reached and sampled for beam energies around 160 GeV per nucleon (see [7]).

### 3. Including Quarks

In order to represent the correct degrees of freedom at high temperatures and densities we include quark fields into our model description following the general approach outlined in [10, 11]. In the following we present results for vanishing chemical potential [6].

The quarks couple to the mean fields of the model. As order parameter for the deconfinement transition we introduce an effective Polyakov loop field  $\Phi$  and its

conjugate  $\Phi^*$  with a potential for the field reading [11]:

$$U = -\frac{1}{2}a(T)\Phi\Phi^* + b(T)\ln[1 - 6\Phi\Phi^* + 4(\Phi^3\Phi^{*3}) - 3(\Phi\Phi^*)^2] \quad (3)$$

where  $a(T) = a_0T^4 + a_1T_0T^3 + a_2T_0^2T^2$ ,  $b(T) = b_3T_0^3T$ . The parameters are fitted to lattice QCD results at zero chemical potential.

The Polyakov loop enters the partition function for the quarks in the following way:

$$\Omega_q = -T \sum_{i \in Q} \frac{\gamma_i}{(2\pi)^3} \int d^3k \ln \left( 1 + \Phi \exp \frac{E_i^* - \mu_i}{T} \right) \quad (4)$$

and

$$\Omega_{\bar{q}} = -T \sum_{i \in Q} \frac{\gamma_i}{(2\pi)^3} \int d^3k \ln \left( 1 + \Phi^* \exp \frac{E_i^* + \mu_i}{T} \right) \quad (5)$$

suppressing quark contributions in the confined phase at  $\Phi = 0$ . In order to ensure that hadronic degrees of freedom vanish at high temperatures we introduce a finite volume correction  $v$  for the hadrons ( $v = 1 \text{ fm}^3$ ) extending the approach discussed in [12]:

$$v_{Quark} = 0, \quad v_{Baryon} = v, \quad v_{Meson} = v/8 \quad (6)$$

assuming, for mesons, for simplicity, a radius with a value of half the effective radius of the baryons. At this stage we do not include more subtle effects in the description of the excluded volume effects like possible Lorentz contractions [9] or temperature-dependent values of the volume [13]. An alternative approach to suppress hadrons at high temperatures and densities by generating a mass shift for the hadronic particles at non-zero values of  $\Phi$  is discussed in ref. [8].

Using a modified chemical potential  $\tilde{\mu}_i$  for particle  $i$

$$\tilde{\mu}_i = \mu_i - v_i P \quad (7)$$

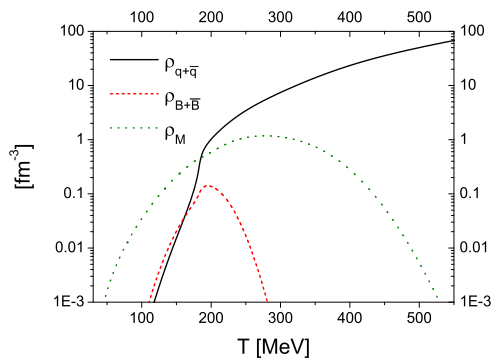
where  $P$  is the sum over all partial pressures, and by correcting all energy, particle, and entropy densities ( $\tilde{e}_i$ ,  $\tilde{\rho}_i$  and  $\tilde{s}_i$ ) by the volume exclusion factor:

$$f = \frac{V'}{V} = \left( 1 + \sum_i v_i \rho_i \right)^{-1} \quad (8)$$

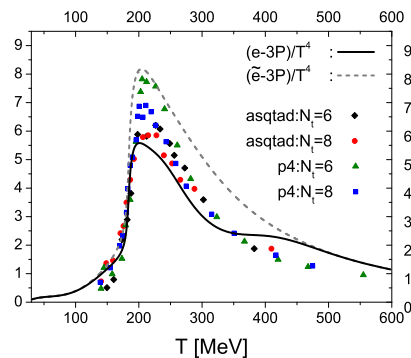
one obtains a thermodynamically consistent set of equations.

Extremizing the grand canonical potential in mean-field approximation we determine the fields, densities, and thermodynamical quantities as function of temperature [6]. Fig. 3 shows the resulting hadronic and quark densities as function of temperature. The critical temperature in the model is  $T_c = 183 \text{ MeV}$ , defined as maximum in the change of the scalar condensate during the cross-over phase transition. The results show contributions from mesons beyond  $T_c$ . The relative densities of quarks and hadrons, however, drop relatively fast with temperature.

Looking at the interaction measure  $\epsilon - 3p$  in Fig. 4 we see reasonably good agreement with lattice calculations. The wiggle at  $T \approx 300 \text{ MeV}$  is generated by the vanishing of the excluded volume effect of the remaining mesons. The strength and position of this effect depends rather strongly on the parameters used for the excluded volume



**Figure 3.** Particle number densities for different particle species as function of  $T$ . The solid line shows the total number density of quarks and antiquarks. The dotted line refers to the total meson density and the dashed line to the number density of baryons and antibaryons.

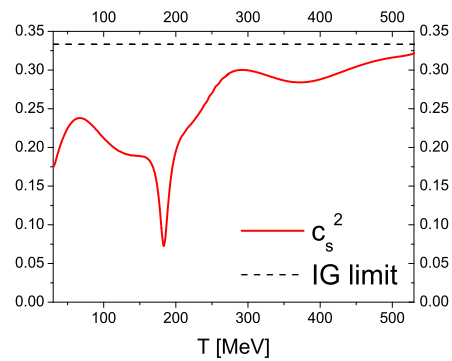


**Figure 4.** Interaction measure  $(\epsilon - 3p)/T^4$  as a function of  $T$  at vanishing chemical potential. The dashed line depicts the interaction measure using the energy density  $\tilde{\epsilon}$  without volume correction factor for comparison. The lattice data (symbols) for different lattice actions and lattice sizes in temporal direction are taken from [15].

calculation. Note, however, that currently lattice gauge simulations are most likely not accurate enough to realistically include possible hadronic interaction effects at high temperatures.

The effect of the phase transition on the expansion dynamics of the hot system created in a heavy-ion collision can be inferred from the speed of sound. Fig. 5 shows its behavior with rising temperature. One can observe a significant dip while passing through the cross-over phase transition. At very high temperatures the value approaches the ideal gas limit  $c_s^2 = 1/3$ . However, one should keep in mind that a realistic calculation of the heavy-ion dynamics will include an averaging over a range of temperatures and densities contained in the hot system, washing out at least part of the signal related to the dip in the speed of sound.

Microscopic calculations of the expansion in a hydrodynamical approach and fur-



**Figure 5.** Squared value of the speed of sound as function of temperature.

ther studies of the model at finite baryon density are currently in progress and will be reported on in the near future.

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