



**HAL**  
open science

# Parity-violating elastic electron scattering and nuclear structure

O Moreno, E Moya de Guerra, P Sarriguren, J M Udas

► **To cite this version:**

O Moreno, E Moya de Guerra, P Sarriguren, J M Udas. Parity-violating elastic electron scattering and nuclear structure. *Journal of Physics G: Nuclear and Particle Physics*, 2010, 37 (6), pp.64019. 10.1088/0954-3899/37/6/064019 . hal-00600804

**HAL Id: hal-00600804**

**<https://hal.science/hal-00600804>**

Submitted on 16 Jun 2011

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Parity violating elastic electron scattering and nuclear structure

O. Moreno<sup>1</sup>, E. Moya de Guerra<sup>1</sup>, P. Sarriguren<sup>2</sup>, J. M. Udías<sup>1</sup>

<sup>1</sup> Departamento de Física Atómica, Molecular y Nuclear, Universidad Complutense de Madrid, E-28040 Madrid, Spain

<sup>2</sup> Instituto de Estructura de la Materia, CSIC, Serrano 123, E-28006 Madrid, Spain

## **Abstract.**

We discuss parity violating elastic electron scattering as a complementary tool in the race for more precise determinations of neutron densities in nuclei. Isovector and isoscalar densities and form factors in  $N > Z$  and  $N = Z$  stable nuclei are discussed taking  $^{208}\text{Pb}$  and  $^{28}\text{Si}$  as examples. Distorted wave calculations of parity violating asymmetries are shown and are compared to plane wave impulse approximation. The extraction of the ratio between neutron and proton monopole form factors is discussed. The isospin mixing produced by Coulomb interaction in the ground state of  $N = Z$  nuclei with Skyrme selfconsistent mean fields is also discussed.

PACS numbers: 25.30.Bf, 21.10.Gv, 21.60.Jz

## 1. Introduction

Knowledge of the neutron density distributions in atomic nuclei is of fundamental importance to understand basic aspects of the nuclear structure, but contrary to what happens with proton distributions, the experimental information available on neutrons is still clearly insufficient. The neutron distributions, and in particular the neutron radii of heavy nuclei are essential nuclear-structure observables that still remain elusive. Improving our knowledge of the neutron density distributions in nuclei is one of the strongest challenges in present nuclear structure activities.

An accurate measurement of the neutron densities in heavy nuclei would have significant implications for nuclear structure. First of all, one should bear in mind that because of the uncertainties in the neutron density, modern effective nuclear forces are typically constructed without any constraint on them. However, a correct parametrization of the isovector channel of the effective nuclear force is essential for the description of phenomena in exotic neutron-rich nuclei such as halos [1] or neutron skins [2]. Once these interactions are constrained to reproduce a neutron radius in a stable nucleus such as  $^{208}\text{Pb}$ , they can make improved predictions for a variety of unstable nuclei. The isospin dependence of the energy functional in nuclear matter would be also constrained by this information. In particular, it would lead to an improved neutron equation of state with important consequences in astrophysics [3] (including the structure of neutron stars). Precise neutron density distributions in nuclei are also required to make progress in atomic parity non-conservation (PNC) experiments [4]. As the accuracy of these experiments improves, they will need more and more precise information on neutron densities because the parity violating interaction is basically proportional to the overlap between the electrons in the atomic orbits and the neutrons [5].

Electron-nucleus scattering has been in the past an excellent tool for studying the nuclear structure. Much reliable information on electromagnetic form factors and charge density distributions has been accumulated for stable nuclei [6] and it is expected that the new facilities in GSI [7] and RIKEN [8] will provide a good opportunity to extend the study of the charge density to unstable nuclei as well. Unfortunately, a measurement of the neutron density distribution to a precision and detail comparable to that of the proton one is hardly possible. Progress on our knowledge of the neutron densities has been limited by the use of hadronic probes that are subject to large and controversial uncertainties because of the not well known reaction mechanism. Neutron density measurements are usually performed using probes having different sensitivities to protons and neutrons. The methods used include hadron scattering, antiprotonic atoms, as well as excitations of the giant-dipole resonance (GDR) and the spin-dipole resonance (SDR). The latter have been especially used to determine neutron skin radii.

Neutron radii were extracted firstly from Coulomb energy differences [9] and from neutron pickup reactions ( $p, d$ ) and ( $d, t$ ) [10], but since these reactions are mainly sensitive to the tail of the neutron density, model assumptions were needed for the

interior density. Hadron scattering and in particular proton-nucleus elastic scattering at intermediate energies [11], is a well-established method for probing nuclear matter density distributions of stable nuclei. Although the reaction is surface-dominated, it is sensitive to both the surface and the interior neutron density. This reaction produces large cross sections, but suffers from the complexity of the strong interaction. Therefore, it is difficult to describe the reaction mechanism and to perform a model-independent analysis of the data. In general, present hadron scattering interpretation involves a strong model dependence of the results. Difference between neutron and proton density distributions at large radial distances, on the nuclear surface, has been determined with antiprotonic atoms [12], where a slow antiproton is captured into an atom. These differences have also been determined by exciting the GDR [13]. This has been done in inelastic alpha scattering to the GDR, where the cross section of the process depends strongly on the ratio  $(R_n - R_p)/R$ . Unfortunately, the cross section of the GDR excitation is very small relative to those of other overlapping resonances. Similarly, the cross sections of the isovector SDR has been used to extract the neutron skin thickness [14]. This resonance is excited in  $(p, n)$ , or  $({}^3\text{He}, t)$  charge exchange reactions using inverse kinematics.

To summarize, there have been many measurements of neutron densities with strongly interacting probes. Unfortunately, the measured neutron densities are still model-dependent and the error bars are much larger than those of proton densities due to the uncertainties inherent to the strong interaction. As a result, no existing measurement of neutron densities or radii has an established accuracy of 1%. As an example, in  ${}^{208}\text{Pb}$ , while electron scattering experiments have determined the charge radius to better than 0.001 fm, realistic estimates place the uncertainty in the neutron radius at about 0.2 fm.

This situation has led to a further consideration of leptonic probes to investigate neutron distributions. Elastic magnetic electron scattering from odd-N nuclei is sensitive to the neutron magnetic moment and information about the odd neutron density can be extracted. However, information about the whole neutron density cannot be directly measured since most of the neutrons in a heavy nucleus are coupled to spin zero and make no contribution to the magnetization. An exciting new possibility could be the direct measurements of the neutron density form factors from the asymmetry in parity-violating (PV) elastic polarized electron scattering [15]. The electroweak experiments can be both accurate and model independent and the data can be interpreted with as much confidence as electromagnetic (EM) scattering. PV electron-nucleus scattering arises from the interference of EM and weak neutral amplitudes and it is a clean and powerful tool for measuring the spatial distribution of neutrons in nuclei with unprecedented accuracy.

## 2. Parity violating electron scattering

### 2.1. Introduction

The first motivation for studying PV electron scattering from nuclei was to use it as a tool to extract information on the weak neutral current (WNC) and thereby to test the validity of the Standard Model (SM) in the low-energy regime [16, 17]. This possibility lies in the fact that the PV asymmetry acquires a very simple, model-independent expression in terms of basic coupling constants because nuclear structure effects cancel out when certain conditions are met. Later on, the emphasis shifted to test nucleon and nuclear structure with such probes. The first experiments in this line were aimed to probe strange quark contributions to the form factors of the nucleon. The SAMPLE experiment [18] measured the strange magnetic form factor at low momentum transfer. The HAPPEX experiment made the most precise measurement yet of strange quarks in the proton [19]. The results from these experiments suggest that strange quarks do not make large contributions to the nucleon electric or magnetic form factors. The contributions from strange quarks are less than 1 percent of the proton charge distribution and no more than 4 percent of its magnetic moment. They are actually compatible with zero [20].

The present interest on PV electron scattering is focused on one hand on the study of the isospin mixing in nuclei. This is so because understanding the sensitivity of PV electron scattering to the nuclear isospin mixing is crucial to determine the precision up to which the SM constants can be deduced and to what extent strangeness effects in the WNC can be studied. On the other hand, the focus is made on the study of neutron densities. This is because the  $Z^0$  couples predominantly to neutrons. The coupling of the  $Z^0$  with the proton depends on the small factor  $(1 - 4 \sin^2 \theta_W)$ , while the coupling with the neutron is more than ten times larger. This situation is opposite to the EM charge coupling where electrons couple to protons and neutrons with strengths 1 and 0 respectively. This property of the weak interaction can be exploited to provide information about the spatial distribution of neutrons in the nuclear ground state. Indeed, this idea was proposed as part of the original study made in Ref. [15], namely that a measurement of the PV-asymmetry in elastic electron-scattering can provide a direct measurement of the Fourier Transform of the neutron density. In fact, this measurement is now being carried out in the Parity Radius Experiment (PREX) [21] at Jefferson Laboratory which has the goal of measuring the neutron radius of  $^{208}\text{Pb}$  to a 1% precision, using PV elastic electron scattering. This is an electroweak alternative to the hadronic program and promises to measure the neutron radius of  $^{208}\text{Pb}$  accurately and model independently. Another advantage of the electroweak program is related to the possibility of calibrating proton-nucleus scattering to reproduce the neutron density in a stable nucleus. This is analogous to use beta decay to calibrate the charge-exchange reactions as a tool to extract the Gamow-Teller strength. Then, proton scattering could be used in a wide variety of other nuclei, including radioactive beams with hydrogen targets in inverse kinematics. In fact, the nuclear matter distribution in  $^6\text{He}$  and  $^8\text{He}$

has been already determined at GSI by using small angle proton scattering in inverse kinematics [22].

It has also been proposed [23] that PV electron scattering can be used to determine the type (skin or halo) of neutron distribution in neutron-rich stable nuclei. In particular, asymmetries for skin-type neutron distributions are larger than those of halo-type neutron distributions.

## 2.2. Formalism

Polarized electron scattering from unpolarized nuclei can be used to study parity violation, since both electromagnetic and weak interactions contribute to the process via  $\gamma$  and  $Z^0$  exchange, respectively. The PV asymmetry is given by [15]

$$\mathcal{A} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}, \quad (1)$$

where  $d\sigma^+(d\sigma^-)$  is the cross section for electrons longitudinally polarized parallel (antiparallel) to their momentum and the asymmetry is proportional to the interference between the  $\gamma$  and  $Z^0$  amplitude. For a  $J^+ = 0^+$  target in Plane Wave Born Approximation (PWBA) the asymmetry  $\mathcal{A}$  can be written as

$$\mathcal{A} = a \frac{F_W(q)}{F_{ch}(q)}, \quad (2)$$

with  $a$  a linear function of  $Q^2$  proportional to the ratio between neutron and proton numbers:

$$a = \frac{G_F Q^2 N}{4\pi\alpha\sqrt{2} Z}, \quad (3)$$

where  $G_F$  and  $\alpha$  are the Fermi and fine-structure coupling constants, respectively, and  $Q^2 = -q_\mu^2 = 4\epsilon^2 \sin^2(\theta_e/2)$  is the four-momentum transfer in the scattering process.  $F_W(q)$  and  $F_{ch}(q)$  are the elastic weak and EM charge form factors, respectively, containing the dependence on the nuclear structure. These form factors are monopole Coulomb-type and are defined as follows,

$$F_{ch}(q) = G_{E_p}(q)F_p^0 + \frac{N}{Z}G_{E_n}(q)F_n^0, \quad (4)$$

and

$$F_W(q) = G_{E_p}(q) \left[ F_n^0 - \frac{Z}{N}(1 - 4\sin^2\theta_W)F_p^0 \right] - G_{E_n}(q) \left[ F_n^0(1 - 4\sin^2\theta_W) - \frac{Z}{N}F_p^0 \right], \quad (5)$$

where  $G_{E_p}$  and  $G_{E_n}$  are the charge form factors of the proton and the neutron ( $G_{E_p}(q=0) = 1$  and  $G_{E_n}(q=0) = 0$ ) and  $\theta_W$  is the Weinberg angle. The monopole form factors for point nucleons are

$$F_p^0(q) = \frac{1}{Z} \int d^3r j_0(qr) \rho_p(r), \quad F_n^0(q) = \frac{1}{N} \int d^3r j_0(qr) \rho_n(r). \quad (6)$$

In these expressions the neutron and proton densities are normalized to the numbers of their corresponding type of nucleons and  $F_{p,n}^0(q=0) = 1$ . Therefore, we see that the asymmetry, aside from  $\mathcal{A}$ , depends mainly on the ratio of neutron to proton nuclear form factor ( $F_n^0(q)/F_p^0(q)$ ):

$$\mathcal{A} = \mathbf{a} \left[ \frac{\frac{F_n^0(q)}{F_p^0(q)} - \frac{Z}{N}(1 - 4 \sin^2 \theta_W) - \frac{G_{E_n}(q)}{G_{E_p}(q)} \left[ \frac{F_n^0(q)}{F_p^0(q)} (1 - 4 \sin^2 \theta_W) - \frac{Z}{N} \right]}{1 + \frac{N}{Z} \frac{G_{E_n}(q)}{G_{E_p}(q)} \frac{F_n^0(q)}{F_p^0(q)}} \right]. \quad (7)$$

At the  $q$ -values that are of our concern here, the charge form factor of the neutron ( $G_{E_n}$ ) can be neglected and we can write

$$\mathcal{A} = \mathbf{a} \left[ \frac{F_n^0(q)}{F_p^0(q)} - \frac{Z}{N}(1 - 4 \sin^2 \theta_W) \right]. \quad (8)$$

In a first approximation the radial distribution of mass can be assumed to follow that of charge so that the radial dependence of the neutron density is that of the proton density and they are both scaled by the  $N/Z$  factor ( $\rho_n(r) = \rho_p(r) N/Z$ ). In this case  $F_n^0 = F_p^0$  for all  $q$  and one gets:

$$\mathcal{A}^0 = \mathbf{a} \left[ 1 - \frac{Z}{N}(1 - 4 \sin^2 \theta_W) \right], \quad (9)$$

or

$$\mathcal{A}^0 \approx \mathbf{a} \quad \text{for } Z/N < 1. \quad (10)$$

For  $N = Z$  nuclei Eq. (9) leads to the usual expression:

$$\mathcal{A}_{N=Z}^0 = \mathbf{a} \, 4 \sin^2 \theta_W, \quad (11)$$

as obtained in the exact SU(2) isospin limit in Refs. [15, 24]. In nuclei, isospin is not an exact symmetry, but still at  $q = 0$ ,  $F_n^0 = F_p^0$ . For  $q > 0$  the assumption  $F_n^0 = F_p^0$  amounts to assume that the nuclear ground state is a good zero-isospin eigenstate. Then only isoscalar matrix elements contribute and the WNC and EM form factors become proportional. Since isospin is not an exact symmetry in nuclei the actual PV asymmetry deviates from the linear  $Q^2$  dependence by a correction  $\Gamma$ , where

$$\mathcal{A} = \mathcal{A}^0 [1 + \Gamma(q)]. \quad (12)$$

The deviation  $\Gamma$  accounts not only for the effects of nuclear isospin mixing but also for the strangeness content in the PV asymmetry,  $\Gamma = \Gamma_I + \Gamma_s$  [24]. The last measurements [20] give a strangeness density  $\rho_s$  consistent with zero at the  $q$  values of interest here, and therefore we will discuss only  $\Gamma = \Gamma_I$  in the next section.

Since  $F_p^0$  is well known from standard (parity conserving) electron scattering, a precise information on  $F_n^0$  (hence on  $\rho_n$ ) can be obtained by accurate measurements of  $\mathcal{A}$  at different  $q$  values. In the region  $0 < q \lesssim 5R^{-1}$ , that requires  $q$  values of the order of  $A^{-1/3}$  GeV at most, the elastic cross section is sufficiently large to measure  $\mathcal{A}$  accurately [24].

We note that in general a direct measurement of the  $q$  dependence of  $F_n^0/F_p^0$  is obtained from the difference between the exact asymmetry  $\mathcal{A}$  and the approximated one  $\mathcal{A}^0$  as:

$$\frac{F_n^0(q)}{F_p^0(q)} = 1 + \frac{[\mathcal{A} - \mathcal{A}^0]}{a}. \quad (13)$$

This equation provides the means to obtain the experimental  $q$ -dependent ratio  $(F_n^0(q)/F_p^0(q))_{\text{exp}}$  from the experimental asymmetry values,  $\mathcal{A} = \mathcal{A}_{\text{exp}}$ , substituting  $\mathcal{A}^0$  and  $a$  by the known linear functions of  $Q^2$  in Eq. (3) and Eq. (9).

As already mentioned there are several corrections to the asymmetry expressed by Eq. (8) that could interfere the neutron radius measurement. The corrections have been studied in Ref. [25], concluding that they should not be an issue for the interpretation of the experiment because they are either negligible or well under control. These corrections include effects from strange quarks, neutron electric form factors, parity admixtures in the ground state, dispersion corrections, meson exchange currents, radiative corrections, contributions from excited states, and target impurities. By far the largest correction to the asymmetry in heavy nuclei comes from Coulomb distortions of the electron wave function. Distortion effects can be included exactly [24, 25, 26] by solving numerically the Dirac equation for an electron moving in the Coulomb potential (vector) plus the small weak neutral potential (axial-vector). The axial-vector potential is given by

$$W_W(r) = \frac{\alpha}{\sin^2(2\theta_W)} \frac{2\pi}{m_Z} \int_0^\infty \rho_W(r') \frac{r'}{r} \left( e^{-m_Z|r-r'|} - e^{-m_Z(r+r')} \right) dr' \quad (14)$$

where  $m_Z$  is the mass of the  $Z^0$  boson and  $\rho_W(r)$  is the weak density (inverse Fourier transform of the weak form factor of Eq. (5)). For massless electrons, one has positive and negative helicity states that scatter from a  $V + A$  and  $V - A$  potentials. Thus, one obtains the cross sections  $d\sigma^+$  ( $d\sigma^-$ ) entering in Eq. (1) from the phase shifts which result from the numerical solution of the Dirac equation, involving a partial wave expansion. For heavy nuclei, distortion corrections can be large but they can be calculated with an accuracy significantly better than the 3% experimental error expected for the asymmetry. When Coulomb distortion effects are taken into account we can still define a distorted wave (DW) neutron to proton form factor ratio,

$$\left( \frac{F_n^0(q)}{F_p^0(q)} \right)_{DW} = 1 + \frac{[\mathcal{A}^{DW} - \mathcal{A}^0]}{a} \quad (15)$$

which is the actual ratio to be compared to experiment.

In what follows we show results of the asymmetries calculated from densities obtained within selfconsistent mean field approximation, with and without Coulomb distortion effects.

### 2.3. Selfconsistent mean field approach for quasiparticles

The nuclear structure calculation involved in the asymmetry, which basically concerns the proton and neutron form factors, is performed within a selfconsistent deformed mean

field approach with pairing correlations. To generate the ground-state wave function we use a Skyrme density-dependent nucleon-nucleon interaction (SLy4 [27]). The Hartree-Fock (HF) equations are solved iteratively and at the end of each iteration we solve the BCS equations to generate the occupation probabilities in a consistent way. We obtain the single-particle levels, occupation numbers, and wave functions. The latter are expanded in a harmonic oscillator basis using 12 major shells. More details can be found in Refs. [24, 28, 29].

In this approach the collective isospin mixing effect of the Coulomb force is included non-perturbatively in the isospin-non-conserving HF mean field, along with other collective effects such as pairing and deformation. As a result, the HF+BCS ground state is made up of quasiparticles with rather complex admixtures of harmonic oscillator wave functions in many different major shells. So far, this approximation is known to provide optimal descriptions of charge form factors [30, 31] of heavy nuclei and can be expected to be reliable for the theoretical description of the asymmetry.

### 3. Calculation of asymmetries

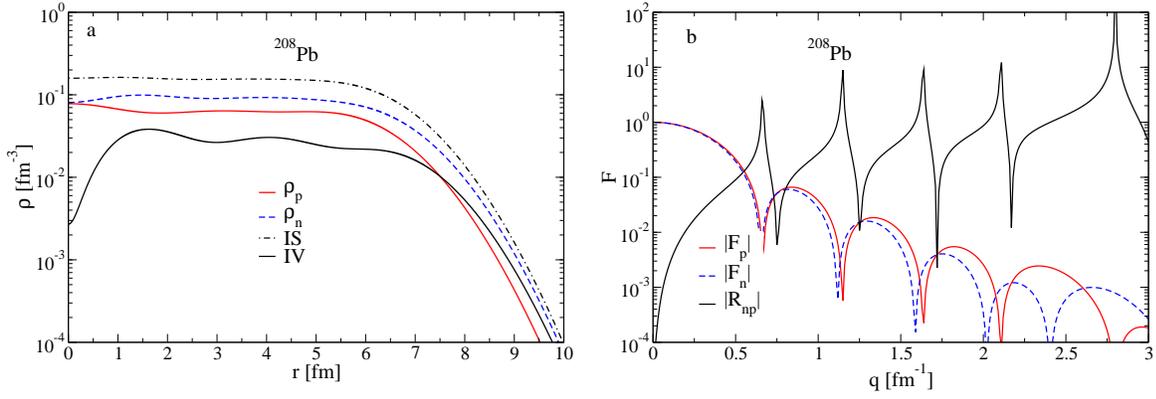
As a paradigmatic example of  $N > Z$  nuclei we show results for  $^{208}\text{Pb}$ . In Fig. 1a we show the proton  $\rho_p$  and neutron  $\rho_n$  ground state densities of  $^{208}\text{Pb}$  from a HF(SLy4) calculation. Also shown are the isoscalar (IS) ( $\rho_{IS} = \rho_p + \rho_n$ ) and isovector (IV) ( $\rho_{IV} = \rho_n - \rho_p$ ) densities.

In Fig. 1b we show the corresponding proton  $F_p^0$  and neutron  $F_n^0$  form factors in PWBA (see Eqs. (6)) together with  $\mathcal{R}_{np}(q)$  defined as:

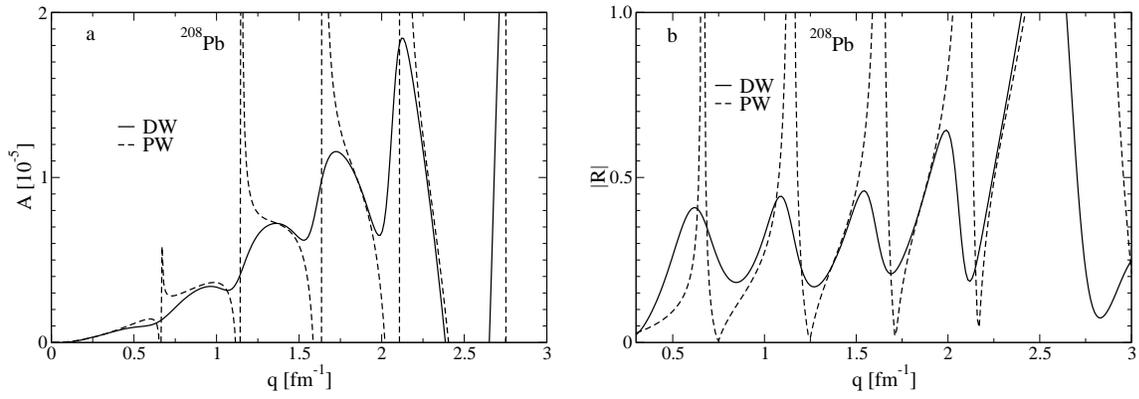
$$\mathcal{R}_{np}(q) = \frac{F_n^0(q)}{F_p^0(q)} - 1. \quad (16)$$

In Fig. 2a we show the PV asymmetry in  $^{208}\text{Pb}$  obtained from both PWBA (dashed line) and DWBA (solid line). The DWBA calculation is performed for an incident electron energy of 850 MeV. It is worth noticing how the PWBA singularities become smooth functions of  $q$  in the distorted case. In this kinematics, the expected asymmetry amounts to  $8.4 \cdot 10^{-7}$  at  $q = 0.45 \text{ fm}^{-1}$ , which is the momentum transfer chosen at PREX [21]. A word of caution should be made with regards to the effective transverse momentum that should be used when comparing DWBA or experimental data to PWBA calculations (see for instance Ref. [32] and references therein). A displacement in  $q$  of the DWBA peaks relative to the PWBA peaks is clearly seen in Fig. 2. The Coulomb distortion is also responsible for the anomalous behaviour of  $\mathcal{R}_{np}$  in DWBA as  $q \rightarrow 0$ . Actually, the isospin mixing effect is best explored when comparing the DWBA asymmetry to that obtained also in DWBA but taking  $\rho_n = \rho_p N/Z$  (see figures 2 and 3 in Ref. [24]).

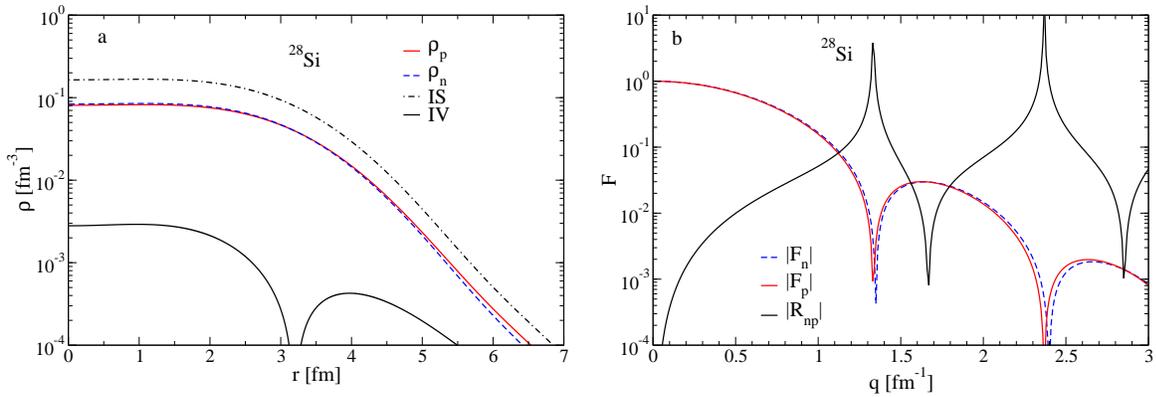
As an example of an  $N = Z$  nucleus we show results for  $^{28}\text{Si}$ . For this nucleus the calculations are done with the same Skyrme interaction using constant pairing gaps  $\Delta_{p,n} = 1 \text{ MeV}$ . Figs. 3 and 4 contain similar results as Figs. 1 and 2, but for  $^{28}\text{Si}$ .



**Figure 1.** Left panel: proton, neutron, IS and IV densities of  $^{208}\text{Pb}$  in its ground state. Right panel: proton and neutron form factors together with  $\mathcal{R}_{np}$ , in PWBA.

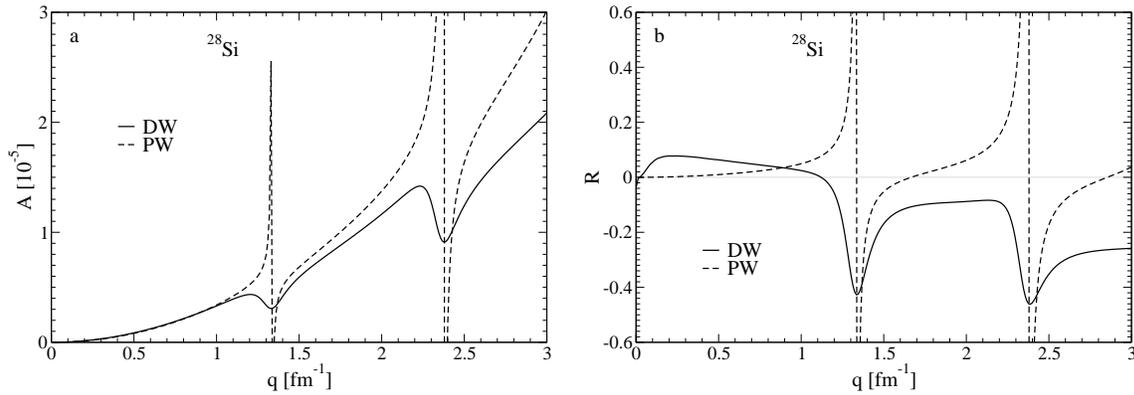


**Figure 2.** Left panel: Plane wave (PW) and distorted wave (DW) PV asymmetries of  $^{208}\text{Pb}$  in its ground state. Right panel: for the same nucleus, the quantity  $\mathcal{R}_{np}$  in plane wave (PW) and distorted wave (DW).



**Figure 3.** Same as in Fig. 1, but for the  $N = Z$  nucleus  $^{28}\text{Si}$ .

The effects expected on the elastic parity-violating electron scattering at momentum transfers extending up to about  $1.5 \text{ fm}^{-1}$  from the isospin-mixing in the nuclear ground-state wave functions of a set of  $N = Z$  nuclei, including  $^{28}\text{Si}$ , have been studied in



**Figure 4.** Same as in Fig. 2, but for the  $N = Z$  nucleus  $^{28}\text{Si}$ .

Ref. [24]. The small differences between the proton and neutron density distributions yield both isoscalar and isovector ground-state Coulomb monopole matrix elements and produce modifications in the PV asymmetry from the model-independent result obtained in the absence of isospin-mixing.

Kinematic ranges where potential future measurements might be undertaken have been discussed in Ref. [24] by studying both the deviations in the PV asymmetry  $\Gamma$  and the experimental figure-of-merit. Isospin mixing will have a measurable effect on the asymmetries in the range  $0.5 \leq q \leq 1.0 \text{ fm}^{-1}$ .

#### 4. Isospin mixing in $N = Z$ nuclei

As seen in the previous section the PV asymmetry in  $N = Z$  nuclei is affected by the isospin mixing in the ground state. Isospin impurities are discussed in the literature in different schemes. In previous works [33] we evaluated isospin mixing by computing the mean value of the  $T^2$  operator in the ground state. In Refs. [34, 35] isospin mixing is evaluated in terms of the overlaps between spherical neutron and proton single particle wave functions with the same quantum numbers. The admixture of  $T = T_0 + 1$  in the ground state with isospin  $T_0$  is given in terms of those overlaps [34, 35],

$$P(T = T_0 + 1) \approx \frac{1}{T_0 + 1} \sum'_{nlj} N_{nlj} C_{nlj}, \quad (17)$$

where  $N_{nlj}$  is the number of protons in the orbit  $(nlj)$  and  $C_{nlj}$  are the deviations from unity of the overlaps,

$$C_{nlj} = 1 - \int R_{nlj}^p(r) R_{nlj}^n(r) r^2 dr. \quad (18)$$

In a more general framework where deformation and pairing are included, as it is our case, a better formulation of the isospin mixing in the ground state wave function of  $N = Z$  nuclei is given as

$$P_T = 1 - \langle \Psi_p | \Psi_n \rangle, \quad (19)$$

where  $\Psi_{p,n}$  are the proton and neutron wave functions in the selfconsistent HF+BCS ground state. This expression takes into account all the possible overlaps between proton and neutron single particle wave functions, as can be seen in the explicit expressions for the BCS overlaps [36].

In Table 1 we show the isospin mixing probabilities (percentage) for various  $N = Z$  nuclei obtained in this work using Eq. 19. Our HF(SLy4)+BCS calculations for  $^{12}\text{C}$  and  $^{28}\text{Si}$  with  $\Delta_{p,n} = 1$  MeV give spherical ( $^{12}\text{C}$ ) and oblate ( $^{28}\text{Si}$ ,  $\beta = -0.25$ ) solutions. For the doubly closed nuclei  $^{40}\text{Ca}$  and  $^{100}\text{Sn}$  the spherical ground states have zero pairing gaps. Also given in the table are the results obtained by Hamamoto *et al.* [34] (HF(Sk3) and HF(SG2)), by Satula *et al.* [37] before (BR) and after (AR) rediagonalization, as well as the values corresponding to the formulation in Bohr and Mottelson [38], p. 173. In these works all the nuclei are spherical and no pairing correlations are included.

**Table 1.** Isospin mixing probabilities (percentage) for various  $N = Z$  nuclei obtained in this and in other works (see text).

	$^{12}\text{C}$	$^{28}\text{Si}$	$^{40}\text{Ca}$	$^{100}\text{Sn}$
This work SLy4	0.041	0.28	0.58	3.9
Hamamoto <i>et al.</i> Sk3	$\sim 0.05$	-	$\sim 0.6$	$\sim 3.9$
Hamamoto <i>et al.</i> SG2	$\sim 0.05$	-	$\sim 0.6$	$\sim 4.8$
Satula <i>et al.</i> SLy4 BR	-	$\sim 0.35$	$\sim 0.6$	$\sim 3.9$
Satula <i>et al.</i> SLy4 AR	-	$\sim 0.45$	$\sim 0.8$	$\sim 6.2$
Bohr and Mottelson	0.007	0.063	0.165	1.9

Our HF+BCS estimates are in accordance with other HF calculations [34, 35] and are larger than the estimates given by the BM isospin mixing, obtained from a collective model description of the isovector giant monopole resonance.

## 5. Conclusions

The accurate knowledge of the neutron density in the nucleus is a challenge to the present understanding of the nuclear structure and its determination in a model independent way is still an open problem. In this work we have shown that parity violating elastic electron scattering is an attractive alternative to the use of hadronic probes as an instrument to get information on neutron density distributions. The reason for that is that, contrary to the strong force, the electroweak interaction is perfectly known and therefore information on the nuclear structure can be extracted in a clean way. On the other hand, while the standard parity conserving electron scattering is sensitive to the nuclear charge distribution, parity violating electron scattering is mainly sensitive to the neutron distributions. Thus, these experiments will allow to measure a nuclear weak-charge density distribution and finally to determine the neutron distribution with a highly improved accuracy as compared to hadronic probes, which could be calibrated

afterwards to reproduce more precisely that information. Accurate measurements of the PV asymmetries in  $N = Z$  nuclei will also highlight the issue of isospin mixing that still remains an open problem too.

By a direct comparison to theory, these measurements will represent a critical test to the nuclear models and will have a deep impact on nuclear structure, as well as on atomic physics and astrophysics.

## Acknowledgements

This work was supported by Ministerio de Ciencia e Innovación (Spain) under Contract FIS2008-01301. J.M.U. acknowledges support from INTAS Open Call grant No 05-1000008-8272, and Ministerio de Ciencia e Innovación (Spain) under grants FPA-2007-62616 and FPA-2006-07393, and UCM and Comunidad de Madrid under grant Grupo de Física Nuclear (910059).

## References

- [1] Tanihata I *et al* 1985 *Phys. Rev. Lett.* **55** 2676; 1986 *J. Phys. G* **22** 157; 1995 *Prog. Part. Nucl. Phys.* **35** 505
- [2] Suzuki T *et al* 1995 *Phys. Rev. Lett.* **75** 3241
- [3] Horowitz C J 2005 *Eur. Phys. J. A* **24** 167
- [4] Wood C S, Bennett S C, Cho D, Masterson B P, Roberts J L, Tanner C E and Wieman C E 1997 *Science* **275** 1759
- [5] Bennett S C and Wieman C E 1998 *Phys. Rev. Lett.* **82** 2484; Pollock S J and Welliver M C 1999 *Phys. Lett. B* **464** 177
- [6] de Vries H, de Jager C W and de Vries C 1987 *At. Data Nucl. Data Tables* **36** 495
- [7] *Technical Proposal for the Design, Construction, Commissioning, and Operation of the ELISE Setup*, GSI Internal Report, (Dec. 2005).
- [8] Suda T and Wakasugi M 2005 *Prog. Part. Nucl. Phys.* **55** 417
- [9] Nolen J A and Schiffer J P 1969 *Phys. Lett.* **29B** 396
- [10] Körner H J and Schiffer J P 1971 *Phys. Rev. Lett.* **27** 1457
- [11] Ray L *et al* 1979 *Phys. Rev. C* **19** 1855; Hoffmann G W *et al* 1981 *Phys. Rev. Lett.* **47** 1436; Ray L and Hoffmann G W 1985 *Phys. Rev. C* **31** 538
- [12] Trzcinska A *et al* 2001 *Phys. Rev. Lett.* **87** 082501
- [13] Krasznahorkay A *et al* 1994 *Nucl. Phys. A* **567** 521
- [14] Krasznahorkay A *et al* 1999 *Phys. Rev. Lett.* **82** 3216; 2004 *Nucl. Phys. A* **731** 224
- [15] Donnelly T W, Dubach J and Sick I 1989 *Nucl. Phys. A* **503** 589
- [16] Feinberg G 1975 *Phys. Rev. D* **12** 3575; Walecka J D 1977 *Nucl. Phys. A* **258** 349
- [17] Donnelly T W and Peccei R D 1979 *Phys. Rep.* **50** 1; Musolf M J, Donnelly T W, Dubach J, Pollock S J, Kowalski S and Beise E J 1994 *Phys. Rep.* **239** 1
- [18] Mueller B *et al* 1997 *Phys. Rev. Lett.* **78** 3824
- [19] Aniol K A *et al* 1999 *Phys. Rev. Lett.* **82** 1096; 2004 *Phys. Rev. C* **69** 065501; 2006 *Phys. Rev. Lett.* **96** 022003
- [20] Acha A *et al* 2007 *Phys. Rev. Lett.* **98** 032301
- [21] <http://hallaweb.jlab.org/parity/prex/>
- [22] Alkhasov G D *et al* 1997 *Phys. Rev. Lett.* **78** 2313
- [23] Dong T, Chu Y, Ren Z and Wang Z 2009 *Phys. Rev. C* **79** 014317

- [24] Moreno O, Sarriguren P, Moya de Guerra E, Udias J M, Donnelly T W and Sick I 2009 *Nucl. Phys. A* **828** 306
- [25] Horowitz C J 1998 *Phys. Rev. C* **57** 3430; Horowitz C J, Pollock S J, Souder P A and Michaels R 2001 *Phys. Rev. C* **63** 025501
- [26] Rufa G 1982 *Nucl. Phys. A* **384** 273
- [27] Chabanat E, Bonche P, Haensel P, Meyer J and Schaeffer R 1998 *Nucl. Phys. A* **635** 231
- [28] Sarriguren P, Gaidarov M K, Moya de Guerra E and Antonov A N 2007 *Phys. Rev. C* **76** 044322
- [29] Moya de Guerra E, Sarriguren P, Caballero J A, Casas M and Sprung D W L 1991 *Nucl. Phys. A* **529** 68
- [30] Vautherin D and Brink D M 1972 *Phys. Rev. C* **5** 626; Vautherin D 1973 *Phys. Rev. C* **7** 296
- [31] Negele J W 1989 *Adv. Nucl. Phys.* **19** 1
- [32] Nishimura M, Moya de Guerra E and Sprung D W L 1985 *Nucl. Phys. A* **435** 523
- [33] Álvarez-Rodríguez R, Moya de Guerra E and Sarriguren P 2005 *Phys. Rev. C* **71** 044308
- [34] Hamamoto I and Sagawa H 1993 *Phys. Rev. C* **48** R960
- [35] Colò G, Nagarajan M A, Van Isacker P and Vitturi A 1995 *Phys. Rev. C* **52** R1175
- [36] Šimkovic F, Pacearescu L and Faessler A 2004 *Nucl. Phys. A* **733** 321
- [37] Satula W, Dobaczewski J, Nazarewicz W and Rafalski M 2009 *Phys. Rev. Lett.* **103** 012502
- [38] Bohr A, Mottelson B R 1975 *Nuclear Structure*, W.A. Benjamin, New York