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GLOBAL STABILITY ANALYSIS OF FLOW THROUGH AN ANEURYSM

Shyam Sunder Gopalakrishnan, Benoît PIER & Arie BIESHEUVEL
Laboratoire des mécanique des fluides et d'acoustique (CNRS - Université de Lyon).
École centrale de Lyon, 36 avenue Guy-de-Collongue, 69134 Écully, France.

The global linear stability analysis of flow through a model fusiform aneurysm is carried out numerically. A gaussian profile was used to model the wall channels to account for the bulge, that gradually straightens out to a uniform circular pipe in the upstream and downstream direction. The model configuration considered in the study is characterised by the bulge length and the maximum diameter of the bulge. These parameters for our model (*Model A*) were $3.0D$ and $2.0D$ consistent with the one used in [1] and *Model 3* in [2], where D is the inlet pipe diameter. A steady parabolic velocity profile was imposed at the inlet along with a swirl prescribed using a sinuous function ($w_0 \sin(\pi r/R)$ where w_0 is a constant parameter used to quantify the inlet swirl and R is the inlet pipe radius) for the azimuthal flow component. The global stability analyses were carried out on base flows that were axisymmetric on which non-axisymmetric perturbations were allowed to grow. For *Model A* in the absence of inlet swirl ($w_0 = 0$), the flow was found to be weakly unstable to global eigenmodes of azimuthal modenumbers 4 and 5 at a Reynolds number (based on the maximum centerline velocity and pipe diameter) $Re \approx 9700$. The least stable eigenmodes are concentrated at the downstream end of the bulge and were characterised by their slow oscillatory nature. The swirl prescribed at the inlet was varied by changing w_0 and was found to have a stabilizing effect on the stability properties with the flow becoming unstable at a higher Reynolds number. The results obtained are found to be consistent with those presented in [1].

Motivation

Aneurysms are localized permanent arterial dilations caused due to disease or other complex processes that result in the weakening of the arterial walls [3]. Among these, fusiform aneurysms, that are commonly found in the abdominal aorta and are characterised by a bulge about the vessel centerline are the focus of the present study. In the recent years, there has been an increase in the studies carried out on blood flows from a fluid mechanics perspective. In particular, the studies on flows in constricted pipes designed to model stenosis have shed light on flow behaviour and stability in such geometries [4]. However, the research on flow through aneurysmal channels has been limited [2, 5]. These studies comprised of experiments [2] and numerical simulations [5] in axisymmetric aneurysm models investigate the role of wall shear stresses in the rate of enlargement of aneurysms.

The present study aims to explore the stability of the flow in aneurysms for steady inlet flows and to quantify their behaviour by studying the nature of instabilities inherent in these geometries. The analyses were carried out in the same spirit as in the recent article by Sheard & Blackburn [1].

Methodology

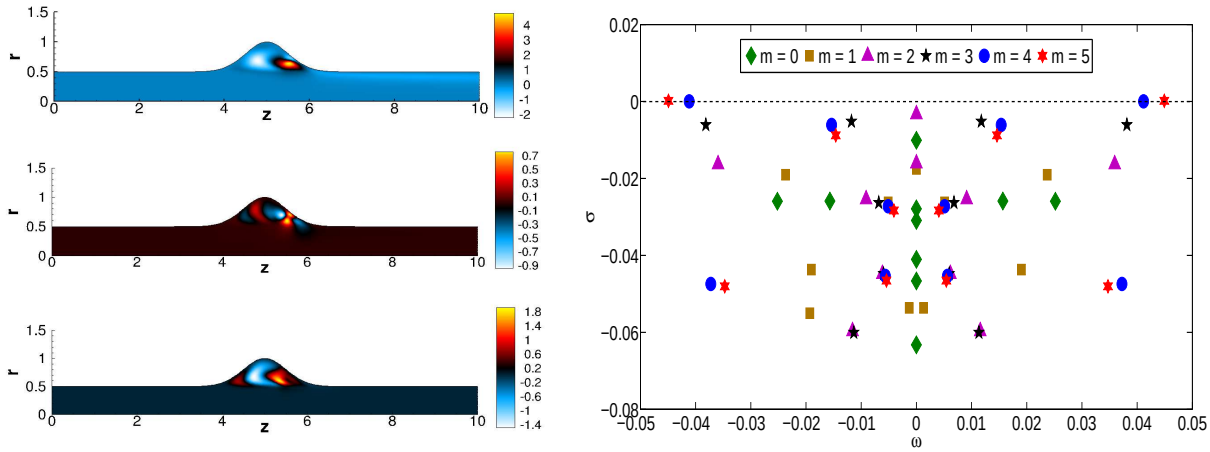
We use the incompressible Navier–Stokes equations written in cylindrical coordinates to simulate the flows considered in this study. Fully developed Poiseuille flow with maximum centerline velocity U was input upstream at the inlet. To compute the base flow, we adopt the procedure used by Marquet *et al.* [6] wherein a time-dependent simulation of the two-dimensional Navier–Stokes equations is first used to obtain an approximate solution of the steady solution at low Reynolds number where it is stable to two-dimensional perturbations. A Newton iteration is then applied to solve the stationary Navier–Stokes equations, starting the procedure with the approximate steady solution as an initial guess. The Newton algorithm requires the inversion of a non-symmetrical sparse matrix which is carried out by a direct method, the UMFPACK library being used to perform a sparse LU factorisation.

For the linear stability analysis, the base flows were assumed axisymmetric due to the general agreement in literature that blood flow in abdominal aortic aneurysms is dominated by axisymmetric flow structures [3]. The three-dimensional perturbation field was discretized in the azimuthal direction using Fourier modes, which are individually decoupled. This permits the stability analyses of individual azimuthal modenumbers (m) to be computed separately on the same axisymmetric domain as used for the base flow computations. The resulting eigenvalue problem was solved using a 'shift-and-invert' strategy as in [6].

The two-dimensional stationary Navier–Stokes equations and the general eigenvalue problem are solved numerically by a finite element method. The spatial discretization is a mixed finite-element formulation using P2-P1 Taylor–Hood elements: six-node quadratic triangular elements with quadratic interpolation for velocities (P2) and three-node linear triangular elements for pressure (P1). The meshes as well as the discrete matrices resulting from the variational formulation of the problems and projected onto those meshes are generated with the software FreeFem++ (http://www.freefem.org).

Preliminary Results

The computations shown in figure 1 were carried out on *Model A* for $w_0 = 0$. Figure 1a shows the leading global modes characterized by their axial, radial and azimuthal velocity components (top to bottom) for azimuthal modenumber $m = 4$ at $Re = 9700$. The spatial distribution of the global modes shows that they are localized at the downstream end of the bulge. Figure 1b shows a typical eigenvalue spectrum obtained from the linear stability analysis at $Re = 9700$ for various values of m. The temporal growth rate of the eigenmodes σ is plotted in the y-axis and their frequency ω in the x-axis. The least stable eigenmodes are observed for $m = 4, 5$ with a non-zero frequency.



(a) (b)

Figure 1: (a) Global modes visualized by their axial, radial and azimuthal (top to bottom) velocity components. Parameter settings: *Model A*, $m = 4, Re = 9700, w_0 = 0$. (b) Typical eigenvalue spectrum in the (ω, σ) plane for different azimuthal modenumbers m. Parameter settings: *Model A*, $Re = 9700, w_0 = 0$.

We intend to present the results obtained from our stability analyses along with the effect of varying inlet swirl on *Model A* during the conference.

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