A study of homogeneous turbulence within baroclinic context

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A STUDY OF HOMOGENEOUS TURBULENCE WITHIN BAROCLINIC CONTEXT

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We present direct numerical simulations and linear models for homogeneous turbulence submitted to the combined distorsions of constant vertical mean shear, vertical solid body rotation and vertical stable stratification, representative of situations in geophysical flows. Depending on the parameters, optimally reduced to the Richardson number and a baroclinic parameter, the baroclinic instability is triggered and modifies the dynamics and structure of the flow. We characterize turbulence in these different states, which present specific anisotropic features, using physical-space quantities (energies, correlation lengths, velocity structure functions) and spectral statistics (directional poloidal/toroidal/potential energy spectra).

1 Introduction to the baroclinic context in turbulent flows

The baroclinic context is characterized by a superposition of three coupled phenomena, required for the existence of baroclinic instability: (1) the Coriolis force, caused by earth rotation in geophysical flows; (2) stable stratification due to density gradients in the atmosphere, which lead to buoyancy forces in the vertical direction; (3) high vertical velocity gradients, as at the altitude of the tropopause in atmospheric flows, in the form of jet streams, which, in first approximation, are modelled by homogeneous shear. Due to admissibility conditions [1, 8], a spanwise density gradient is necessarily created and added to the previous vertical one. Under these conditions, tilting of isopycnal surfaces is induced [14], and, although the mean density gradients are stable in the sense that they lead to positive restoring forces, the misalignment of isopycnal surfaces with the horizontal plane can lead to unstable dynamics [14, 15], hence the baroclinic instability.

The first works on baroclinic instability where done by Charney (1947) [16] using quasi-geostrophic equations in the β-plane approximation. He derived a necessary condition for instability to occur formulated simply by an inequality implying the Rossby radius of deformation. Further contribution by Eady (1949) [17] using the f-plane approximation was later added. Eady considered a simpler model — a Couette-like configuration — of the atmosphere and obtained an instability condition resting on a critical wavenumber [14]. Following Eady, we also place the study within the f-plane approximation. We replace the two-boundary plates approximation by an infinite Couette flow to model shear effects [10].

2 DNS results of baroclinic turbulence

Investigations of homogeneous turbulent flows submitted to the separate effects of homogeneous shear, stable vertical stratification or rotation were done in the past decade [2, 3, 4, 5, 6]. Here, we focus on the following new approach: Direct Numerical Simulation (DNS) of homogeneous turbulence including the complete coupled three previous distorsions — the so-called 'baroclinic' context. Unlike the previously mentioned contexts, the baroclinic instability is a reservoir of energy which does not impose to add an artificial forcing to the simulations, or to have to deal with decaying flows. The aim of our study is first to provide a detailed characterization of the Eulerian properties of developped baroclinic turbulence. The parametric space is very large, since it not only includes the properties of turbulence itself, Reynolds number, cut-off wavenumbers, but also the choice of the intensities of rotation, stratification and shear. A reduction of the parametric space is obtained by considering the Richardson number defined as $Ri = N^2/S^2$ and the baroclinic parameter $\varepsilon = Sf/N^2$ which is also the mean slant angle of the isodensity surfaces. Here, $S = \partial_y U_x$ is the mean velocity gradient, $N = (-g\partial_y\overline{\rho}/\rho_0)^{1/2}$ is the Brunt-Väisälä frequency and $f = 2\Omega$ is the rotation rate. We shall investigate ranges of these parameters between $[0:2]$ for Ri and $[0:1]$ for ε. The Navier-stokes equations in the Boussinesq approximation are solved using pseudo-spectral Direct Numerical Simulation. Periodicity is assumed in the three directions. Under the action of shear, the mesh is deformed and a periodic remeshing is needed using the algorithm by Rogallo [10] (the anisotropic adaptation of the Orszag-Patterson algorithm) in spectral space (p, q, r). De-alisasing is done following Delorme [12] method. The code used in this study is a MPI-based parallel code : the turbulence box is cut into slabs following the algorithm by Coleman et al. [11]. Fast Fourier transforms are done using the library FFTW by Frigo and Johnson [13]. Time-advancing is done according to the third order Runge-Kutta (RK3) method. Lastly, the rotational form of the non-linear term has been implemented. We start by considering two-point spectra of Eulerian velocity and buoyancy fields, as well as the evolution of additional scalars which are introduced in the flow, with different Schmidt numbers. The time evolution of quantities such as the deviatoric

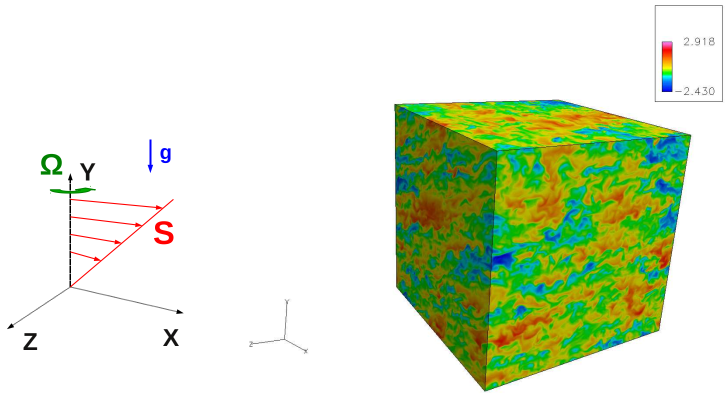

Figure 1: Sketch of the mean flow (left). Buoyancy distribution (right). DNS with 512x768x512 degrees of freedom with Richardson number $Ri = 0.5$ and baroclinicity parameter $\varepsilon = 0.2$. $Re_\lambda(0) = 44.1$ and $\frac{Sk}{\epsilon_\nu} = 2.55$. With ϵ_ν the kinetic energy dissipation. $\tau = tS = 7$.

part of the Reynolds stress tensor, the componentality and dimensionality tensors [7, 9], potential and kinetic energy directional spectra are analized to bring out and understand the complex anisotropic structure of the flow. A link with extended structure functions is also proposed, in order to assess the possible application, or disagreement, of scalings available for isotropic turbulence in the context of Kolmogorov theory. We will also study two-point correlations for characterizing the turbulent structures, and their properties. Large scale structures will be quantified with directional correlation length obtained from the velocity correlation tensor, whereas two-point vorticity correlations put to the fore smaller turbulent structures. The Lagrangian properties are also studied, and related to Eulerian data, especially considering the dual characterization of anisotropy. To refine our study, we thus obtain results concerning mixing properties of the flow by looking at both the stratifying agent concentration (see for instance the distribution of buoyancy on figure 1) and passive scalars advection. We also put the emphasis on the analysis of possibly unstable dynamics of turbulence within the baroclinic context, in a second part. Inertial transfers will be investigated, considering energy exchanges between the kinematic and the potential modes, but also the interscale and directional energy drain. This structuring of the flow, of nonlinear, irreversible nature since it is due to quadratic terms in the equations, is also compared to the predictions of linear theory (often called Rapid Distorsion Theory, RDT), which provides lots of valuable information on the spectral energy distribution reorganization by the mere body forces (Coriolis and buoyancy) and shear. Since RDT is also computationally much lighter than DNS, it also permits a preliminary opening of the parametric space, allowing to choose the most relevant parameters for the high resolution simulations.

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