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HYDRODYNAMIC INSTABILITIES IN THE ECCENTRIC TAYLOR–COUETTE–POISEUILLE FLOW

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1 Objectives

Since Taylor's first theoretical results obtained almost a century ago [5], researchers have dedicated a relentless effort in modelling the extremely rich dynamical behaviour displayed by a fluid trapped between two concentric cylinders in differential rotation. Taylor–Couette centrifugal instability discovery has lead to the development of hydrodynamic instability theory and the study of transition to turbulence, thanks to a reduced geometric complexity that made analytical studies tractable back in the early 20th century. Since then, increasing complexity has been considered through additional ingredients modifying the canonical configuration: effect of eccentricity, axial flow, non-Newtonian behaviour, etc. Asymptotic analyses and experiments have progressively been complemented by an increasing computational power that allowed a larger parameter space to be explored quantitatively [1, 3, 2].

Only recently, numerical computations reached the degree of efficiency allowing to tackle hydrodynamic stability issues impacting a more realistic industrial configuration, namely here: the three-dimensional flow of mud that is drained upwards in an eccentric annular domain, between a rotating flexible drillstring and the rock face of an oil well, during drilling operations. In that configuration, the inner cylinder is rotating so as to drive the drill bit at the bottom of the well. Mud is pumped down through the drillstring then flows back to the surface through the annular domain. This mud flow is intended to carry the rock cuttings out, avoid their sedimentation when drilling is stopped, limit friction, and finally maintain the right pressure level in the annular domain to prevent gases trapped in the rock to leak inwards the well and cause damage.

In the resulting eccentric Taylor–Couette–Poiseuille flow, how does eccentricity and axial flow impact the centrifugal instability thresholds? In such an open flow, how big of a flow rate is needed to convect the most unstable perturbations downstream and prevent them to overwhelm the laminar basic state? Finally, what new states can be expected after onset of instability? Another topic of interest is the impact of the fluid on drillstring vibrations: under the influence of the hydrodynamic forces, what will be the trajectory of the drillstring (periodic, quasi-periodic, chaotic, ...)?

2 Methodology & results

A DNS solver, specifically designed for eccentric annular domains, is currently being developed to solve the three-dimensional time-dependent incompressible Navier–Stokes equations in primitive variables (assuming a Newtonian behaviour). Considering non-dimensional variables, two control parameters appear in the equations: a Reynolds number Re based on the inner cylinder velocity and a measure of the gap width, and a non-dimensional pressure gradient G_z .

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla \pi + G_z = \frac{1}{Re} \Delta \mathbf{u}$$

$$\nabla \cdot \mathbf{u} = 0$$

Bipolar coordinates are relevant to the geometry and yield a separable variable Laplacian operator. Using an appropriate conformal transformation (modified bipolar coordinates, Wood [6]) together with a dilatation on the "pseudo-radial" coordinate, the eccentric annular region is mapped onto the rectangular domain $D = \{(\xi, \phi) \in [-1, 1] \times [0, 2\pi]\}$, where all fields are periodic in the "pseudo-azimuthal" coordinate ϕ . The transformations use two dimensionless geometric parameters, an aspect ratio η based on the two cylinder radii, and an eccentricity parameter e ranging from axisymmetry to the limit of touching cylinders.

Pseudo-spectral approximations are used for enhanced accuracy, with Fourier modes in the periodic "pseudo-azimuthal" direction ϕ , and Chebyshev collocation on Gauss–Lobatto points over the "pseudo-radial" direction ξ . Different multi-step time schemes are tested, all treating the convective term explicitly and the diffusion term semi-implicitly. A projection method is used to decouple pressure and velocity, similar to the modified Goda's algorithm introduced by Raspo et al. [4]. This algorithm is used to calculate the axially invariant base flow for



each setting of control parameters (Re, G_z, η, e). Subsequently, the linear and non-linear stability of these base flows will be investigated, using a Fourier decomposition along the axial z -coordinate.

We will first present a series of results validating our code for axially invariant flow, and benchmarking its efficiency against existing solvers. Then an exploration of the parameter space will be carried out in order to identify the flow patterns pertaining to this system. For configurations relevant to the oil industry, hydrodynamic forces acting on the inner cylinder will be calculated in order to evaluate the stability of the drillstring depending on its radial position.

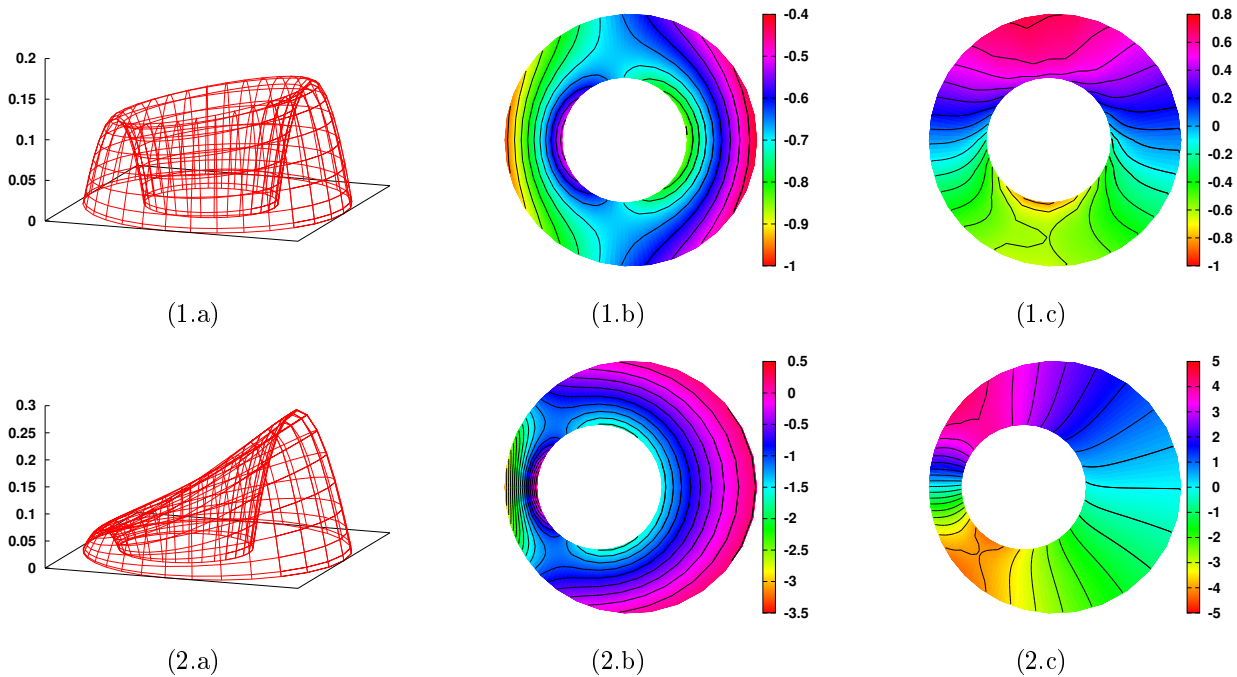


Figure 1: Base flows ((a) axial velocity, (b) axial vorticity, (c) pressure) for (1) weak and (2) moderate eccentricity.

References

- [1] D. L. Cotrell and A. J. Pearlstein. The connection between centrifugal instability and Tollmien–Schlichting-like instability for spiral Poiseuille flow. *J. Fluid Mech.*, 509:331–351, 2004.
- [2] M. P. Escudier, P. J. Oliveira, and F. T. Pinho. Fully developed laminar flow of purely viscous non-Newtonian liquids through annuli, including the effects of eccentricity and inner-cylinder rotation. *Int. J. Heat Fluid Flow*, 23:52–73, 2002.
- [3] D. Martinand, E. Serre, and R. M. Lueptow. Absolute and convective instability of cylindrical Couette flow with axial and radial flows. *Phys. Fluids*, 21(104102), 2009.
- [4] I. Raspo, S. Hugues, E. Serre, A. Randriamampianina, and P. Bontoux. A spectral projection method for the simulation of complex three-dimensional rotating flows. *Computers & Fluids*, 31(4–7):745–767, 2002.
- [5] G.-I. Taylor. Stability of a viscous liquid contained between two rotating cylinders. *Phil. Trans. Roy. Soc. Lond. A*, 223:289–343, 1923.
- [6] W. W. Wood. The asymptotic expansions at large Reynolds numbers for steady motion between non-coaxial rotating cylinders. *J. Fluid Mech.*, 3:159–175, 1957.