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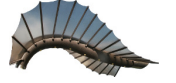
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TRANSITION FROM QUASI-2D TO 3D IN A ROTATING ELECTROMAGNETICALLY FORCED DIPOLAR FLOW STRUCTURE

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1 Introduction

Background rotation is assumed to enforce the two-dimensionality of flows. If the rotation is fast enough, the flow far away from the boundaries becomes independent of one coordinate. Furthermore, the characteristics of the boundary layers that form next to the boundaries perpendicular to the rotation axis (known as Ekman boundary layers) allow to model the evolution of decaying rotating flows using the two-dimensional vorticity equation

$$\frac{D\omega}{Dt} = \lambda_{Ek}\omega, \quad (1)$$

where D/Dt is the material derivative, ω is the vorticity component parallel to the rotation axis of the system, and λ_{Ek} is a constant known as the Ekman damping coefficient. If a flow obeys eq. (1), the flow can be said to be quasi-two-dimensional [1]. Equation (1) is *a priori* only valid if the Coriolis acceleration is much stronger than the advective acceleration (i.e. the Rossby number Ro is small) and if the flow evolves freely.

In the current work, we determine experimentally the limit of applicability of the linear Ekman theory, on which eq. (1) is based, to flows driven by a non-conservative body force. In other words, we determine the transition in the parameter space between a quasi-two-dimensional and a three-dimensional flow. In particular, we study a dipolar flow structure driven by time-independent electromagnetic forcing in a shallow fluid layer subjected to background rotation, and we characterize the response of the flow as a function of the parameters of the problem. The limit for the two-dimensionality of the flow is reflected as a change in the scaling of the response of the flow to the magnitude of the electromagnetic forcing, and against the common belief, it does not depend exclusively on the Rossby number.

2 Methods

The experiments were carried out in a water tank with a base of $34 \times 30 \text{ cm}^2$ placed on top of a rotating table, which rotates at a frequency Ω . The tank is filled with a salt solution with a concentration of 178 g l^{-1} with a kinematic viscosity $\nu = 1.50 \cdot 10^{-6} \text{ m}^2\text{s}^{-1}$ and a density $\rho = 1190 \text{ kg m}^{-3}$ to a depth H and covered with a transparent perspex lid to avoid free-surface deformations. To force the flow, two titanium electrodes are placed along two opposite sides of the tank, and three $30 \times 10 \text{ cm}^2$ rectangular magnets are placed 1.1 cm underneath the tank bottom. The magnitude of the magnetic field of each these magnets is 0.018 T measured just above their centre. The magnet at the centre has its North Pole facing up, while the two side magnets have their North Pole facing down. A constant electric current I is applied through the fluid using a power supply with a precision of 10^{-2} A .

Particle Image Velocimetry (PIV) is used to measure the horizontal velocity field of the flow in a plane at mid-depth. The fluid is seeded with $106\text{--}150 \text{ }\mu\text{m}$ polymethylmethacrylate (PMMA) particles, which are illuminated at mid-depth with a laser sheet produced by a double pulsed Nd:YAG laser.

Numerical simulations of the flow were performed using a finite-element code (COMSOL). These simulations serve two purposes: (1) to obtain the three-dimensional velocity field of the flow and (2) to reach regions of the parameter space that are inaccessible experimentally.

2.1 Non-dimensional parameters

Dimensional analysis yields three independent dimensionless control parameters:

$$Ch \equiv \frac{IBH}{\rho\nu^2}, \quad Ek \equiv \frac{\nu}{\Omega H^2}, \quad \delta \equiv \frac{H}{L}, \quad (2)$$

where the Chandrasekhar number Ch characterizes the EM forcing and represents the ratio of the Lorentz force to the viscous force; the Ekman number Ek characterizes the system's rotation rate and represents the ratio

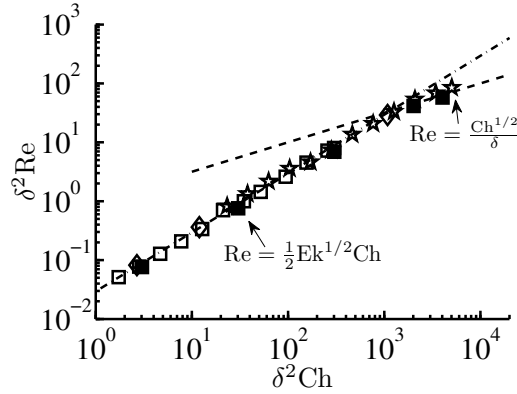
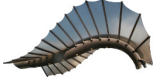


Figure 1: Response of the flow to electromagnetic forcing. Results from experiments (empty symbols) and simulations (filled symbols) for $Ek = 3.5 \cdot 10^{-3}$ and $\delta = 0.0353$ (squares), $\delta = 0.0588$ (diamonds), $\delta = 0.0941$ (stars). The dashed lines represent theoretical approximations obtained from dimensional analysis.

of viscous forces to the Coriolis force; and the aspect ratio of the container δ characterizes the geometry of the flow domain. B is the magnitude of the magnetic field at mid-depth above the center of a magnet.

In addition, as the response parameter of the problem, we define the Reynolds number

$$Re \equiv \frac{UL_x}{\nu}, \quad (3)$$

which represents the ratio of inertia forces to viscous forces, with U the spatial average of the horizontal velocity at mid-depth. In other words, $Re = Re(Ch, Ek, \delta)$. The Rossby number can be written in terms of the other dimensionless parameters as $Ro = \delta^2 Ek Re / 2$.

3 Results

Figure 1 presents the response of the flow as function of the forcing for experiments and simulations with $Ek = 3.5 \cdot 10^{-3}$ and different values of δ . Note that the axes have been multiplied by δ^2 , resulting in the collapse of the curves for different δ -values. As can be seen, there are two scaling regimes for all Ek -values. For small values of $Ch\delta^2$ there is a linear regime where $Re \propto Ch$, while for large values of $Ch\delta^2$, there is a non-linear regime where $Re \propto Ch^{1/2}$. The transition reflects the growing importance of the advective acceleration, and is similar to the transition found in the case of shallow flows without background rotation [2].

From dimensional analysis, it can be estimated that the transition between the two different regimes occurs when

$$\frac{\delta^2 Ch Ek}{4} = \frac{Ro}{Ek^{1/2}} \approx 1, \quad (4)$$

showing that it does not depend exclusively on the Rossby number.

For strong background rotation (small Ek -values), viscous dissipation is dominated by Ekman damping in the linear regime. However, numerical simulations show that the Ekman boundary layers are deformed in the non-linear regime. Hence, linear Ekman damping is no longer a good approximation in this regime, and the flow cannot be considered as quasi-two-dimensional. This can be true even for small Ro -values. The deformation of the Ekman boundary layers is then not due to non-linearities in the bulk of the fluid. Instead, the balance between viscous forces and the Coriolis force, which essential for the existence of Ekman boundary layers, is broken due the electromagnetic forcing within the boundary layer.

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