Transition characteristics of a swirling annular flow

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Instabilities and transition in three-dimensional flows with rotation

TRANSITION CHARACTERISTICS OF A SWIRLING ANNULAR FLOW

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1 Introduction and scope of the work:

The subject of this paper is transition of a **swirling annular flow** induced by the Tollmien-Schlichting mechanism which is characterised by waves propagating with a certain speed in the shear flow. One of the outstanding characteristics of this flow is the helical shape of its streamlines. Flows with such streamlines are very difficult to successfully simulate with CFD, as the Reynolds stress tensor has a strong anisotropy [1]. The object of this work is to try to gain insight into the physical features both in qualitative and quantitative terms through studying the bifurcation characteristics of this flow on the verge of transition. The focus of our attention is on the shape of the Reynolds stress tensor, for which the relevant properties of the bifurcating solution of the nonlinear equation of motion for disturbances to the basic flow are examined. A set of (nonlinear) differential equations governing the dynamics of disturbances in a swirling flow is derived for the flow in the annular gap between concentric circular cylinders. The analysis is undertaken for the swirling flow when an axial pressure gradient (parameterized by Re, Reynolds number), a rotation of the outer cylinder (parameterized by S_a, rotation number) and the axial translation of the inner cylinder (parameterized by T_w, translation number) act simultaneously to maintain the swirling flow.

2 Outline of the method of Stewartson and Stuart to gain an understanding of the influence of flow parameters in this problem

The starting point of our work is a linear stability analysis for the flow described above. To this end it is necessary to derive the nonlinear equations of motion for the flow in which the pressure is eliminated as an unknown. The system of equations derived is referred to as the **extended Orr-Sommerfeld and Squire equations** which together with the continuity equation lead to the following equation system which is written in matrix notation $\mathscr{D}\mathbf{v} = \mathscr{N}$:

$$\mathscr{D} = \begin{pmatrix} D_{OSr} & D_{OS\varphi} & D_{OSx} \\ D_{Sqr} & D_{Sq\varphi} & D_{Sqx} \\ D_{Cor} & D_{Co\varphi} & D_{Cox}. \end{pmatrix}, \mathbf{v} = \begin{pmatrix} v_r \\ v_\varphi \\ v_x \end{pmatrix}, \mathscr{N} = \begin{pmatrix} \mathscr{N}_{OS} \\ \mathscr{N}_{Sq} \\ \mathscr{N}_{Co} \end{pmatrix} \tag{1}$$

Here D_{OS}, D_{Sq}, D_{Co}, $\mathbf{v}$ and $\mathscr{N}$ represent the operator in the classical Orr-Sommerfeld equation, Squire equation, continuity equation, velocity disturbance and nonlinear terms in the radial, azimuthal, and streamwise directions respectively.
When the velocity disturbance is considered small the nonlinear terms can be neglected and the equation system above becomes the classical eigenvalue problem with which it is possible to determine the neutral stability surface of the flow.

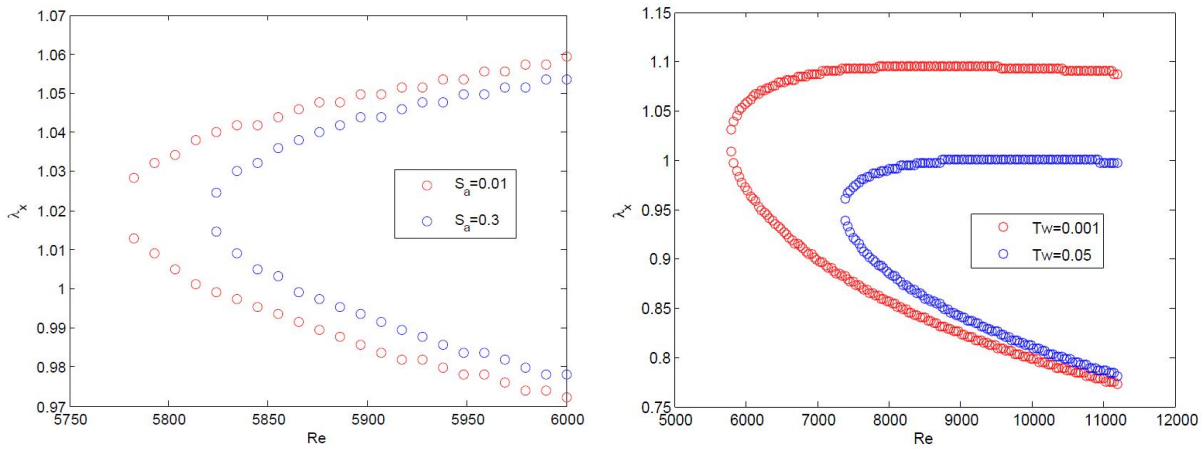

Figure 1: Neutral surfaces for a swirling annular flow.

The diagrams above demonstrate the stabilizing effect of both the rotation of the outer cylinder from $Sa = 0.01$ to $Sa = 0.3$ and the axial translation of inner cylinder from $Tw = 0.001$ to $Tw = 0.05$.
The figures of projections of these neutrally stable curves were evaluated numerically through MATLAB-programs.
Following the steps developed by Stewartson and Stuart for a weakly nonlinear theory, elementary waves centered around the neutral stability curve which were determined in the linear stability theory are used as part of a wave packet which for large times is described as superposition of several elementary waves moving with the group velocity and whose amplitude varies slowly. The slow variation of the amplitude is designated here as **B**. The wave packet has the following form:

$$\mathbf{v} = A_N(y)\exp[i\Theta_N]exp[\omega_{Ni}t] + c.c.. \tag{2}$$

$$\Theta_N(r,\varphi,x) = (\lambda_{Nx}x + n_{N\varphi}\varphi + \omega_N t) \tag{3}$$

Here the frecuency ω_N is permitted to be complex, the wavenumber λ_{Nx} is real whereas n_φ may assume only real integer values. The subscript N denotes values on the surface of neutral stability and i the imaginary part. The above form of the wave involves the identification of a small amplitude parameter ϵ_A with the temporal growth rate ω_{Ni} of the wave packet.
When the departure of the parameters of the flow (Re, Sa and Tw) is small relative to those of the neutral stability surface, the growth rate ω_{Ni} can be approximated by its Taylor expansion through the flow parameters [2].

$$\epsilon_A = \omega_{Ni} = d_{Re}|Re - Re_N| + d_{S_a}|S_a - S_{aN}| + d_{T_w}|T_w - T_{wN}| \tag{4}$$

With the help of this parameter it is possible to define new scales in the problem solving the nonlinear problem with the method of multiple scales. An asymptotic expansion is introduced for the velocity perturbation v as well as the extension of the partial derivatives of the equations in the new scales [3].

$$\mathbf{v} \simeq \epsilon_A^{\frac{1}{2}}\mathbf{v}_1 + \epsilon_A\mathbf{v}_2 + \epsilon_A^{\frac{3}{2}}\mathbf{v}_3 + o(\epsilon_A^{\frac{3}{2}}) \tag{5}$$

By substitution of (5) in (1) yields a new system of equations which is sorted according to the order of the approximations. In the compact matrix-operator notation these may be written as follows:

$$\begin{aligned} O(\epsilon_A^{\frac{1}{2}}) &: \mathscr{D}\mathbf{v}_1 = (RHS)_1 = 0. \\ O(\epsilon_A) &: \mathscr{D}\mathbf{v}_2 = (RHS)_2 \\ O(\epsilon_A^{\frac{3}{2}}) &: \mathscr{D}\mathbf{v}_3 = (RHS)_3 \end{aligned} \tag{6}$$

The solution for $\mathbf{v}_1$ is sought in a product form as follows:

$$\begin{aligned} \mathbf{v}_{1r} &= B_r(New\ scales)A_{Nr}(y)\exp[i\Theta_N] + c.c \\ \mathbf{v}_{1\varphi} &= B_\varphi(New\ scales)A_{N\varphi}(y)\exp[i\Theta_N] + c.c \\ \mathbf{v}_{1x} &= B_x(New\ scales)A_{Nx}(y)\exp[i\Theta_N] + c.c \end{aligned} \tag{7}$$

Formulating the solvability condition for the equations to the orders $O(\epsilon_A)$ and $O(\epsilon_A^{\frac{3}{2}})$ in (6) , which involves determining the solution of adjoint problem of original eigenvalue problem, one of the three equations necessary for the evolution equation of the amplitude B can be obtained.
The remaining equations for B can be obtained through the continuity equation and determining the direction of flow of kinetic energy in the fluctuation motion, matching the ratio of the partial derivatives of the kinetic energy to new scales with the ratio of the partial derivatives of ω_{Ni} to the wave numbers of the problem.
Besides it is expected that the form of the Reynolds stress tensor in the wall-normal direction is given by components of the tensor product of (A_{Nr}; $A_{N\varphi}$; A_{Nx}), with itself.

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