DEVELOPMENT OF HELICAL VORTEX THEORY

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The main goal of the current communication is to attract attention to the thriving theory of helical vortices which are 2-D from Euler's consideration and looks from this point of view like "plane" flows (fig. 1a) but with additional uniform rotation of a plane pattern of the flow along fixed axis (fig. 1b). However, from Lagrange's point of view the 2-D helical vortices induce strong 3-D motion of fluid particles (fig. 1c) [1].

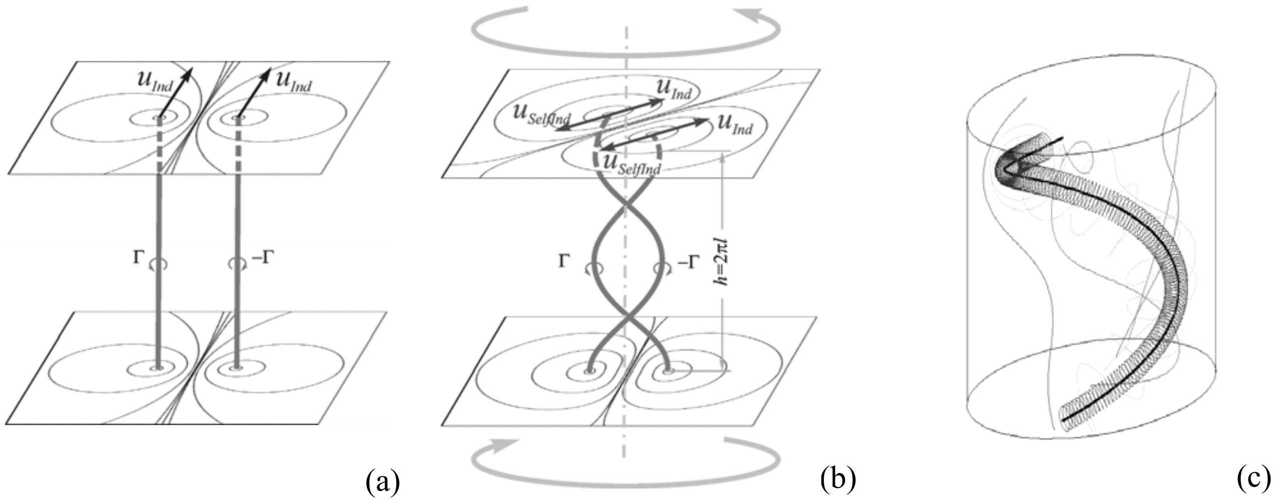

Figure 1. Examples of 2-D flows with rectilinear (a) and helical vortices (b);
3-D paths of fluid particles induced by single helical vortex filament in a cylinder (c).

At present the theory of 2-D helical vortex dynamics are based on various analytical components, which holds true for all values of the helix pitch, such as (i) the 2-D Biot-Savart law for helical filaments represented by Kapteyn series [2] or rewriting in a form with singularity separation [3] (fig. 2); (ii) solutions of helical vortex tubes with finite core, governed by series expansion of helical multipoles [4]; (iii) relations between the induction of vortex filaments and the self-induced velocity of helical vortex tubes [5] resulting in a closed analytical solution of the helix motion [6]; (iv) analytical representation of Goldstein's solution for the circulation of a helical vortex sheet in equilibrium [7]; (v) Kelvin's N-gon stability problem of point vortices generalized to multiple helical vortices [6, 8].

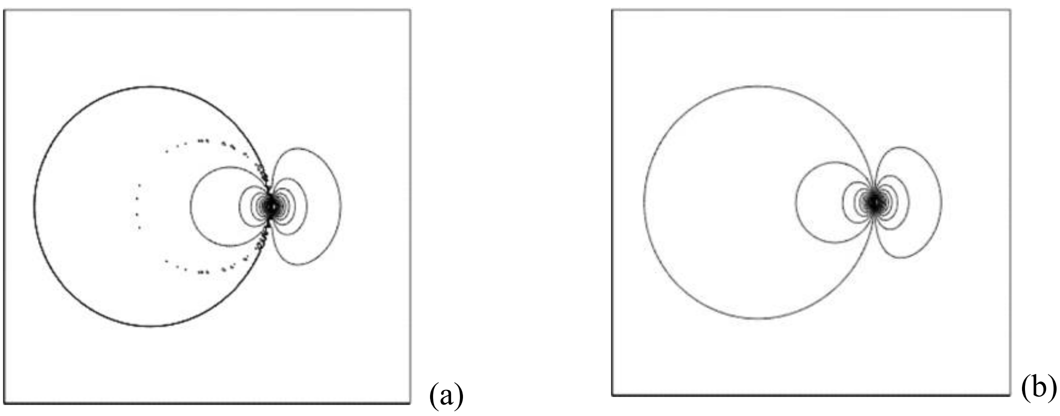

Figure 2. Isolines of the azimuthal velocity field induced in cross-section by helical vortex filament:
(a) inaccurate simulation by finite part of the Kapteyn series with unreal singularities on cylinder supporting the helix;
(b) accurate simulation after separation of the singularity along the helix filament in the Kapteyn series [3].

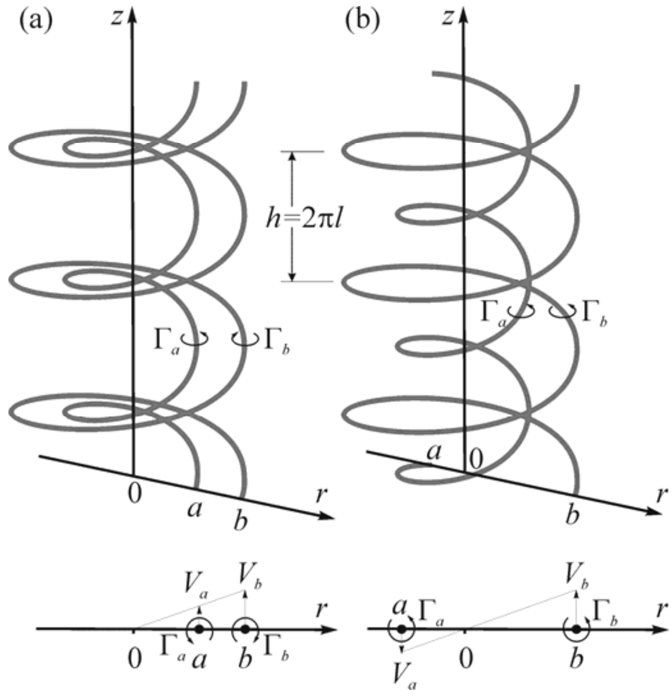

Figure 3. Two possibilities for equilibrium of helical vortex pairs and its prototypes from point vortex pairs: (a) centre or axis of rotation lies on the same hand from both point vortices; (b) centre of or axis rotation lies between ones

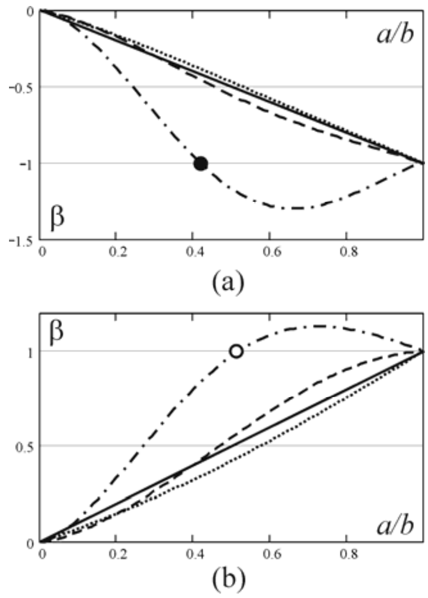

Figure 4. The circulation ratio β of the two helical vortices in equilibrium rotation as function of their relative proximity for different values of helical pitch h: 1.5 (dash-dotted lines); 2(dashed lines); 5 (dotted lines); infinity (solid lines). Circle and point indicate equilibrium of the helical pairs with different levers.

An important result of the 2-D helical vortex theory is that the vorticity submitted to the helical symmetry does not vary along a trajectory of fluid particles. It coincides with conclusion of point vortex dynamics but in contrast of the plane one, where the total vortex motion consists of the mutual induction of other point vortices, for helix case in additional to an induced flow field due to induction from the other helical vortices on the surroundings a self-induction of the helical vortex should be included too. In result of this difference unexpressed applications of the 2-D helical vortex dynamics were established. For example figs. 3 and 4 demonstrate how the two helical vortices (vortex pair) with the same or opposite circulations but different levers can be in equilibrium rotation which is impossible for two point vortices. In the presentation it was also shown how the assumption of helical symmetry in the context of 2-D helical vortices can be exploited to analyze and model various cases of rotating flows.

References

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