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DYNAMICS OF FLOWS WITH HELICAL SYMMETRY

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1 Context and objectives

Rotating devices such as propellers, wind turbines, helicopter rotors are known to develop a system of helical vortices in their wake. These structures result from the rapid roll-up of the vorticity sheet continuously generated at the trailing edge of the rotating blades. Very often, these flows display a helical symmetry, meaning that they are, at least locally, invariant through combined axial translation of distance Δz and rotation of angle $\theta = \Delta z/L$ around the same z-axis. The helix pitch $2\pi L$ is then a spatial period of the flow. In the literature, the analytical [1, 2] and numerical [3] works describing stationary helical vortices have been mostly restricted to helical vortex filaments and patches in inviscid situations with the vorticity always pointing along helical lines of pitch $2\pi L$. However, wake vortices form through the roll-up of the trailing vorticity sheet, and viscous diffusion eventually leads to continuous distributed vorticity distributions within the vortex cores, such as Gaussian. Moreover, a distribution of axial velocity may also be present, which has always been disregarded in the literature.
The objective of the current study is to simulate the viscous dynamics of helical vortex systems without the above restrictions. We present here an original DNS code [4] which solves the incompressible Navier–Stokes equations with enforced helical symmetry at fixed pitch $2\pi L$. The three-dimensional equations are reduced to a modified two-dimensional unsteady problem which can be solved via a generalised vorticity/streamfunction formulation. The code is thus able to take 3D vortex curvature and torsion effects into account while the resolution is of a 2D type, allowing for finer grids, higher Reynolds numbers and longer integration times.

2 Results

In this framework, we study the long-time (or equivalently far-wake) dynamics of systems with 2 and 3 helical vortices regularly spaced along a cylinder of radius R. Figure 1 shows the time evolution of the distance between two interacting helical vortices (when cut in a plane perpendicular to their common axis) for different values of the reduced pitch L. At high pitch ($L/R > 2$), one recovers the classical picture of pure 2D vortex merging ($L = \infty$), yet with larger evolution time scales. At smaller pitch (see $L/R = 0.8$) a quite different type of vortex merging occurs which involves neighbouring spires, as shown in figures 2 and 3. The conditions on vortex pitch and core size leading to one type of merging or to the other as well as the associated transition are investigated. Helically symmetric flows are also produced by natural instability of axisymmetric jets, wakes and trailing vortices. DNS of the nonlinear saturation of such instabilities are presented here in the case of the Batchelor vortex. We retrieve the different behaviours observed when the swirl parameter is varied [5], and we present new results concerning the saturation of viscous centered modes [6].

References

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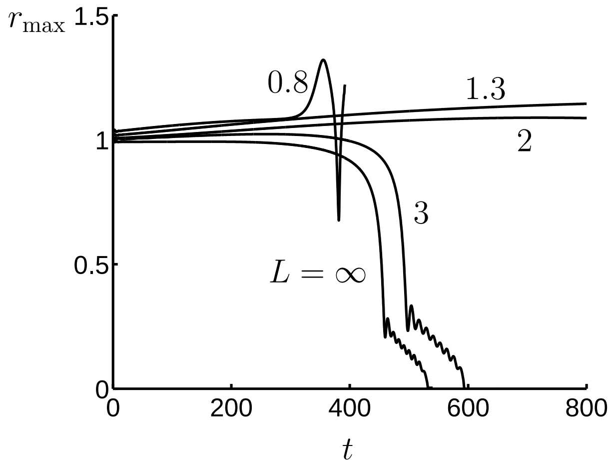

Figure 1: Merging of two helical vortices at $Re = \Gamma/\nu = 10000$ for different values of the reduced pitch L: time evolution of the distance $r_{\max}$ of one of the vortices from the axis. Initially, the helical vortices have circulation $\Gamma = 1$, core size $a = 0.2$ and are on a cylinder of radius $R = 1$.

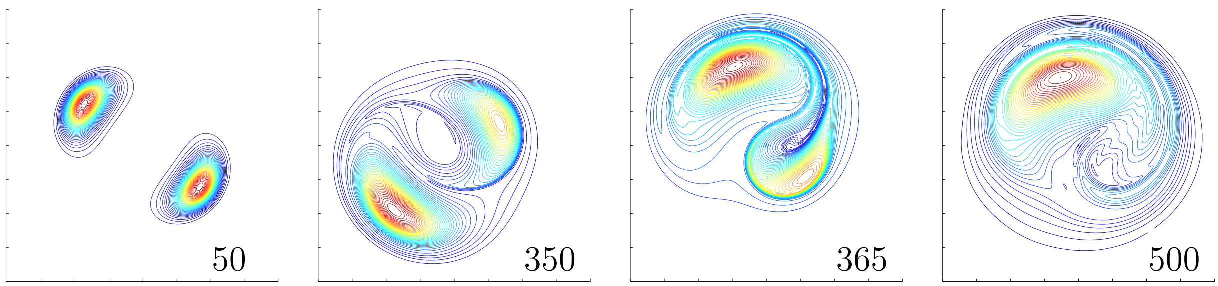

Figure 2: Merging of two helical vortices at pitch $L = 0.8$ at $Re = 10000$. Initially, the helical vortices have circulation $\Gamma = 1$, core size $a = 0.2$ and are on a cylinder of radius $R = 1$. Snapshots of the helical vorticity component for different times in a plane perpendicular to the helix z-axis.

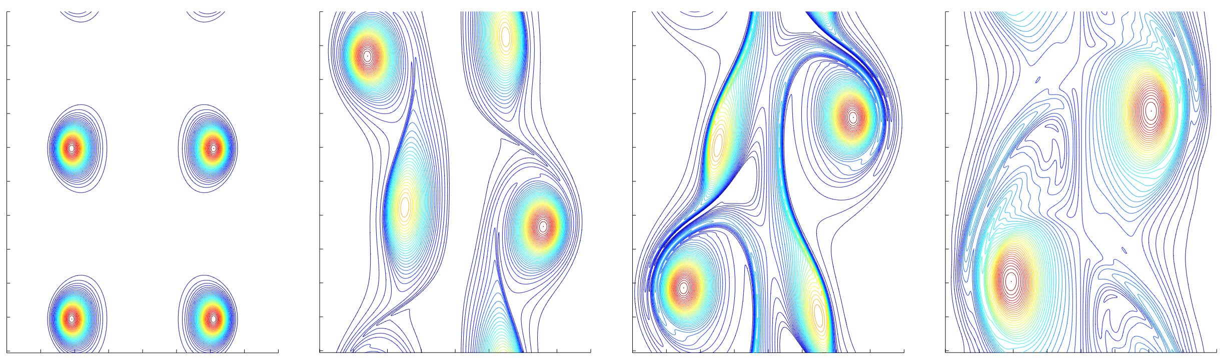

Figure 3: Same as 2 but viewed in a meridian plane.