Flow instabilities in a vertical differentially rotating cylindrical annulus with a radial temperature gradient

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FLOW INSTABILITIES IN A VERTICAL DIFFERENTIALLY ROTATING CYLINDRICAL ANNULUS WITH A RADIAL TEMPERATURE GRADIENT

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1 Introduction

Flow in a differentially rotating cylindrical annulus with a radial temperature gradient is encountered in many industrial applications [1]. It has been investigated experimentally and numerically by few authors [2, 3, 4, 5, 6]. For small values of the control parameter,away from the boundaries, the base flow has two velocity profiles each of which is unstable. The rotation induces circular Couette flow which is potentially unstable to centrifugally driven perturbations leading to longitudinal vortices. The radial temperature gradient induces a baroclinic vertical flow (ascending near the hot surface and descending near the cold one). This flow has a velocity profile with an inflexion point and it is potentially unstable to transverse oscillatory perturbations. The present study is concerned with experimental and linear stability analysis of this flow when the driving forces (control parameters) are increased in magnitude.

2 Experimental setup and stability analysis

The experimental setup consists of two coaxial cylinders of height $L = 55.9\,cm$ and a gap $d = 0.5\,cm$. The aspect raio of the system is $\Gamma = 111.8$ and the radius ratio$\eta = 0.8$. The inner cylinder is rotating at angular frequency Ω while the outer cylinder is fixed. Inside the inner cylindrical tube was circulating a water low maintained at controlled temperature T_1 and the outer cylinder was immersed into a large thermal bath maintained at controlled temperature T_2. The working fluid is a deionized water with a kinematic viscosity $\nu = 10^-2\,cm^2/s$ at $T = 295K$. The flow is described by three control parameters : the Prandtl number $Pr = \nu/\kappa$, the Grashof number $Gr = \alpha\delta T g d^3/\nu^2$ which measures the magnitude of the temperature gradient on the flow and the Taylor number $Ta = (\Omega a d/\nu)(d/a)^{1/2}$ related to rotation and therefore counting for centrifugal effects. The flow was visualized by adding a suspension of Kalliroscope AQ1000 in 2% by volume. To visualize the temperature fields, we seeded the flow with SR25C5W thermochromic liquid crystals from Hallcrest of about by 0.05%. They occur in form of encapsulated spheres of mean diameter $75m$ and their response time to a temperature change is about $3ms$. These particles have also been used to determine the velocity components, using a Basler IEEE-1394 camera, by tacking pictures of the cross section with a time delay of 82 ms.

2.1 Experimental results

A radial temperature gradient imposed on the cylindrical surfaces of the flow annulus induces a large convective cell with particules ascending near the hot wall and descending near the cold one. For a fixed value of Grashof number, we have increased the rotation rate until we obtained a bifurcation to a new state formed of pattern of helicoidal vortices (Figure 1-a). All critical states are represented in the diagram (Gr, Ta) in Figure 1-b. For chosen state, we have measured the velocity field and the temperature in the cross section (Figure 2).

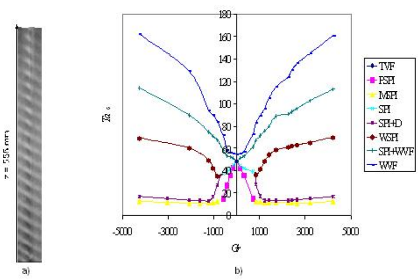

Figure 1: a) Pattern obsevred for $Ta = 24$ and Gr=706. b) Diagram of bifurcations in the plane Gr, Ta).

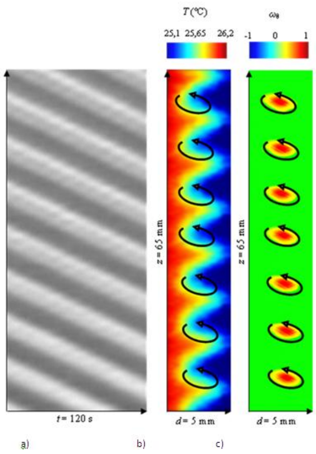

Figure 2: a) Space-time diagram of the pattern obsevred for Ta= 24 and Gr=706. b) Temperature distribution in the gap, c) Vorticity distribution in the gap.

2.2 Linear stability analysis

The flow is governed by the Navier-Stokes equations, energy equation and mass conservation equations written in cylindrical coordinates together with no-slip boundary conditions and isothermal cylindrical surfaces. Neglecting the end effects near the top and bottom plates, the base flow velocity profile is $\vec{v} = V(r)\vec{e}_\theta + W(r)\vec{e}_z$ where $V(r)$ and $W(r)$ are the circular Couette profile and the baroclinic axial velocity component respectively; their expressions can be found in [5]. We have performed linear stability of this flow assuming an infinite length of the system. We found that critical modes are nonaxisymmetric oscillating vortices. The variation of critical values of Ta with Gr are in a very good agreement with experimental ones. The corresponding state diagrams are well superimposed.

3 Conclusion

We have performed thourough investigation of stability of the flow between differentially rotating annulus with a radial temperature gradient. We have found a good agreement between experimental results, those from linear stability analysis and from numerical simulations [7].

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