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LOCAL AND GLOBAL STABILITY OF ROTATING DISK BOUNDARY LAYERS

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The von Kármán boundary-layer flow above a rotating disk can be shown to be absolutely unstable, by deploying a local linear analysis where the base flow is artificially simplified by taking it to be homogeneous along the radial direction [5]. Making use of the same approximation to represent the base flow, absolute instability has also been identified for the whole family of rotating disk boundary layers that, through the variation of the Rossby number, continuously connect the Kármán flow to the Bödewadt flow [6]. Moreover, absolute instability still persists if suction is introduced at the disk surface [7] or there is an applied axial magnetic field [4], though its onset is postponed to higher Reynolds numbers. Thus absolute instability is pervasive for rotating disk boundary layers. However, we will illustrate how the results obtained using a localized analysis for the homogenized version of the base flow cannot just be taken, in an immediate manner, to predict the correct global behavior for the genuine radially inhomogeneous base flow. The relationship between the local and global stability properties turns out be rather more complex and interesting than many researchers might have anticipated.

For the case of the von Kármán boundary-layer flow, direct numerical simulations of linearized disturbance development in the genuine flow showed that the predicted absolute instability does not in fact give rise to any unstable linear global modes [1]. This is despite the fact that the temporal growth rates for the absolute instability display a marked increase with the radial distance from the rotation axis [2]. The apparent disparity between the radially increasing strength of the absolute instability and the absence of any global instability can be understood by considering analogous behaviour in impulse solutions of the linearized complex Ginzburg-Landau equation [3]. These solutions show that *detuning*, arising from the radial variation of the temporal frequency of the absolute instability, may be enough to globally stabilize disturbances. Depending on the precise balance between the radial increase in the growth rates and the corresponding shifts in the frequencies, it is possible for an absolutely unstable flow to remain globally stable.

Globally stable behaviour was found to persist in numerical simulations where mass injection was introduced at the rotating disk surface, modifying the base flow. More interestingly, it was discovered that globally unstable behaviour is promoted when suction is applied [8]. This contrasts with the locally stabilizing influence of suction that had previously been identified, using the approximation of a radially homogenized base flow. For the inhomogeneous flow, impulsively excited disturbances display an increasingly rapid temporal growth at the radial position of the impulse, albeit without any selection of a dominant frequency, as would be more usual for an unstable global mode. A similar form of global destabilization, overcoming locally stabilizing effects, was reproduced when an axial magnetic field was applied.

We will discuss numerical simulation results to illustrate the new type of instability, and show how it can be successfully described using impulse solutions of the linearized Ginzburg-Landau equation. More generally, we will review the subtle connections that can persist between the local and global linear behavior, for a range of the rotating disk boundary layer flows that have been mentioned above. For example, the figure depicts the space-time development of a disturbance that was generated in a rotating disk boundary layer with Rossby number $Ro = 0.4$. Results for both the artificially homogenized version of the flow and the genuine flow are shown. The flow parameters were chosen so that, for the homogenized version of the base flow, the disturbance is at the margin of absolutely instability. In marked contrast to what might have been predicted by a naive use of local stability results, it may be very clearly seen that there is a strong form of global stability in the genuine flow. The disturbance is not even convectively unstable, although it does display transient growth.

The previously unsuspected complexity of the relationships between the local and global stability properties of linearized disturbances in rotating disk boundary layers can be expected to have important consequences for any nonlinear developments. It is conceivable that, in some instances, the character of the linearly unstable disturbance may be such as to discount the possibility of any analytical approach to describing the emergence of nonlinearity that, in the interests of tractability, tries to postpone the incorporation of the effects of the inhomogeneity of the base flow. For the case of the rotating disk boundary layer with suction, for example, destabilization due to the radial inhomogeneity may need to be accounted for right from the very outset of the analysis, irrespective of the smallness of the initial disturbance amplitude.

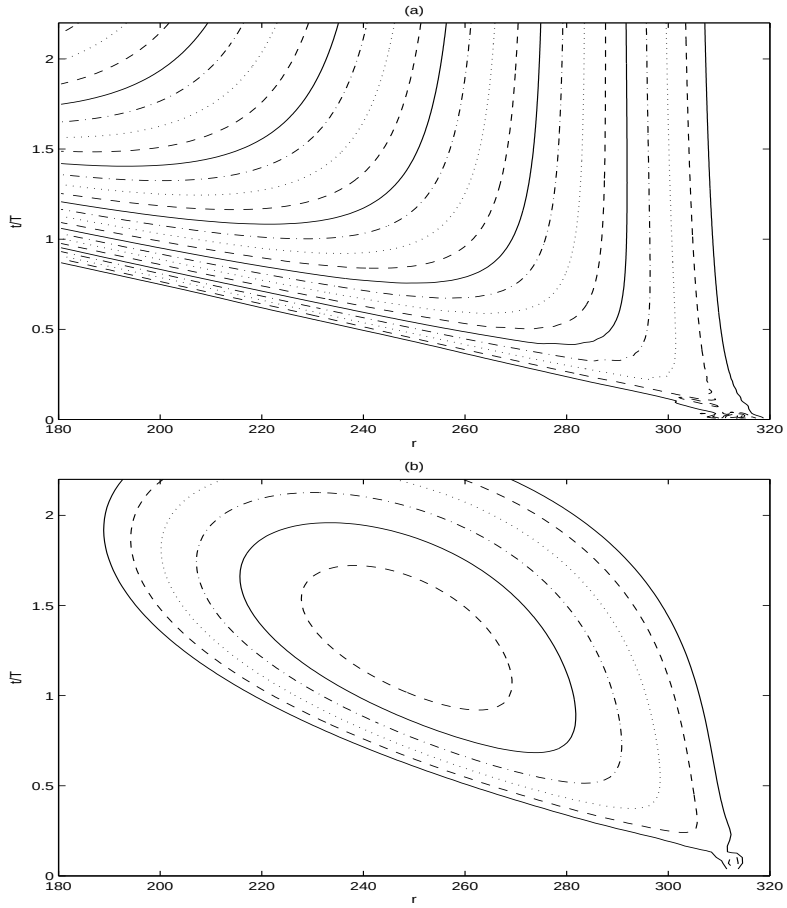

Figure 1: Spatio-temporal development of the azimuthal vorticity at the disk surface for an impulsively excited disturbance with $Ro = 0.4$ and azimuthal mode number $n = 20$. The disturbance was excited at $r_e = Re/Ro = 313$. (a) Homogenized flow with $Re = 125$ (b) Radially inhomogeneous flow.

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