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Enhancing performance in order picking processes by Dynamic Storage Systems

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Abstract

Increasing productivity and reducing labor cost in order picking processes are two major concerns for most warehouse managers. Particularly picker-to-parts order picking methods lead to low productivity as order pickers spend much of their time traveling along the aisles. To enhance order picking process performance, an increasing number of warehouses adopt the concept of *Dynamic Storage* where only those products needed for the current order batch are dynamically stored in the pick area, thereby reducing travel time. Other products are stored in a reserve area. We analyze the stability condition for a Dynamic Storage System with online order arrivals and develop a mathematical model to derive the maximum throughput a DSS can achieve and the minimum number of worker hours needed to obtain this throughput, for order picking systems with a single pick station. We discuss two applications of dynamic storage in order picking systems with multiple pick stations in series. In combination with simulation modeling, we are able to demonstrate that dynamic storage can increase throughput and reduce labor cost significantly.

Keywords: Order picking; Storage; Warehousing; AS/RS.

1. Introduction

Order Picking (OP), the process of picking products from storage locations to fill customer orders, is a crucial function in warehouses. OP is very labor intensive. According to Tompkins (2003), about 55% of all operating costs in a typical warehouse can be attributed to order picking. Achieving high throughput, that is the number of orders that can be picked by an OP system in a certain time period, with minimum worker hours (i.e., the total amount of hours used by order pickers in

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4 picking), is one of the major goals for OP systems design and operation. In a
5
6 conventional OP system, all the Stock Keeping Units (SKUs) ordered by customers
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8 during the entire day or a picking shift (which normally takes several hours) are
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10 located in the pick area with every SKU in a separate storage bin. Due to the large
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12 size of the pick area, order pickers spend much time on traveling to the pick locations,
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14 leading to low throughput and low productivity. This paper introduces the concept of
15
16 Dynamic Storage (DS). In a Dynamic Storage System (DSS), customer orders are
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18 batched in groups of B orders before they are released to the picking system. Only
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20 those products needed for the current picking batch are retrieved from a reserve area
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22 and stored in the pick area, just in time. All products are stored in bins, which we
23
24 assume to all have the same sizes. In a DSS the needed storage space of the pick area
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26 depends on the type and the number of products contained in the batch and hence it is
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28 a function of the batch size. Products ordered in two consecutive batches may differ.
29
30 Automated Storage and Retrieval (S/R) machines reshuffle the products in the pick
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32 area before the picking process for a batch of orders starts. Depending on the products
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34 ordered by the current and the previous batches, products in the pick area need to be
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36 swapped, appended, or condensed (see section 2.3 for a detailed explanation).

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38 The major advantage of such a DSS over a conventional OP system is the higher
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40 throughput that can be achieved since only a small fraction of the SKUs are stored in
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42 the pick area, which reduces the order pickers' travel time. Also the number of worker
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44 hours needed for picking reduces compared to conventional OP systems due to the
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46 smaller pick area. Between two pick batches the storage and reshuffle machines need
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48 some time to reshuffle the SKUs in the pick area. Depending on the time and
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50 frequency of this process (for example once per shift, or even more frequent) order
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52 pickers may be assigned to do other warehousing activities. However, this is only
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54 possible if the reshuffle time is sufficiently long for order pickers to carry out other
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56 warehousing activities. This can be realized by choosing the appropriate batch size,
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58 which is discussed in section 2.6. Additionally, since the reshuffle work is done by
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60 S/R machines, a DSS can eliminate potential worker congestion in the picking aisles
caused by manual replenishment. DS has become very popular in Europe with its high

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4 labor cost. All major warehousing solution providers now sell such systems. Figure 1
5 shows one of the earliest DSS implementations with some pick stations at the
6 warehouse of *Nedac Sorbo*, a large non-food store merchandiser in the Netherlands.
7
8 The bulk storage area is situated behind the pick area. S/R machines are used to
9 automatically replenish and reshuffle the products in the pick area.
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14 < Insert Figure 1 here >
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16 We could not find any literature on dynamic storage. De Koster et al. (2007) point out
17 that research on DS is a virgin area and mention several research topics related to it.
18 The most related topic is the forward-reserve allocation problem, which basically
19 discusses the separation of the bulk stock (reserve area) from the pick stock (forward
20 area). Hackman and Rosenblatt (1990) develop a model to decide which products
21 should be assigned to the pick area and how much space must be allocated to each of
22 the products given a fixed storage capacity of the forward area, with an objective to
23 minimize the total costs for order picking and replenishment. Frazelle et al. (1994)
24 extend the problem and the solution method of Hackman and Platzman (1990) by
25 treating the size of the forward area as a decision variable. Van den Berg et al. (1998)
26 consider a warehouse with busy and idle periods where reserve-picking is also
27 allowed. Assuming unit-load replenishments, they develop a knapsack-based heuristic
28 to find an allocation of products to the forward area that minimizes the expected total
29 labor time related to order picking and replenishment during a busy period.
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44 This paper differs from the above literature in both the model and the objectives. One
45 of the advantages of DS is that all the picks are carried out from the pick area
46 (forward area), therefore, our model does not consider direct picking from the bulk
47 area. We assume items will not be depleted during the pick cycle. Our objectives are
48 to compare the maximum throughput a DSS can achieve and the associated labor time
49 needed to a conventional order picking system. Although the number of DSS
50 implementations increases rapidly, this paper is the first to model and analyze it.
51
52 Through mathematical and simulation modeling, we illustrate that a DSS can
53 simultaneously improve order picking throughput and reduce the number of picking
54 worker hours by comparing its performance with a conventional order picking system,
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4 where all the products are stored in the pick area. Of course the performance
5 improvement is at the expense of an automated reshuffle system. Our model can
6 therefore be used to evaluate the justification of such an investment. The paper is
7 organized as follows. In section 2, we develop an approximation model to analyze the
8 performance of a DSS with a single station. We compare the results with simulation
9 and show the model is accurate. In section 3, we discuss two applications of DSS in
10 order picking systems with multiple pick stations. We draw conclusions in section 4.
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18 **2. Performance of a DSS with a single pick station**

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22 In this section, we illustrate the throughput improvement and worker hour reduction
23 brought by a DSS for order picking with a single pick station. The layout of the
24 station is illustrated in Figure 2. The products are stored in identical bins and are
25 located along the picking rack. To simplify the discussion, we assume each SKU is
26 equally likely to be ordered. Hence products are stored randomly (or uniformly) in the
27 pick area. Random storage has the advantage of high space utilization and has been
28 used by many warehousing researchers (see De Koster et al., 2007). In a DSS also
29 other storage policies might be applied, for example, class-based storage or cubed-
30 per-order-index (COI) storage. In view of the compact size of the forward storage area
31 of a DSS the performance advantage of such strategies will be slight as compared to
32 random storage.
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45 The bulk storage area is located behind the picking face, where a S/R machine is used
46 to replenish the product bins to the pick area. We also assume random storage in the
47 bulk area and the machine Pickup/Deposit (P/D) station is located at the lower left-
48 hand corner of the rack. Without loss of generality, the picker's home base is located
49 at one end of the pick area. The picker travels along the picking rack to the product
50 locations, picks the required quantity and then returns to his home base. We assume
51 all the required products in an order are picked in one tour, and an order picker picks
52 one order per picking tour. It is possible to consider the case that the picker's home
53 base is located in the middle of the pick area. In such a case, the picker first travels
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4 one side of the home base to pick the products, then turns round, travels to the other
5 side to pick the products, and then returns to the home base. The calculation of travel
6 time will be different in this case, the further analysis remains the same.
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9
10 < Insert Figure 2 here >

11 We also assume orders arrive online, which means the order information is only
12 available upon arrival at the order picking process. We first discuss the stability
13 condition for the DSS, and then develop a mathematical model to find the maximum
14 throughput the DSS can achieve and on top of that, the optimal batch size B to
15 minimize the total number of worker hours needed to pick a certain number of orders.
16 The throughput improvement and the saving on worker hours are compared to a
17 benchmark system, where all the products in the warehouse are located in the pick
18 area.
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29 **2.1. Stability conditions for a DSS**

30 We suppose customer orders arrive at the warehouse according to a Poisson process at
31 a rate λ . They are batched in first-come-first-served sequence in groups of size B , and
32 then released simultaneously to the picking system. Figure 3 illustrates the relation
33 between batch-forming times, order service times (travel time + picking time),
34 reshuffle times, and the potential waiting times to start reshuffling (since the S/R
35 machine can only start to reshuffle products when the whole batch of orders is
36 available to the DSS. This explains waiting time W_i in Figure 3.). Reshuffling and
37 order picking occur in alternating sequence at the station. To obtain a stable system,
38 we require the mean cycle time (reshuffle time plus service time of a batch of orders)
39 for processing a batch of orders to be smaller than the mean batch-forming time. Our
40 main objective is to determine the maximum throughput the DSS can achieve; on top
41 of that, we are going to find the optimal batch size to minimize the total number of
42 worker hours needed to pick a certain number of orders.
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56 <Insert Figure 3 here>

57 **2.2. Mathematical formulation**

58 The parameters and notations used in the formulation are as follows:
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4 O_n : probability that an order contains n order lines. $n \leq N$.

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7 N : the maximum order size.

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9 M : the total number of products in the warehouse.

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11 λ : customer order arrival rate to the DSS.

12
13 d : number of servers at the pick station.

14 Variables:

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16 B : batch size.

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19 Z : number of products needing to be reshuffled between 2 batches each
20 consisting of B orders.

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23 R : reshuffle time between 2 batches each consisting of B orders.

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25 \bar{Y} : the number of lines in an order.

26
27 Y : the number of lines in a batch of B orders.

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29 Y_i : the number of lines in the i^{th} batch.

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32 $X_m = \begin{cases} 1, & \text{if product } m \text{ should be stored in the picking area.} \\ 0, & \text{otherwise.} \end{cases}$

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36 $X_{i,m} = \begin{cases} 1, & \text{if product } m \text{ is ordered by the } i^{\text{th}} \text{ batch.} \\ 0, & \text{otherwise.} \end{cases}$

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40 se : the service time of an order in the DSS.

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42 S : service time of a batch of orders in the DSS.

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44 The maximum throughput a DSS can achieve depends on the batching strategy. If
45 batches are large, many products need to be stored in the picking area, increasing
46 picking times per order, and the benefits of using a DSS will be slight. If batch sizes
47 are too small, too much time is lost in replenishment and a certain required throughput
48 may become infeasible. In this paper we take a design approach: the batch size is a
49 decision variable that can be used to maximize the possible throughput of a system.
50
51 Maximizing the throughput capabilities of a station may eliminate the need for
52 multiple parallel stations or may allow to handle peak loads. The maximum
53 throughput achieved by a DSS during a certain time period is proportional to the
54 customer order arrival rate λ . Equivalently, we have to minimize the inter-order
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arrival time $1/\lambda$. We therefore need to solve the following model,

$$\min \quad 1/\lambda \quad (1)$$

$$st: \quad E[R] + E[S] < B * \frac{1}{\lambda} \quad (2)$$

where equation (2) is the stability condition. Both λ and B are decision variables. In order to solve the model, we need to obtain the expression of the mean reshuffle time between two batches, $E[R]$, and the mean service time of a batch of B orders, $E[S]$, both of which are functions of the batch size B . We derive $E[R]$ in section 2.3 and $E[S]$ in section 2.4. The model solution procedure is explained in section 2.5.

2.3. Mean reshuffle time between two batches

To obtain the expression for $E[R]$, we first derive the expression of the expected number of products stored in the pick area for a batch of B orders at the start of the reshuffle process, $E[K]$, then we analyze the expected number of products needing to be reshuffled between two consecutive batches, $E[Z]$.

We can calculate the expected number of order lines in an order $E[\bar{Y}]$ by

$$E[\bar{Y}] = \sum_{n=1}^N n * O_n \quad (3)$$

The expected number of lines in a batch of B orders can be obtained by

$$E[Y] = B * E[\bar{Y}] \quad (4)$$

since orders are independent of each other.

The expected value of X_m , i.e., the probability that product m should be stored in the pick area for an order batch of B orders, is calculated as

$$E[X_m] = E[E[X_m | Y]] = E[(1 - (1 - \frac{1}{M})^Y)] \quad (5)$$

where $(1 - \frac{1}{M})^Y$ is the probability that product m is not ordered in a batch of Y order lines.

To calculate $E[X_m]$, we need to calculate the probability distribution of Y , which is a B -fold convolution of \bar{Y} . It is often more convenient to use the moment generating function of Y ,

$$\phi(t) = E[e^{tY}] \quad (6)$$

Where $\phi(t)$ is a function of batch size and the order profile O_n . As an example, if the number of lines in an order follows a shifted Poisson distribution of $1 + \text{Poisson}(a)$, then $Y = B + \text{Poisson}(a * B)$ in distribution and $\phi(t) = e^{tB} * e^{aB(e^t - 1)}$.

We let

$$w = \log\left(1 - \frac{1}{M}\right) \quad (7)$$

and put it into equation (5). We then have

$$E[X_m] = E[1 - e^{-wY}] = 1 - \phi(w) \quad (8)$$

The expected number of products stored in the pick area is

$$E[K] = \sum_{m=1}^M E[X_m] = M * [1 - \phi(w)] \quad (9)$$

We suppose the picking rack is h layers high, and each storage bin is l meters long.

The length of the pick area can then be approximated as

$$L = l * E[K] / h \quad (10)$$

Next we calculate the expected number of products needing to be reshuffled between two batches of orders, $E[Z]$.

We distinguish 3 cases based on the relationship between the number of products

stored in the i^{th} batch, $\sum_{m=1}^M X_{i,m}$, and the $i+1^{\text{st}}$ batch, $\sum_{m=1}^M X_{i+1,m}$.

$$\text{Case 1: } \sum_{m=1}^M X_{i,m} = \sum_{m=1}^M X_{i+1,m}$$

The two batches have the same storage space in this case. Figure 4 illustrates the locations taken by the products in the i^{th} batch, the products ordered by the $i+1^{\text{st}}$ batch,

the products needing to be reshuffled and the final product locations in the $i+1^{st}$ batch after reshuffling. Products ordered by both batches (4 and 6 in Figure 4, illustrated in bold print) remain at their positions. Products ordered by the i^{th} batch but not by the $i+1^{st}$ batch (products 1, 2, 3, and 5 in Figure 4) are swapped with those products ordered by the $i+1^{st}$ batch but not by the i^{th} batch (products 7, 8, 10, and 9 in Figure 4).

In this case, $\sum_{m=1}^M X_{i,m}(1-X_{i+1,m})$, the number of products ordered by the i^{th} batch but not by the $i+1^{st}$ batch, equals $\sum_{m=1}^M X_{i+1,m}(1-X_{i,m})$, the number of products ordered by the $i+1^{st}$ batch but not by the i^{th} batch. Therefore, the number of products needing to be reshuffled between 2 batches is $\sum_{m=1}^M X_{i+1,m}(1-X_{i,m})$.

< Insert Figure 4 here >

$$\text{Case 2: } \sum_{m=1}^M X_{i,m} < \sum_{m=1}^M X_{i+1,m}$$

The i^{th} batch has smaller storage space than the $i+1^{st}$ batch and $\sum_{m=1}^M X_{i+1,m}(1-X_{i,m})$ is larger than $\sum_{m=1}^M X_{i,m}(1-X_{i+1,m})$ in this case. Except for the $\sum_{m=1}^M X_{i,m}(1-X_{i+1,m})$ products

needing swapping ($1 \leftrightarrow 9$, and $5 \leftrightarrow 10$ in Figure 5), $\sum_{m=1}^M X_{i+1,m}(1-X_{i,m}) - \sum_{m=1}^M X_{i,m}(1-X_{i+1,m})$ products (products 12 and 7 in Figure 5) need to be appended to the

pick area. The number of products needing to be reshuffled in this case is also

$$\sum_{m=1}^M X_{i+1,m}(1-X_{i,m}).$$

< Insert Figure 5 here >

$$\text{Case 3: } \sum_{m=1}^M X_{i,m} > \sum_{m=1}^M X_{i+1,m}$$

As illustrated in Figure 6, in this case, the storage space in the i^{th} batch is larger than

the storage space in the $i+1^{st}$ batch. $\sum_{m=1}^M X_{i,m}(1-X_{i+1,m})$ is larger than

$\sum_{m=1}^M X_{i+1,m}(1-X_{i,m})$. $\sum_{m=1}^M X_{i+1,m}(1-X_{i,m})$ products need to be swapped between the two

batches. To reduce the travel distance for order pickers in picking the $i+1^{st}$ batch, we need to move those products ordered by both batches but located between

the $\sum_{m=1}^M X_{i+1,m} + 1^{st}$ closest location from the depot and the $\sum_{m=1}^M X_{i,m}^{th}$ location to the first

$\sum_{m=1}^M X_{i+1,m}$ slots. We select those slots occupied by products ordered in the i^{th} batch but

not ordered in the $i+1^{st}$ batch. As illustrated in Figure 6, we need to switch product 10 with product 7. This process is called *condensing*. The products ordered in the i^{th}

batch but not in the $i+1^{st}$ batch and located outside the closest $\sum_{m=1}^M X_{i+1,m}$ locations from

the depot (product 8 in Figure 6) and the products being swapped out of the pick area for the $i+1^{st}$ batch (product 7 in Figure 6) will be moved to the reserve area. Moving

times of these products are not included in the reshuffle time as they can be carried out by the S/R machines parallel to the order picking process. Therefore, in this case,

the number of products needing to be reshuffled before the start of the picking process

is $\sum_{m=1}^M X_{i+1,m}(1-X_{i,m}) + CD$, where CD is the number of products needing to be

condensed between two batches. In the Appendix, we compare the expected value of

CD and the expected value of $\sum_{m=1}^M X_{i+1,m}(1-X_{i,m})$ to show that the impact of CD is

negligible for practical parameter settings.

< Insert Figure 6 here >

In conclusion, in all these cases, the number of products needing to be reshuffled between the i^{th} batch and the $i+1^{st}$ batch can be approximated by the number of products ordered in the $i+1^{st}$ batch, but not in the i^{th} batch, i.e.,

$$Z = \sum_{m=1}^M [X_{i+1,m} * (1 - X_{i,m})] \quad (11)$$

Using conditional probability and combining equations (6) and (7), the expected number of reshuffle products between two batches is calculated as

$$\begin{aligned} E[Z] &= M * E[E[X_{i+1,m} | Y_{i+1}]] * E[E[(1 - X_{i,m}) | Y_i]] \\ &= M * E[1 - (1 - \frac{1}{M})^{Y_{i+1}}] * E[(1 - \frac{1}{M})^{Y_i}] \\ &= M * (1 - E[e^{-wY_{i+1}}]) * E[e^{-wY_i}] \\ &= M * [1 - \phi(w)] * \phi(w) \end{aligned} \quad (12)$$

where Y_{i+1} and Y_i are the number of order lines in batches $i+1$ and i respectively.

The mean reshuffle time between two batches is approximated as:

$$E[R] = rs * \lceil E[Z] \rceil \quad (13)$$

where $\lceil * \rceil$ means rounding up to the nearest integer value, and rs is the reshuffle time for one product.

Before analyzing rs , we first need to understand the movement of the S/R machines in a reshuffle process. As we discussed above, a reshuffle process may be a swap process, an appending process, or a condensing process. Since the Appendix shows the impact of condensing can be neglected, we focus on the swap and the appending process. In a swap process, the S/R machine first travels to the location of the product needing to be removed from the pick area, picks the product bin, travels to an empty location in the reserve area, stores the product bin, travels to the location in the reserve area of the product needing to be stored in the pick area, retrieves the product bin, travels to the previously emptied storage slot in the pick area, and stores it there. In an appending process, the S/R machine travels to the location in the reserve area of the product needing to be stored in the pick area, picks the products bin, travels to the end of the pick area, and drops the bin into the first empty slot next to the end of the pick area.

For swapping bins, the S/R machine carries out double cycles. For appending bins, it carries out single cycles. In both cases, we approximate the cycle time of the S/R

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4 machine by formula 10.74 for dual command cycles of Tompkins et al. (2003), which
5 is a worst case approximation in the case of appending bins. The formula is listed
6 below:
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$$9 \quad T_{DC} = E(DC) + 4T_{P/D} \quad (14)$$

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12 where, T_{DC} is the expected dual command cycle time; $E(DC)$ is the expected travel
13 time from the storage location to the retrieval location during a dual command cycle;
14 and $T_{P/D}$ is the time required to either pickup or deposit the load. $E(DC)$ is
15 calculated by,
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17

$$18 \quad E(DC) = \frac{T}{30} [40 + 15Q^2 - Q^3] \quad (15)$$

19
20
21 where T (the scale factor) designates the longer (in time) side of the rack, i.e., the
22 maximum of the travel time required to travel horizontally from the machine's P/D
23 station to the furthest location in the bulk storage area and the travel time required to
24 travel vertically from the P/D station to the furthest location in the bulk storage area.
25
26 Q (the shape factor) designates the ratio of the shorter (in time) side to the longer (in
27 time) side of the rack.
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31 We will show in the following sections that even under this assumption, a DSS can
32 lead to significant improvements on order throughput and worker time needed.
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35 **2.4. Mean service time for a batch of B orders**

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37 To analyze the mean service time to pick a batch of B orders, $E[S]$, we first derive
38 the expression for $E[se]$, the mean service time to pick an order. $E[se]$ is the
39 summation of the mean travel time and the mean picking time, which is proportional
40 to the number of lines to be picked.
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44 The mean travel time to pick an order, $E[tr]$, is calculated as,
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$$E[tr] = (2 * \sum_{n=1}^N L * (\frac{n}{1+n}) * O_n) / v \quad (16)$$

Where, L is obtained from equation (10), N is the maximum number of lines contained in an order, and v is the picker's travel speed. $L * \frac{n}{1+n}$ is the expected travel distance given that n lines are picked and all the products are uniformly located in the interval $[0, L]$.

The mean service time of an order at the pick station is,

$$E[se] = E[tr] + E[\bar{Y}] * pk \quad (17)$$

where $E[\bar{Y}]$ is obtained from equation (3) and pk is the picking time for an order line (constant).

The mean service time of a batch of B orders can now be approximated as

$$E[S] = \lceil B/d \rceil * E[se] \quad B > d \quad (18)$$

where d is the number of workers in the pick station.

When the batch size is not larger than d , the mean service time of a batch of B orders is modified to

$$E[S] = E[\max(se | B)] \quad B \leq d \quad (19)$$

which is the expected value of the largest service time of an order among B orders, because every picker picks at most one order and the batch is finished when the last picker finishes his order. We approximate it as

$$E[S] = E[\max(se | B)] \approx 2 * (\frac{E[Y_{\max}]}{1 + E[Y_{\max}]} * L) + E[Y_{\max}] * pk \quad B \leq d \quad (20)$$

where $Y_{\max} = \max\{\bar{Y}_1, \bar{Y}_2, \dots, \bar{Y}_{B-1}, \bar{Y}_B\}$ is the maximum number of lines to be picked in an order among a batch of B orders. The first term in the equation above is an approximation of the mean travel time and the second term the expected value of picking time. The cumulative distribution function (CDF) of Y_{\max} is calculated by taking the B^{th} power of the CDF of \bar{Y}_i .

$$\begin{aligned}
 F_{Y_{\max}}(y) &= P\{(\bar{Y}_1 \leq y) \cap (\bar{Y}_2 \leq y) \cap \dots \cap (\bar{Y}_{B-1} \leq y) \cap (\bar{Y}_B \leq y)\} \\
 &= \{F_{Y_i}(y)\}^B
 \end{aligned}
 \tag{21}$$

where \bar{Y}_i are the number of lines in the i^{th} order in the batch. The last equality follows from the fact that \bar{Y}_i are i.i.d random variables.

The mean value of Y_{\max} , $E[Y_{\max}]$, can be obtained from its probability mass function

$$p_{Y_{\max}}(y) = \{F_{Y_i}(y)\}^B - \{F_{Y_i}(y-1)\}^B.$$

Putting the results into equation (20), we obtain the value of $E[S]$ for $B \leq d$.

In the next subsection, we discuss the procedures to obtain the maximum throughput modeled in equation (1) and (2) and the optimal batch size to minimize the total number of worker hours required for picking a batch of B orders while achieving the maximum throughput.

2.5. Solution to obtain the maximum throughput

The value of $1/\lambda$ is the inter-arrival time between two orders with units of seconds. To solve the model expressed in equations (1) and (2), we start from $1/\lambda$ equal to 1 second, and increase this value with a step size of 1. For each value of $1/\lambda$, we determine the feasible range of B satisfying the stability condition in equation (2). The optimal value of λ , λ_{opt} , achieving the maximum throughput, is the largest value of λ , for which a feasible value of B exists. Picking a batch of B orders requires $B * E[se]$ worker hours. The expression of $E[se]$ is given by equation (17). So the optimal batch size B_{opt} to minimize the total worker hours is the minimum feasible value that B can take.

The total number of orders finished within a certain time period T in a DSS (suppose T starts with a new cycle), Fi , is approximated as

$$Fi = \left\lfloor \frac{T}{B_{opt} * \frac{1}{\lambda_{opt}}} \right\rfloor * B_{opt} + \left\lfloor \frac{\max(\Phi, 0)}{E[se]} \right\rfloor * d
 \tag{22}$$

where $\lfloor * \rfloor$ means rounding down to the nearest integer, and

$$\Phi = T - \left\lfloor \frac{T}{B_{opt} * \frac{1}{\lambda_{opt}}} \right\rfloor * B_{opt} - E[R] \quad (23)$$

The first term in equation (22) is the number of orders finished by completed batches in a time interval of length T , and the second term is an approximation of the number of orders completed in the current processing batch at time T .

2.6. Numerical validation and comparison with a benchmark system

We use an example to validate the analytical model. The data used in the example is listed in Table 1. In order to obtain the reshuffle times per trip as listed in Table 1, we assume the layout of the bulk storage rack is square-in-time (SIT), which means the S/R machine's horizontal travel time from the depot position to the furthest location in the storage aisle equals the vertical travel time from the depot position to the highest location in the storage aisle. The size of the storage slot, the moving speeds of S/R machines, and the storage and retrieval times of a product bin are listed in Table 2. With the value given in Table 1 for the total number of products in the warehouse, we can obtain the layout of the bulk storage area as listed in Table 2. As we discussed in section 2.3, the reshuffle time for a product is approximated by the average cycle time of a swap process using equation (14) and (15). The resulting reshuffle time per trip with S/R machine is listed in Table 1.

We first derive the value of λ_{opt} from the model described above. From λ_{opt} , we find the optimal batch size B_{opt} to minimize the total worker hours. For the optimal batch size B_{opt} , we calculate the average number of products stored in the pick area, the average number of products needing to be reshuffled between two batches, the average length of the pick area, the mean service time $E[S]$, the mean reshuffle time $E[R]$, for a batch of B_{opt} orders, the mean service time for an order $E[se]$, and the number of orders completed in the DSS in a time period T , Fi . These results are

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4 compared with simulations.

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6 We built a simulation model in AUTOMOD 10.0. The model takes the parameters
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8 listed in Table 1 and the values of B_{opt} and λ_{opt} obtained above from our analytical
9
10 model as inputs. To obtain the products contained in a batch of orders, we first
11
12 generate the order lines for each order in the batch according to the distribution in
13
14 Table 1. For each order line, a product is chosen randomly from all the products in the
15
16 warehouse. The products in a batch are randomly stored in the pick area. Products are
17
18 swapped, appended, or condensed based on the products ordered by two consecutive
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20 batches. We take the cycle time for a swapping, an appending, and a condensing
21
22 process from the calculation results of equation (14) and (15). To obtain the number of
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24 orders finished in a time interval T of 20 days, we use 10 runs of 20 days with 2 days
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26 initialization time for each run. These runs ensure that the 95% confidence interval of
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28 the number of orders finished in 20 days is less than 1% of the average. All the other
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30 simulation results are obtained by using 1 run of 50 days with 2 days of initialization
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32 period. The simulation time span ensures the 95% confidence interval of all the
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34 measurements below 1% of their averages. The results are listed in Table 3. We can
35
36 conclude that the proposed method is accurate enough for practical purposes.
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41 < Insert Table 1 here >

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43 < Insert Table 3 here >
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46 We next use this model to show the improvements a DSS can bring on throughput,
47
48 and on the number of worker hours to finish P orders compared with a benchmark
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50 system. The pick area of the benchmark system has similar layout as the DSS, but all
51
52 the products in the warehouse are stored at the pick area. Further layout parameters,
53
54 the picker parameters, and the order profiles used in the benchmark system are
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56 identical to the DSS and are listed in Table 1. When an order arrives at the benchmark
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58 picking system, it is processed immediately by an order picker if available. It is
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60 queued otherwise. We use AUTOMOD 10.0 to build a simulation model to obtain the

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4 performance of the benchmark system with the same orders, the same number of runs
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6 and the same initialization period for each run as used in the DSS. To obtain the
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8 maximum order arrival rate of the benchmark system, we set the order inter-arrival
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10 time (i.e., $1/\lambda$) at 1 second, and then increase it with a step size of 1 second, until the
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12 system gets stable (i.e., the utilizations of order pickers are less than, but close to 1).
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14 The largest value of λ making the system stable is the maximum order arrival rate
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16 that can be handled by the benchmark system. The results are compared with the
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18 performance of the DSS using the formulations in the previous sub-sections. The total
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20 worker hours used to pick P orders is the product of P and $E[se]$, the mean service
21
22 time for an order picker to finish an order. Since both the DSS and the benchmark
23
24 system will pick the same number of orders P , it suffices to compare the values of
25
26 $E[se]$ in the comparison. The comparison results are listed in Table 3. We find the
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28 maximum throughput of the DSS to be substantially larger than the benchmark system.
29
30 Also, the number of worker hours needed for picking reduces significantly in the DSS.
31
32 The improvement on throughput and the saving on worker hours are due to the shorter
33
34 travel distance (time) in picking tours since we only store the products needed for the
35
36 current batch of orders in the forward pick area.
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39 We find from Table 3 that with the optimal batch size, B_{opt} , the average reshuffle time
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41 between two batches with two reshuffle machines is around 576 seconds, during
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43 which time period it might not be possible to assign order pickers to do other
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45 warehousing activities apart from simple cleaning work. In practice, we can choose a
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47 larger batch size which still meets the stability condition to make the reshuffle time
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49 long enough for pickers to do other warehousing activities. As an example of this case,
50
51 the system stability condition still holds when the batch size is 70. The reshuffle time
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53 between two batches is then 1040 seconds (around 17 minutes) which is sufficient for
54
55 other warehousing activities. The saving on worker hours in picking is still 75.4%.
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59 **3. The application of DSS to OP systems with multiple stations**

60 We analyzed the application of a DSS and its performance for an order picking system

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3
4 consisting of a single station. In practice, an order picking system generally consists
5
6 of multiple pick stations with buffers in front of them. In a conventional multi-station
7
8 OP system, pick stations are connected by conveyors. All the products in the
9
10 warehouse are distributed evenly over the racks at the stations. Within a station,
11
12 storage is random. A customer order is assigned to an order bin when it arrives at the
13
14 warehouse. As illustrated in Figure 7, order bins are released to the conveyor system
15
16 and will be diverted to a pick station if items need to be picked there and if the buffer
17
18 at that station is not full. Each station has one or more order pickers. Picked order bins
19
20 will be pushed back to the main conveyor system and travel downstream. Order bins
21
22 that are not able to enter a station due to a full buffer will cycle in the system until
23
24 room is available at the station's buffer.

25
26 < Insert Figure 7 here >
27

28 Dynamic storage has been implemented by several warehouses. In most applications
29
30 we studied, products are reshuffled only once per shift with fairly large batch sizes. In
31
32 such systems, the advantages of DS have not been explored fully since the pick area is
33
34 still large. In the following sub-sections, we propose two alternatives for DS in a
35
36 multi-station OP system and compare their performances with the conventional OP
37
38 system described above. The analytical model developed in section 2 can be used to
39
40 analyze the performance in the first alternative. For the second alternative, we resort
41
42 to simulation.

43 44 **3.1. Application of DSS in multi-station OP systems- alternative 1**

45 In this alternative, the order batch size is a multiple of the number of pick stations.
46
47 Orders are assigned to stations sequentially before they are released to the OP system
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49 and each station therefore has the same number of orders to pick for a batch of orders.
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51 As an example, we suppose that there are 2 stations and the batch size is 6. We assign
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53 order 1, 3, and 5 to station 1 and order 2, 4, and 6 to stations 2. Each station has its
54
55 own S/R machine. We assume each S/R machine can retrieve every product and a
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57 product can be assigned to multiple stations. At each station, we just in time store
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59 those products needed for the assigned orders in the pick area and use an S/R machine
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4 to reshuffle products between batches. This implies each order visits only one pick
5 station. Since the numbers of products in orders are identical, independent random
6 variables, the operations at stations are identical. We can therefore analyze each
7 station independently using the methods described in the previous section. The
8 throughput of the whole system equals approximately the throughput of a single
9 station multiplied by the number of pick stations. Figure 8 shows the layout of this OP
10 system and the assignment of orders to stations with an OP system of 5 stations.
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17
18 **< Insert Figure 8 here >**
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20 We use an example to compare the performance of the DSS to a benchmark
21 conventional OP system as we discussed at the beginning of this section. The data
22 used by the DSS and the benchmark system is listed in Table 4 and Figure 7. Again,
23 we assume the bulk storage area is SIT and has random storage. The reshuffle
24 machines work the same way as we described in previous sections at stations. We use
25 similar methods as in section 2.6 to obtain the reshuffle time per trip. The results are
26 shown in Table 4. The performance of the benchmark system is obtained from
27 simulation models built in AUTOMOD. We first use 1 run of 50 days with 2 days
28 initialization time to obtain the maximum order arrival rate of the benchmark system
29 using the method described in section 2.6. We then input this value into the simulation
30 model to obtain the number of orders finished in a time interval of length T and the
31 number of worker hours used to finish a certain number of orders. We use 10 runs of
32 20 days for the simulation to ensure that the 95% confidence interval of these two
33 measurements are less than 1% of the average values. For each experiment, we use 2
34 days initialization time.
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50 **< Insert Table 4 here >**
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52 The comparison between the two systems is shown in Table 5. We note that all the
53 results for the DSS in Table 5 are obtained from the mathematical models developed
54 in the previous sections. Comparing to the performance of the benchmark system, the
55 improvement on throughput and the saving on the number of worker hours in the DSS
56 is substantial. The saving on worker hours in picking in Table 5 is obtained from
57 using the optimal batch sizes. In practice, to allow order pickers to have enough time
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4 to handle other warehousing activities during the reshuffle time period between two
5
6 batches, we can choose larger batch sizes while not violating the stability conditions.
7

8 < Insert Table 5 here >
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10 3.2. Application of DSS in multi-station OP systems- alternative 2

11 The application of DSS in this alternative has similar system layout as the
12 conventional benchmark OP systems illustrated in Figure 7. To make it work properly,
13 its operating rules are defined as follows.
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- 18 1. The products in the bulk storage are divided equally over pick stations.
19 Products stored at a station at a specific moment can only be chosen from the
20 station's bulk storage assortment.
21
22
- 23 2. A product can only be stored at one station, i.e., product splitting is not
24 allowed.
25
26
- 27 3. The products stored at a station at the start of an order batch depend on the
28 products ordered in the batch and the station's assortment.
29
30
- 31 4. Each station has its own S/R machine to reshuffle products between batches.
32
33
- 34 5. A next order batch is released to the system for picking when picking of the
35 previous batch has finished at all stations. No cross-batch picking is allowed in
36 the system.
37
38
- 39 6. Reshuffling at stations is not synchronized. Each station starts reshuffling after
40 a batch of orders has formed and the picking process for the previous batch
41 has finished at the station.
42
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- 44 7. At each pick station, reshuffling and picking are carried out sequentially.
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48 To have a stable system in this multiple station case, we require that the mean cycle
49 time (the time period from the batch of orders entering the system until the picking for
50 this batch has finished at all stations) of a batch of orders in the system is smaller than
51 the mean batch forming time. Our objective is to find the maximum order arrival rate
52 that the system can handle and the minimum batch size to achieve it.
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56 Some difficulties exist in developing a mathematical model to analyze the DSS
57 performance in this multiple station case. First, the worker hours used to pick a batch
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of orders contain not only the picking time and the travel time, but also the potential waiting time between two orders arriving at a station, which is difficult to derive. Second, because we do not synchronize reshuffles between stations, reshuffling and picking can be carried out simultaneously at different stations in the system; hence, it is difficult to develop the cycle time for a batch of orders in a closed mathematical form.

We therefore use simulation to analyze the performance and compare it with the performance of the conventional benchmark system, as we described at the beginning of this section with parameters listed in Table 4 and Figure 7. The DSS has the same parameters as the benchmark system and the stations' assortments are identical to that of the benchmark system. Since products are evenly distributed over stations and the products stored at a station at a specific moment can only be chosen from the station's assortment, the average reshuffle time per trip will take the values listed in Table 1. The simulation model and the results for the benchmark system have already been discussed in section 3.1. For the simulation model of the DSS, we first use 1 run of 50 days with 2 days initialization time to obtain the maximum order arrival rate, λ_{opt} , that the DSS can achieve and the optimal batch size, B_{opt} , to minimize the worker hours while achieving the maximum order arrival rate using the similar methods as we described in section 2.5. We then input λ_{opt} , and B_{opt} into the simulation model, and use 10 runs with 2 days initialization period to obtain the maximum throughput in a certain time period T and the total number of worker hours used to finish a certain number of orders. The mean values of these two measurements can be found in Table 6. The 95% confidence interval of these two measurements are less than 1% of the average values.

< Insert Table 6 here >

Comparing the results between Table 5 and Table 6, alternative 2 yields a higher throughput than alternative 1 with the current settings. This might be attributed to the relatively smaller percentage of the reshuffle time in the total cycle time in alternative 2. In general, the preference of one alternative over the other depends on the settings.

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4 Alternative 1 may be cheaper in investment as it needs fewer conveyors. All the
5 orders visit only one station. The operation at a station is independent of the other
6 stations. A disadvantage of this alternative is the relatively larger percentage of
7 reshuffle times compared with alternative 2 since all the products can be stored in the
8 bulk storage area behind the pick station. Although it is difficult to quantify
9 differences analytically, the existence of alternatives provides warehouse managers a
10 choice to select the appropriate implementation according to their working situation.
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19 **4. Conclusions and future research**

20
21 Improving order throughput and saving labor cost in order picking processes are the
22 two major concerns for warehouse managers. This paper discusses the concept of
23 *Dynamic Storage*, which can improve order throughput and reduce labor cost
24 simultaneously due to shorter travel in picking tours, with the aid of S/R machines. In
25 a DSS, orders are released to the system in batches and only those products for the
26 current picking batch are stored in the pick area. Product reshuffles between batches
27 are done by S/R machines. For a single-station order picking system, we derive a
28 mathematical model to obtain the maximum throughput a DSS can achieve and, on
29 top of that, find the optimal batch size to minimize the required order picking worker
30 hours. The application of DS to multi-station order picking systems is discussed
31 through two alternatives. For both order picking systems, the performance of a DSS is
32 compared with conventional order picking where all the products are stored in the
33 pick area. Through our mathematical and simulation models, we are able to
34 demonstrate that a DSS can substantially improve throughput and reduce labor cost at
35 the same time. Our results confirm and quantify the advantage of these systems over
36 conventional picking systems and explain why so many companies are switching to
37 dynamic storage to enhance performance.
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55 The model lends itself for improvements and further extensions. We performed a
56 worst case analysis by using the dual command cycle time of a swap process as the
57 reshuffle time for a product and treat it as a constant. The approximation would
58 probably be more accurate if we could find the distribution of the reshuffle time and
59
60

input the mean value into the model for analysis.

We assume reshuffling and picking are in sequence at a pick station. This assumption is strong since in reality, picking can start in parallel with the reshuffling process for those orders, whose required products have already been reshuffled into the pick area. The maximum throughput that can be achieved by a DSS in our analysis is therefore underestimated. It would be interesting to find out how much improvement can be achieved by relaxing this assumption. Unfortunately, exact analysis of such an operation is fairly difficult.

In many warehouses, order arrivals are not completely online, which indicates the possibility of sequencing and batching orders in a better way such that the mean reshuffle time for batches and the length of the pick area can be reduced so as to achieve higher throughput for a DSS. De Koster and Yu (2008) present a case study on the impact on makespan of product and order reassignments. This however, requires a fairly different approach and leaves an interesting topic for future research.

Appendix

In this appendix, we compare the expected value of CD , the number of products needing to be condensed in case 3, and the expected value of $\sum_{m=1}^M X_{i+1,m} (1 - X_{i,m})$, the number of products needing to be swapped, to show that the impact of CD is negligible.

CD is a discrete random variable expressed in distribution as,

$$CD = \left(\sum_{m=1}^M X_{i,m} * X_{i+1,m} \right) * \frac{\sum_{m=1}^M X_{i,m} - \sum_{m=1}^M X_{i+1,m}}{\sum_{m=1}^M X_{i,m}} \left| \sum_{m=1}^M X_{i,m} > \sum_{m=1}^M X_{i+1,m} \right. \quad (A.1)$$

Where $\left(\sum_{m=1}^M X_{i,m} * X_{i+1,m} \right)$ is the number of products ordered by both batches and

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$\frac{\sum_{m=1}^M X_{i,m} - \sum_{m=1}^M X_{i+1,m}}{\sum_{m=1}^M X_{i,m}}$ is the probability that a product ordered by both batches is located

between the $\sum_{m=1}^M X_{i+1,m} + 1^{st}$ closest location from the depot and the $\sum_{m=1}^M X_{i,m}^{th}$ location

in the i^{th} batch. The expected value of CD is approximated as,

$$E[CD] = E\left[\sum_{m=1}^M X_{i,m} * X_{i+1,m}\right] * \frac{E\left[\sum_{m=1}^M X_{i,m}\right] - E\left[\sum_{m=1}^M X_{i+1,m}\right]}{E\left[\sum_{m=1}^M X_{i,m}\right]} \left| \sum_{m=1}^M X_{i,m} > \sum_{m=1}^M X_{i+1,m} \right. \quad (A.2)$$

Where

$$\begin{aligned} E\left[\sum_{m=1}^M X_{i,m} * X_{i+1,m}\right] &= M * E[E[X_{i,m} | Y_i]] * E[E[X_{i+1,m} | Y_{i+1}]] \\ &= M * E\left[1 - \left(1 - \frac{1}{M}\right)^{Y_i}\right] * E\left[1 - \left(1 - \frac{1}{M}\right)^{Y_{i+1}}\right] \\ &\approx M * \left(\frac{E[Y_i]}{M} * \frac{E[Y_{i+1}]}{M}\right) \\ &= \frac{E[Y_i] * E[Y_{i+1}]}{M} \end{aligned} \quad (A.3)$$

Where Y_i and Y_{i+1} are the number of order lines in the i^{th} and the $i+1^{st}$ batch and

$Y_i > Y_{i+1}$. The approximation in equation (A.3) is obtained by approximating

$\left(1 - \frac{1}{M}\right)^{Y_i}$ to $1 - \frac{Y_i}{M}$ using Taylor Polynomial.

$$\begin{aligned} E\left[\sum_{m=1}^M X_{i,m}\right] &= M * E[E[X_{i,m} | Y_i]] = M * E\left[1 - \left(1 - \frac{1}{M}\right)^{Y_i}\right] \\ &\approx E[Y_i] \end{aligned} \quad (A.4)$$

Putting equation (A.3) and (A.4) into equation (A.2), we get

$$E[CD] \approx E[Y_{i+1}] * \frac{(E[Y_i] - E[Y_{i+1}])}{M} \quad (A.5)$$

The expected value of $\sum_{m=1}^M X_{i+1,m} (1 - X_{i,m})$ is approximated as,

$$\begin{aligned}
E\left[\sum_{m=1}^M X_{i+1,m}(1-X_{i,m})\right] &= M * E[E[X_{i+1,m} | Y_{i+1}]] * E[E[(1-X_{i,m}) | Y_i]] \\
&= M * E\left[1 - \left(1 - \frac{1}{M}\right)^{Y_{i+1}}\right] * E\left[\left(1 - \frac{1}{M}\right)^{Y_i}\right] \\
&\approx E[Y_{i+1}] * E\left(1 - \frac{Y_i}{M}\right)
\end{aligned} \tag{A.6}$$

Comparing the expected value of $E[CD]$ and $E\left[\sum_{m=1}^M X_{i+1,m}(1-X_{i,m})\right]$, we have,

$$\frac{E[CD]}{E\left[\sum_{m=1}^M X_{i+1,m}(1-X_{i,m})\right]} = \frac{E[Y_i] - E[Y_{i+1}]}{M - E[Y_i]} \tag{A.7}$$

In general, the impact of CD increases with the batch size (large batch sizes lead to large value of $E[Y_i]$, the expected number of order lines for a batch of orders) and decreases with M (total number of products in the warehouse). According to the characteristic of a DSS as discussed in the previous sections, the batch size could not be very large and the value of M is much larger than the value of $E[Y_i]$. Since the difference between $E[Y_i]$ and $E[Y_{i+1}]$ can not be too large as Y_i and Y_{i+1} have identical distribution, the quotient is much smaller than 1. The impact of CD can therefore be neglected. With parameters listed in Table 1, simulation results show $E[CD]$ accounts for less than 3% of $E\left[\sum_{m=1}^M X_{i+1,m}(1-X_{i,m})\right]$, when the batch size is less than 200. The percentage increases with the batch size and reaches around 5% when the batch size is 300 and around 10% when the batch size is 600. The simulation results therefore confirm our assumption.

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List of Tables and Figures



Figure 1: An illustration of a DSS at *Nedac Sorbo*: the S/R machines replenish the pick bins from a bulk storage area behind the walls.

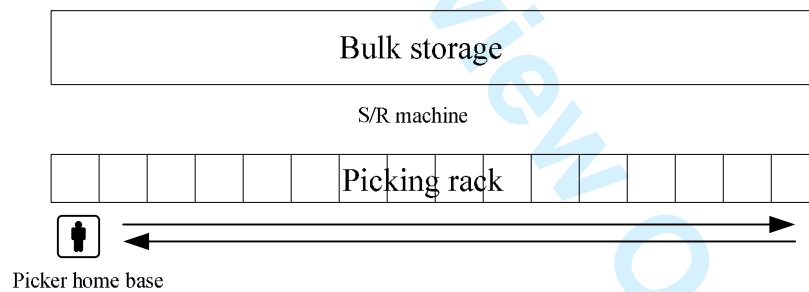
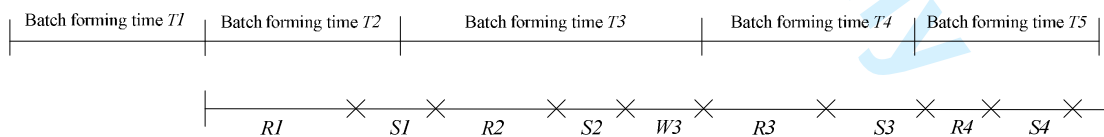


Figure 2: Layout of a pick station



T_i , batch forming time of the i th batch of orders.
 S_i , service time for the i th batch of orders.
 R_i , reshuffling time for the i th batch of orders.
 W_i , waiting time to start reshuffle the i th batch of orders

Figure 3: Illustration of stability requirements for a DSS with a single station.

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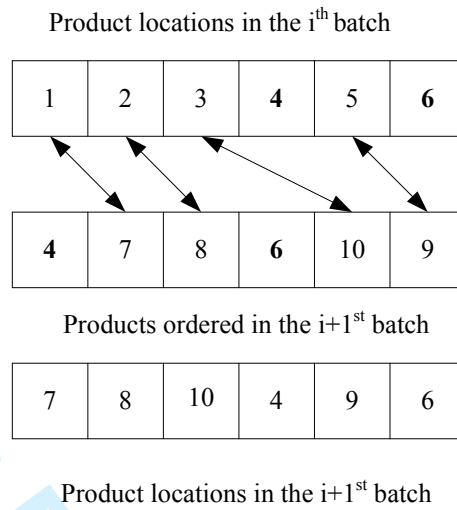


Figure 4: Reshuffle products between two batches in case 1.

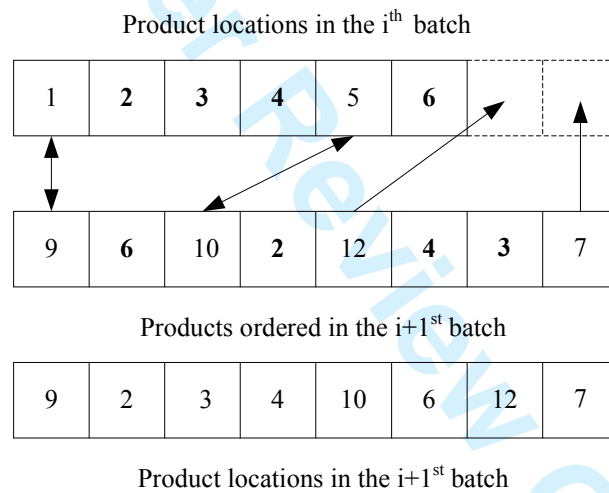


Figure 5: Reshuffle products between two batches in case 2.

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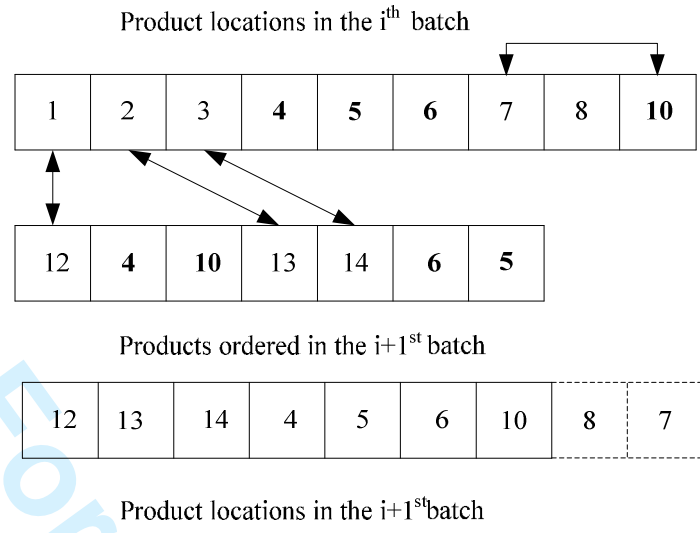


Figure 6: Reshuffle products between two batches in case 3.

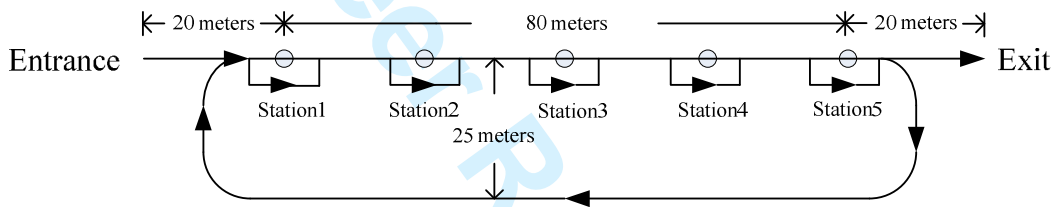


Figure 7: System layout in a conventional OP system with five pick stations.

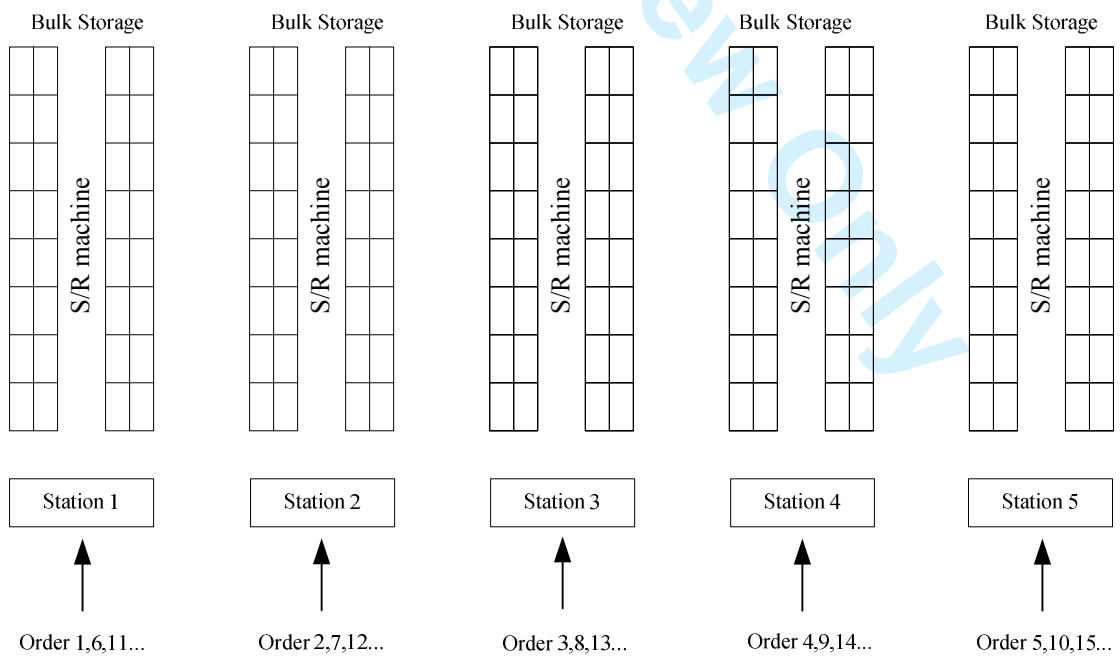


Figure 8: System layout and the order assignments in application alternative 1.

Table 1: Parameters used in the example of a single station.

Parameters	Value
Number of order lines per order	1+Poisson(1)
Total number of products in the warehouse	600
# of storage rack layers	4
Reshuffle time per trip with an S/R machine (constant)	19.2 s
# of pickers	2
Picking time per line (constant)	3 s
Picker's travel speed (constant)	1 m/s
Experiment length T	20 days

Table 2: Parameters used to obtain the reshuffle times per trip for an S/R machine.

Parameters	Value
Length of a storage slot	0.6 m
Height of a storage slot	0.4 m
Length of the bulk storage area (in slots)	40
Height of the bulk storage area (in slots)	15
Horizontal moving speed of a reshuffle machine (constant)	6 m/s
Vertical moving speed of a reshuffle machine (constant)	1.5 m/s
Time required for either retrieval or storage of the load (constant)	4 s

Table 3: Model validation and performance comparison between a DSS and the benchmark system.

	DSS			Benchmark system (fixed storage)
	anal.	sim.	rel.error	
Maximum arrival rate (orders/hour)	85.71			59.02
Optimal batch size	16			
Ave. length of pick area (m)	4.676	4.687	-0.23%	90
Ave. # of products stored in the pick area	32	31.3	2.24%	600
Ave. # of products to be reshuffled between 2 batches	30	29.6	1.35%	
# of orders finished in 20 days	41136	40974	0.40%	28452
Improvement of DSS on throughput	44.5%			
Ave. service time per order (sec)	11.91	12.16	-2.06%	120.5
savings on worker hours	90.1%			
Ave. service time per batch (sec)	95.29	100.77	-5.44%	
Ave. reshuffle time per batch (sec)	576	568.5	1.32%	

Table 4: Parameters used in the example of multiple stations.

Parameters	Values	
	DSS	Benchmark system (fixed storage)
Number of order lines per order		1+Poisson(3)
Number of stations		5
Total number of products in the warehouse		3000
Length of an order bin		0.6 m
Layers of storage rack		4
Length of a storage slot		0.6 m
Height of a storage slot		0.4 m
Length of the bulk storage area (in slots)		88
Height of the bulk storage area (in slots)		34
Moving speed of a reshuffle machine (Horizontal)		6 m/s
Moving speed of a reshuffle machine (Vertical)		1.5 m/s
# of pickers per station		1
Picking time per line		3 s
Picker travel speed		1m/s
# of products stored in a station		600
Buffer capacity between stations		8 order bins
Conveyor moving speed		1m/s
Initialization time		2 days
Reshuffle time per trip	27.84 s	

Table 5: Performance comparison between the DSS application 1 and the benchmark system

	DSS	Benchmark system (fixed storage)
Maximum order arrival rate (orders/hour)	138.46	58.06
Optimal batch size to minimize worker hours	5	
Ave. number of products stored in the picking area	3.99	600
Ave. length of the picking area (m)	0.599	90
# of orders to pick at each station for a batch of orders with optimal batch size	1	
Number of orders finished in 20 days in the system	66460	27944
Improvement of DSS on throughput	137.8%	
Ave. service time per batch (sec)	12.96	
Ave. reshuffle time per batch (sec)	111.36	
Total worker hours used to finish 10000 orders	35.91	866.6
Savings on worker hours over the benchmark system to finish 10000 orders	95.86%	

Table 6: Performance comparison between the DSS application 2 and the benchmark system

	DSS	Benchmark system (fixed storage)
Maximum order arrival rate (order/hour)	150	58.06
Optimal batch size to minimize worker hours	45	
Number of orders finished in 20 days	71690	27944
Improvement of DSS on throughput	156.6%	
Total worker hours used to finish 10000 orders	136.7	866.6
Savings on worker hours over the benchmark system to finish 10000 orders	84.2%	
Ave. # of products stored in the pick area at each station	35.0	600
Ave. storage length of pick area at each station	5.56	90
Mean batch forming time (sec)	1080	
Mean cycle time per batch (sec)	1077	
Mean reshuffle time per batch at stations (sec)	612	