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Order reduced Model of Electrical Systems Using Self-similarity

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Abstract - A new modeling approach for electrical networks is introduced in this paper based on fractal geometry and self-similarity. A particular parameter called fractal dimension is computed using a box counting method. The resulting model, based on a scaling power law, gives an accurate mathematical representation of the frequency behaviour of the network over a large range of frequencies while preserving the strong links between its physical structure and its dynamic performances.

Keywords – Self-similarity, Fractal, Frequency Modeling, Power Systems

1. INTRODUCTION

The liberalization of the electricity market and the needs of more reliable, secure and efficient power systems cause the emergence of a new paradigm: the smart grid. It can be defined by the convergence between the power and information systems infrastructures [1]. This reinforced intrication will enhance the capacity of integration and regulation of the global electric system. But, because of this strong couplings, the smart system will be also complex to design and operate. For that reason, modeling of smart grids is from today a key issue for studies of stability and assessment of new control strategies [2],[3], optimization of power flow [4] or analysis of harmonic distortion [5].

To manage complexity means to be able to describe by a simple mathematical model the global behaviour of the electricity network without writing the equations which rule the operation of each component integrated in the network. Classical approaches are based on aggregation or order reduction methods [6],[7]. For instance, a group of induction motors will be represented by a single aggregated motor instead of describing each motor operation separately. Or a complete, detailed model of a network is identified by a black-box reduced-order model. In the latter case, the mathematical structure of the model does not respect the physical structure of the network and links between model and physical parameters are lost.

The authors propose an alternative approach based on fractal theory and self-similarity. Its purpose is the development of reduced-order but knowledge-based models of power systems (ICT infrastructure is here ignored for simplicity purpose. Of course, it should be reintroduced in future works). The model

will be derived from the self-similar relations which describe the structure of an electrical network. A particular attention will be paid to the dynamic modelling of the system and more specifically links between topology of the network and its frequency response will be underlined.

2. DESCRIPTION AND MODELING OF A REFERENCE NETWORK

2.1. DESCRIPTION

A real power distribution network of 21 Bus is chosen for this study. It has been elected for its exemplarity (medium size and radial distribution network, IEEE referenced test network). The topology is given in Fig. 1. Lines and loads parameters are detailed in [8]. The network is assumed to be balanced so that only a single phase network is studied. No propagation effects are taken into account. Finally, the network is simplified by removing loads and reactance capacitors. Of course, this will limit the validity of our approach but it will better highlight the strong links between the physical structure of the network and its mathematical representation.

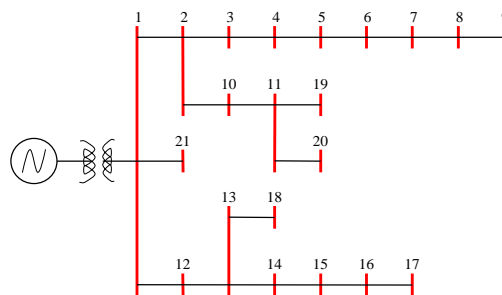


Fig.1. Topology of the 21 Bus-feeder

The frequency response of the network is given in Fig. 2. It exhibits three distinct ranges of behaviours: capacitive, resonant and inductive.

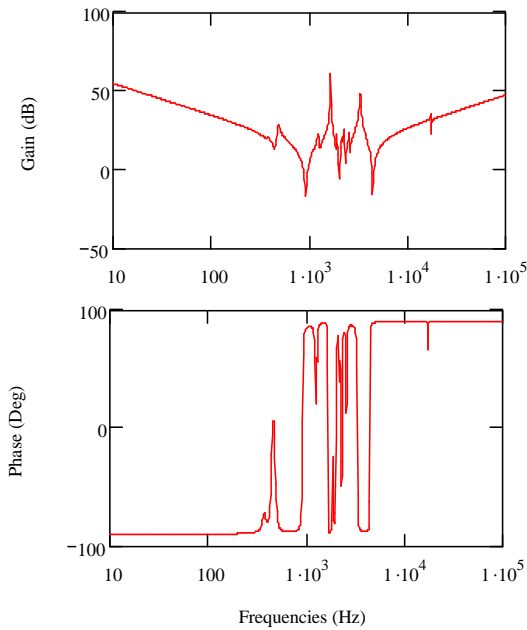


Fig.2. Frequency response of the real network

The resonant zone is bounded at high frequencies by the natural frequency f_H and at low frequencies by the natural frequency f_L . Their numerical values are: $f_H = 4529$ Hz, and $f_L = 450$ Hz. Fig. 3 shows that below f_L , the network behaves as a capacitance equal to the sum of all line capacitance. At high frequencies, the behaviour becomes inductive and depends on the inductance value of the line connected to the source [9].

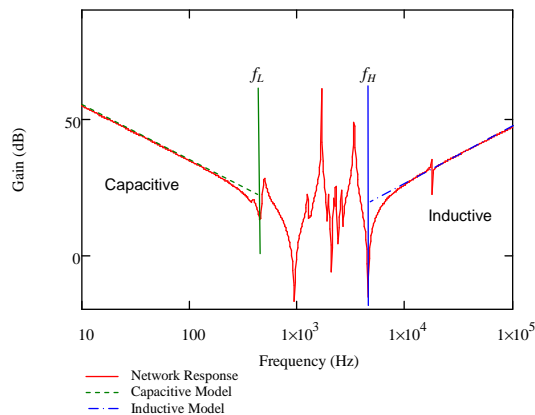


Fig.3. Asymptotic approximation outside the resonance range [9]

2.2. BLACK BOX IDENTIFICATION OF THE REFERENCE NETWORK

The reference network can be represented by an ARMA model whose parameters are identified using a black-box approach [10]. To get a good approximation, a 24-order model is required (see Fig.4).

Even if widely used by control engineers for identifying dynamic processes, this approach has

severe drawbacks. At first, it gives a black-box model which makes difficult any reverse engineering. Indeed, the model does not include any information about the topology and the effective parameters of the system. It is not possible to track into the model which characteristics of the network are responsible of a particular dynamic behaviour. Moreover, the order of the model is not “controlled” during the modelling process. It depends only of the accuracy required from the mathematical representation and it can result in high order models not suitable for analysis or design. For those reasons, knowledge-based models are by far the best way to represent electrical systems.

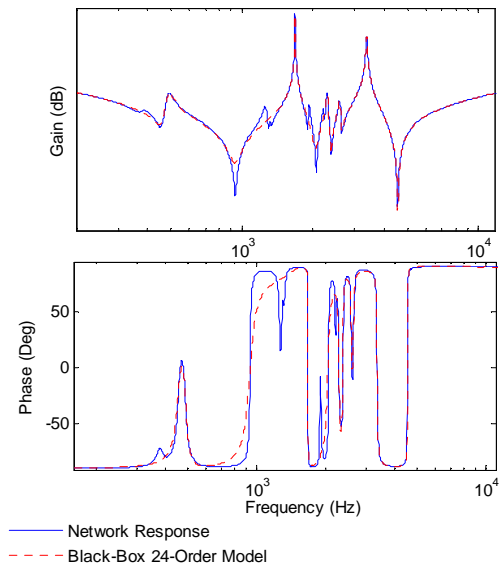


Fig.4. Black-box identification

3. TOWARD A KNOWLEDGE-BASED MODEL

3.1. INVESTIGATION OF THE INTERMEDIATE FREQUENCIES BEHAVIOUR

In order to investigate the intermediate frequencies range (very costly in terms of modelling) the damping factors of the lines are first artificially increased to kill the resonant peaks. A comparison between damped and resonant cases is given in Fig.5. It appears that the damped network can be seen as giving the average behaviour of the real system. It is worth noting that the gain of this damped network follows a power law $\omega^{-\eta}$ and its phase is constant and equal to $-\eta \cdot (\pi/2)$ rad/s. This is an abnormal behaviour, neither inductive, nor capacitive or resistive. This is typical of a dynamic system described by the following power law: $Z(j\omega) = A \cdot (j\omega)^{-\eta}$.

Power laws are often exhibited by complex systems (internet, biological or social networks, polymers, landscapes ...) which results of the self-similar assembly of elementary components [11],[12]. The

exponent η is generally linked with the structural parameters of the assembly. Power law reflects scaling properties of complex systems, meaning that they quantify the dependence between the scale of measurement (e.g. the frequency) and the measured property (e.g. the impedance).

Of course, in the case of the 21-Bus network, this scaling invariance is hidden by the resonances but this is a starting point for considering an alternative and order-reduced mathematical representation.

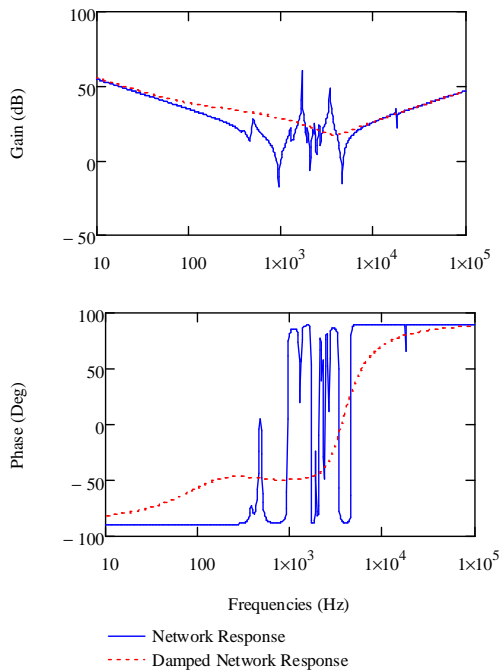


Fig.5. Comparison between damped and reference network responses

3.2. MODELLING OF THE INTERMEDIATE FREQUENCIES BEHAVIOUR

For real networks such as the 21-Bus network, it is obvious that the power law can not be analytically derived from the electrical equations which describe the lines. Indeed, there are too numerous lines and the the scaling invariance is hidden by the resonances at the intermediate frequencies. So, an alternative approach should be proposed for identifying the power law followed by the impedance.

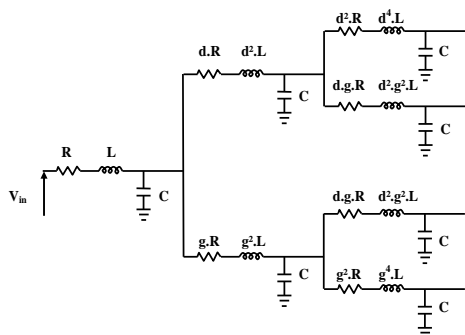


Fig.6. Structure of the ideal network

For that purpose, the intermediate frequency behaviour of an ideal network is first analysed. Its structure is given in Fig.6. It is inspired from a radial distribution network but it is built in such a way that it clearly exhibits a self-similar fractal structure [9].

At low frequencies, the behavior of the network is capacitive while at high frequencies it is inductive. If the network is excited in the intermediate frequencies range by a voltage source whose frequency is equal to the natural frequency of the i^{th} lines, then the impedance of the i^{th} lines is only limited by their resistance R_i while the upstream lines behave as capacitance and the downstream lines are inductive (see. Fig. 7). As the inductive behaviour of the downstream lines prevent any current propagation to the network terminals, the network can finally be reduced to a R-C circuit made of i cells. Its impedance is then given by the following recursive equation:

$$Z(j\omega) \approx R + \frac{1}{j\omega C + \frac{2}{a.Z(j\omega a)}} \quad (1)$$

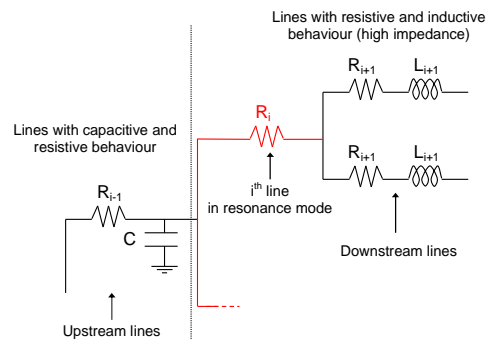


Fig.7. Ideal network behaviour at natural frequency ω_i related to the i^{th} lines

Ref. [9] and [13] have shown that this recursive equation can be rewritten as:

$$Z(j\omega) \propto (j\omega)^{-\eta} \quad (2)$$

Where $\eta = 1 - [\ln(2)/\ln(a)] = 1 - D_f$ and D_f is the fractal dimension of the Cantor fractal tree which has inspired the network of Fig.6.

This proves that, at the intermediate frequencies, a self-similar network exhibits a scaling power law whose exponent is strictly linked to its structure. The principal interest of this mathematical representation is of course its parcimony.

The point now is: do real networks exhibit such self similarities? For the ideal network, the scaling exponent has been given by its fractal dimension. The question then becomes: is it possible to compute a fractal dimension of a real network?

4. A FRACTAL-BASED GEOMETRICAL METHOD FOR COMPUTATION OF THE POWER-LAW EXPONENT

4.1. INTRODUCTION TO FRACTAL COMPUTING

Basically, fractals are mathematical objects which Euclidean measures (length, surface, and volume) are infinite. They are now widely used to better understand and represent physical systems which show self-similarities and scaling relations. The concept was introduced by B. Mandelbrot in the 70s [14].

Contrary to “regular” objects (segments, squares, spheres ...) the dimension of fractals is not an integer but a real number [14],[15]. This number is defined by using the Hausdorff measure.

Assume that F is a fractal object. F is optimally covered by a set of boxes B_i which have a size $d_i \leq \varepsilon$ in such a manner that any points belong to at least one box [14]. The covering is optimal in the sense of being economical. The Hausdorff measure is defined by [15]:

$$Hs(F) = \lim_{\varepsilon \rightarrow 0^+} \sum_{i=1}^N (d_i)^{D_f} \quad (6)$$

D_f is called the Hausdorff dimension. It can be considered as the fractal dimension.

Practically, it is difficult to achieve an optimal covering of the fractal object using boxes of different sizes. So the fractal dimension is approximated by another dimension called the box dimension D_b . This latter one is computed using boxes of constant size d [16]. Then, (6) becomes:

$$H_s(F) = \lim_{d \rightarrow 0^+} N(d) \cdot (d)^{D_b} \quad (7)$$

If the box dimension D_b exists, the Hausdorff measure is constant and D_b is given by the slope of $\log(N(d))$ vs. $\log(d)$ when $d \rightarrow 0$:

$$D_b \propto \lim_{d \rightarrow 0^+} \frac{\log(N(d))}{\log(d)} \quad (8)$$

Even if the box dimension is an approximation, it is a powerful and helpful tool for characterizing fractals. It will now be shown how it can be applied to electrical networks.

4.2. COMPUTING THE FRACTAL DIMENSION OF A POWER NETWORK

Assuming that electrical networks are self-similar objects (at least over a given frequency range), it is proposed to build a fractal analogous. For that

purpose, the Cantor set is chosen because of its radial feature. Each line of the network is then associated to a segment of the Cantor set. The length of the segment is chosen proportional to the value of the natural frequency of the line. In the case of the ideal network, the resulting Cantor set is shown in Fig.8.

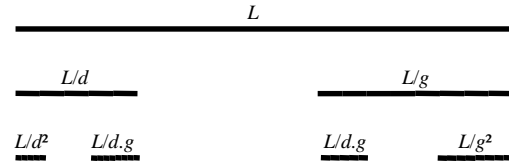


Fig.8. Cantor fractal set analogous

The Cantor set holds both geometric and frequency information. It is then expected that the fractal dimension of the analogous Cantor set be near to the exponent value of the power law characterizing the intermediate frequencies range. The fractal dimension is computed by the box counting method [16]. Because, the lengths of the same step segments can be different, the box size is chosen equal to the mean value of the segment lengths as expressed in (9) (see. Fig.9). i is referred to the step and j is the number of segments at the step $\#i$.

$$\hat{d}_i = \frac{1}{2^i} \sum_j L_{i,j} \quad (9)$$

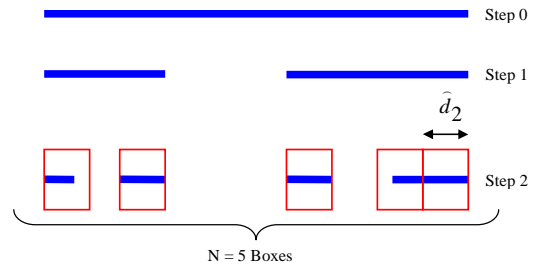


Fig. 9. Box covering at the 3rd step

$\ln(N_i)$ versus $\ln(\hat{d}_i)$ is plotted in Fig. 10. It gives a box dimension $D_b = 0,588$.

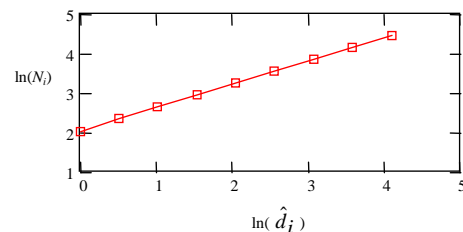


Fig.10. Calculation of the box dimension D_b

The scaling exponent is calculated by: $\eta = 1 - D_b$. The comparison between the scaling law associated to this exponent and the impedance of the ideal

network is shown below. It proves the validity of the proposed approach to identify the power law behavior at the intermediate frequencies.

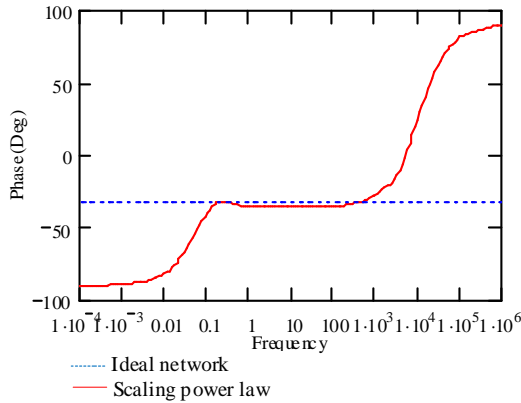


Fig.11. Scaling power law modelling

4.3. APPLICATION TO THE 21-BUS NETWORK

To further validate our approach, the 21-Bus network is now analysed. As shown in Section 3.1, the scaling behaviour is hidden by the resonances at the intermediate frequencies. So, it is proposed to first build an analogous Cantor set of the damped network.

The first segment length is equal to $f_{0,1}$, the natural frequency of the first line of the network. The second step of the Cantor set is built using $f_{1,2}$, $f_{1,21}$ and $f_{1,12}$, the natural frequencies of lines 1-2, 1-21, and 1-12 respectively. The process is repeated until the last line of the network. Then, the fractal dimension is computed by the box counting method. It leads to a value of $\eta_{real}=0,509$.

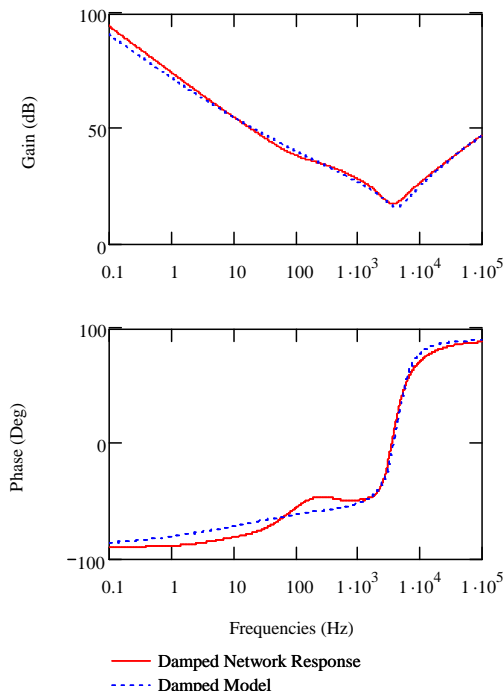


Fig.12. Damped frequency responses

Modeling of the damped network is achieved by summing the frequency power law based on η_{real} to the low frequency part (made of the sum of all the line capacitances) and the high frequency part (equal to the inductance of the line connected to the source). The frequency response given by this model is compared to the response of the damped network in Fig. 12. It clearly shows its accuracy over the whole frequency range.

Only the inclusion of the resonances has now to be done. The real network frequency response in dB is noted $H(j\omega)$ and its damped part is noted $H_{da}(j\omega)$. It is assumed that the resonant part $H_{re}(j\omega)$ is given by:

$$H_{re}(j\omega) = H(j\omega) - H_{da}(j\omega) \quad (16)$$

The damped part is represented by the knowledge-based model $\hat{H}_{da}(j\omega)$ described just before. The resonant part is modelled by a parametric ARMA model $\hat{H}_{re}(j\omega)$ [10]. Its identification leads to a 10-order model. The results of the identification is shown in Fig.13.

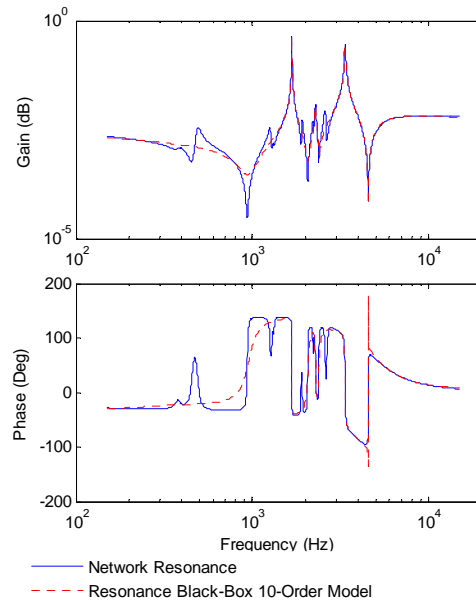


Fig.13. Identification of the resonant part of the 21-Bus network

The complete model $\hat{H}(j\omega)$ combining damped and resonant parts is compared to the frequency response of the 21-Bus network in Fig.14. It clearly appears as a good approximation. To see how this approach is powerful in reducing the order of the models while preserving a good degree of knowledge, we compare it to the full black box 24-order ARMA model as shown in Fig. 14.

Of course, the full ARMA model matches more accurately the frequency response of the network

than the model derived from the power law. The full black-box model gives a maximal error of 2.2 % while the fractal approach gives 8.7 %. But this slight relaxing in accuracy allows dividing the number of parameters by two (from 12 to 24). Last but not least, none of the black-box model parameters give information about the physical structure of the network whereas some parameters of the alternative model are directly derived from physical considerations and give precious information on the network structure.

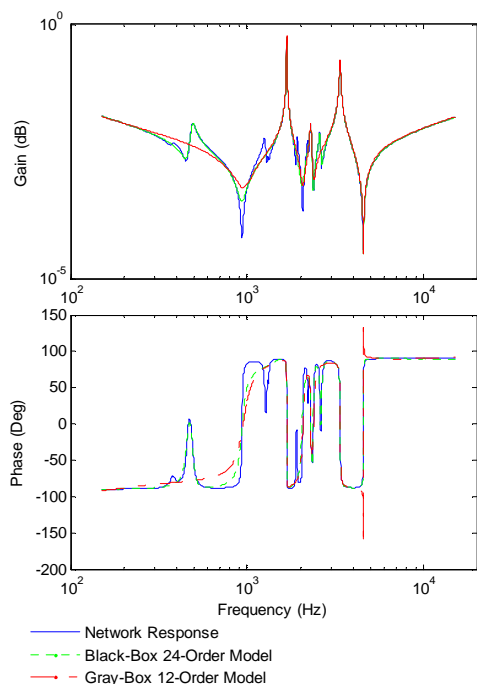


Fig.14. Final comparison of frequency responses including resonances

5. CONCLUSION

In this paper, the authors show how self-similarities can be helpful for reduced-order dynamic modeling of power networks. At intermediate frequencies, a power law has been used to represent a constant phase angle average behaviour. The power exponent is measured by a box counting method applied to a Cantor set analogous to the electrical network. Applications to a 21-Bus power network shows how relevant is the proposed approach to provide reduced models over a wide range of frequencies while preserving a reasonable level of knowledge.

Of course, further studies should be carried out to better include resonances, loads and propagation in the final model. But, these first results prove that it is possible to develop an original, simple and powerful modelling approach which makes strong links between network structure and performance. It

is a first step toward a complete revision of electrical systems as a part of a more complex electricity system made of the convergence of power networks and ICT infrastructures.

6. REFERENCES

- [1] European Commission, European smart grids technology platform – Vision and strategy for Europe's electricity networks of the future, 2006.
- [2] Y.N.Yu, *Electric Power System Dynamics*. Academic Press, 1983
- [3] P. Kundur, *Power System Stability and Control*. McGraw-Hill, 1993
- [4] M. Eremia, *Electric Power System: Electric Networks*. Editura Academiei Române, 2006.
- [5] G.J. Wakileh, *Power System Harmonics*. Springer, 2001.
- [6] G. Troullinos, J. Dorsey, H. Wong, J. Myers, Reducing the order of very large power system models in *IEEE Transactions on Power Systems*, Vol. 3, No. 1, February 1988
- [7] A.J. Germond, R. Podmore, Robin, Dynamic aggregation of generating unit models in *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-97, No. 4, 1978, p. 1060 – 1069.
- [8] H.Yang, G.Richards, Wosrt-Case Analysis of Distribution System Harmonics Using Genetic Algorithms, in *IEEE, Southestcon'92, Proceeding*. Vol.2, pp.530-533.
- [9] A. Amrane, N. Retière, D. Riu. New Modeling of Electrical Power network Using Fractal Geometry, in *The 14th IEEE International Conference on Harmonics and Quality of Power Bergamo Italy*, 26-29 September, 2010
- [10] Levi, E.C. Complex-Curve Fitting, *IRE Trans on Automatic Control*, Vol.AC-4 (1959), pp.37-44.
- [11] B. B. Mandelbrot, How long is the coast of Britain? Statistical self-similarity and fractional dimension. in *Science*: 156, 1967, 636-638
- [12] S. Chaoming, H. Shlomo, A.M. Hernan, Self-similarity of complex networks. in *Nature*, Vol 433, 27 January 2005
- [13] O. Enacheanu, D. Riu, N. Retière. Fractal Modeling of Electrical Networks. in *2nd IFAC Workshop on Fractional Differentiation and its Applications 2006*, Porto, July 2006
- [14] B.Mandelbrot, *The Fractal Geometry of Nature*. W. H. Freeman, 1982.
- [15] T. Nakayama, K. Yakubo, *Fractal Concepts in Condensed Matter Physics*. Solid-State Sciences, NY Springer, 2003.
- [16] K. Falconer, *Fractale Geometry :Mathematical foundation and Application*. John Wiley & Sons, 2003.