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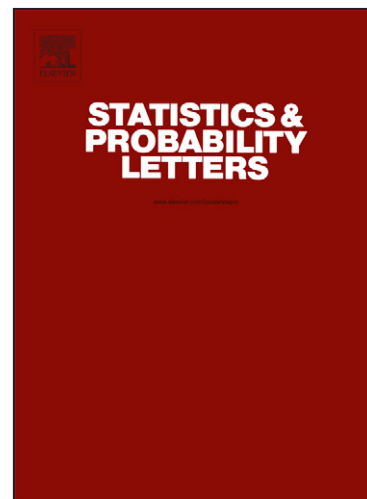
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A Note on the Malliavin Derivative Operator under Change of Variable

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Abstract

The Malliavin derivative operator is classically defined with respect to the standard Brownian motion on the Wiener space $C_0[0, T]$. We define the Malliavin derivative with respect to arbitrary Brownian motions on general probability spaces and compute how the Malliavin derivative of a functional on the Wiener space changes when the functional is composed with transformation by a process which is sufficiently smooth. We then use this result to derive a formula which says how the Malliavin derivatives with respect to different Brownian motions on the same state space are related to each other. This has applications in many situations in Mathematical Finance, where Malliavin calculus is used.

Keywords: Malliavin Calculus, Analysis on Wiener Space, Mathematical Finance

Mathematics Subject Classification: 60H07, 60H40, 91B28

1 Introduction

Malliavin calculus has undoubtedly played a major role in recent applications in Mathematical finance. Its main object, the Malliavin derivative operator, is classically defined on the Wiener space $C_0[0, T]$ with respect to the standard Wiener measure. In Mathematical Finance however one often works with a multitude of measures at the same time, for example when pricing options in incomplete markets. In many cases one would like to have greater flexibility in order to which measure the Malliavin derivative is taken and it is then important to understand how the Malliavin derivatives with respect to these different measures relate to each other. In this article we provide a formula which describes how Malliavin derivative operators with respect to two different Brownian motions relate to each other. We found this formula to be useful in many contexts, see for example [1],[2],[3],[4].

2 The Malliavin Derivative

We follow a similar approach as in [5]. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space which carries a Brownian motion (W_t) and denote with (\mathcal{F}_t) the corresponding Brownian filtration. We assume $\mathcal{F} = \mathcal{F}_T$. Let $F : \Omega \rightarrow \mathbb{R}$ be an \mathcal{F} -measurable functional. In the following we will say what it means that F is Malliavin differentiable. For this let us first consider the case of the classical Wiener space where $\Omega = C_0([0, T])$, $W_t(\omega) = \omega(t)$ for all ω and F is a cylindrical functional of the form $F(\omega) := f(\omega(t_1), \dots, \omega(t_n))$, where $f \in C_b^\infty(\mathbb{R}^n)$ is a smooth function which has bounded derivatives of all orders. Given $h \in L^2[0, T]$ we have that $\int_0^\cdot h(s)ds \in C_0[0, T]$ where the dot indicates that the upper bound of the integral is variable. The subspace of $C_0[0, T]$ generated

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by this kind of functions is called the Cameron-Martin space. The directional derivative of F in direction $\int_0^\cdot h(s)ds$ at ω is given by

$$D_h F(\omega) := \left. \frac{d}{d\epsilon} \right|_{\epsilon=0} F \left(\omega + \epsilon \cdot \int_0^\cdot h(s)ds \right) = \sum_{i=1}^n \frac{\partial f}{\partial x_i}(\omega(t_1), \dots, \omega(t_n)) \cdot \int_0^{t_i} h(s)ds.$$

Now for fixed ω we consider the linear bounded functional on $L^2[0, T]$ given by $h \mapsto D_h F(\omega)$. By the Riesz Representation theorem there is an element $DF(\omega)$ in $L^2[0, T]$ such that

$$D_h F(\omega) = \langle h, DF(\omega) \rangle = \int_0^T h(s) \cdot DF(\omega)(s)ds, \quad \forall h \in L^2[0, T]. \quad (1)$$

In the following we denote $DF(\omega)(s)$ with $D_s F(\omega)$. Let us now consider ω as a variable. The assumption that f has bounded derivatives of all orders ensures that for all $p \geq 1$ we have $DF \in L^p(\Omega, L^2[0, T])$ when considered as an $L^2[0, T]$ valued functional in ω . Assume now that the functional F is not necessarily cylindrical but there exists a sequence of cylindrical functionals F_n such that $F_n \rightarrow F$ in $L^p(\Omega)$ and $DF_n \rightarrow G$ in $L^p(\Omega, L^2[0, T])$. Then we define $DF := G = \lim_{n \rightarrow \infty} DF_n$. One can show that this is well defined, see [5], Lemma 7.1. on page 325. In other words, the operator $D : L^p(\Omega) \rightarrow L^p(\Omega, L^2[0, T])$ defined on the cylindrical functionals is closable. We define the Malliavin derivative operator $D : L^p(\Omega) \rightarrow L^p(\Omega, L^2[0, T])$ as the closure of this operator and denote its domain with $\mathbb{D}_{1,p}$. If $(\Omega, \mathcal{F}, \mathbb{P})$ is not necessarily the classical Wiener space we define the Malliavin derivative operator on $L^p(\Omega)$ as the unique operator $D^{\mathbb{P}}$ which makes the following diagram commutative :

$$\begin{array}{ccc} L^p(C_0[0, T]) & \xrightarrow{W^*} & L^p(\Omega) \\ \downarrow D & & \downarrow D^{\mathbb{P}} \\ L^p(C_0[0, T], L^2[0, T]) & \xrightarrow{(W \times Id)^*} & L^p(\Omega, L^2[0, T]) \end{array}$$

Here W^* and $(W \times Id)^*$ are the maps which are given by composition, i.e. $\tilde{F} \mapsto F = \tilde{F} \circ W$ and $\tilde{G} \mapsto G = \tilde{G} \circ (W \times Id)$, where $\tilde{G}(\omega)(t) \in L^p(C_0[0, t], L^2[0, T])$ is considered as a function $\tilde{G}(\omega, t)$ on $C_0[0, t] \times [0, T]$. It follows from our assumption on \mathcal{F} that these two maps are isometric isomorphisms and therefore $D^{\mathbb{P}}$ exists and is unique. The upper index \mathbb{P} in $D^{\mathbb{P}}$ is to indicate that we work under a measure which is not necessarily the classical Wiener measure. However in case that \mathbb{P} is the classical Wiener measure on $C_0[0, T]$ we omit the upper index. If $F = \tilde{F} \circ W$ then $F \in \text{dom}(D^{\mathbb{P}})$ if and only if $\tilde{F} \in \text{dom}(D)$ and in this case $D^{\mathbb{P}}F = D\tilde{F} \circ (W \times id)$. For any cylindrical functional on Ω of the form $F = f(W_{t_1}, \dots, W_{t_n})$ we have $F = \tilde{F} \circ W$ with $\tilde{F}(\omega) = f(\omega_{t_1}, \dots, \omega_{t_n})$ and therefore

$$\begin{aligned} \langle h, DF \rangle &= \sum_{i=1}^n \frac{\partial f}{\partial x_i}(W_{t_1}, \dots, W_{t_n}) \cdot \int_0^{t_i} h(s)ds \\ &= \left. \frac{d}{d\epsilon} \right|_{\epsilon=0} f \left(W_{t_1} + \epsilon \cdot \int_0^{t_1} h(s)ds, \dots, W_{t_n} + \epsilon \cdot \int_0^{t_n} h(s)ds \right). \end{aligned}$$

This means that the operator $D^{\mathbb{P}}$ is in fact the derivative of F with respect to the Brownian motion W on Ω . We will later need the following definition.

Definition 2.1. For $p \geq 1$ we denote with $\mathbb{L}_{1,p}^{\mathbb{P}}$ the subclass of not necessarily adapted processes (θ_s) in $L^p(\Omega, L^2[0, T])$ such that for each $t \in [0, T]$ we have $\theta_t \in \mathbb{D}_{1,p}$ and there exists a version of the two parameter process $D\theta$ such that for ω a.s. $D\theta \in L^2([0, T] \times [0, T])$ and $\|D\theta\|_{L^2([0, T] \times [0, T])} \in L^p(\Omega)$.

3 The Malliavin Derivative and Transformations

In this section we consider the classical Wiener space $\Omega = C_0[0, T]$ together with the classical Wiener measure. Let $F : C_0[0, T] \rightarrow \mathbb{R}$ and $\omega : [0, T] \rightarrow \mathbb{R}$ be an element of $C_0[0, T]$. Furthermore let $\Theta : C_0[0, T] \rightarrow C_0[0, T]$ be a transformation on the Wiener space. We can think of Θ as a not necessarily adapted stochastic process on $C_0[0, T]$ via $\Theta_t(\omega) = \Theta(\omega)(t)$. For each ω we have $\omega + \Theta(\omega) \in C_0[0, T]$ and we define a new functional on $C_0[0, T]$ by the induced map $(id + \Theta)^*F$ which is given by $\omega \mapsto F(\omega + \Theta(\omega))$. We assume that there exists a not necessarily adapted process (θ_s) s.t. $\Theta_t = \int_0^t \theta_s ds$ and denote the map $(id + \Theta)^*$ with T_θ . The following theorem says that T_θ maps smooth functionals into smooth functionals under the appropriate assumptions.

Theorem 3.1. *Let $p, q \geq 1$ s.t. $\frac{1}{p} + \frac{1}{q} = 1$ and $F \in \mathbb{D}_{1,p}$. Assume $(\theta_s) \in \mathbb{L}_{1,q}$ and $T_\theta : L^p(C_0[0, T]) \rightarrow L^p(C_0[0, T])$ is continuous. Then $T_\theta F \in \mathbb{D}_{1,1}$ and its Malliavin derivative can be computed by the formula*

$$DT_\theta F = T_\theta DF + \int_0^T (D\theta_t \cdot T_\theta D_t F) dt.$$

Proof. We use the notation introduced above. Let us first assume that F is a cylindrical functional of the form $F(\omega) = f(\omega(t_1), \dots, \omega(t_n))$ where $f \in C_b^\infty(\mathbb{R}^n)$. For $h \in L^2[0, T]$ we have :

$$\begin{aligned} (D_h T_\theta F)(\omega) &= \left. \frac{d}{d\epsilon} \right|_{\epsilon=0} T_\theta F \left(\omega + \epsilon \cdot \int_0^\cdot h(s) ds \right) \\ &= \left. \frac{d}{d\epsilon} \right|_{\epsilon=0} F \left(\omega + \epsilon \cdot \int_0^\cdot h(s) ds + \Theta \left(\omega + \epsilon \cdot \int_0^\cdot h(s) ds \right) \right) \\ &= \sum_{i=1}^n \left\{ \frac{\partial f}{\partial x_i}(\omega(t_1) + \Theta_{t_1}(\omega), \dots, \omega(t_n) + \Theta_{t_n}(\omega)) \right\} \\ &\quad \cdot \left. \frac{d}{d\epsilon} \right|_{\epsilon=0} \left(\omega(t_i) + \epsilon \cdot \int_0^{t_i} h(s) ds + \Theta_{t_i} \left(\omega + \epsilon \cdot \int_0^\cdot h(s) ds \right) \right) \\ &= \sum_{i=1}^n \frac{\partial f}{\partial x_i}(\omega(t_1) + \Theta_{t_1}(\omega), \dots, \omega(t_n) + \Theta_{t_n}(\omega)) \cdot \int_0^{t_i} h(s) ds \\ &\quad + \sum_{i=1}^n \frac{\partial f}{\partial x_i}(\omega(t_1) + \Theta_{t_1}(\omega), \dots, \omega(t_n) + \Theta_{t_n}(\omega)) \cdot D_h \Theta_{t_i}(\omega) \end{aligned}$$

Since $(\theta_s) \in \mathbb{L}_{1,q}$ we have $D_h \Theta_t = \int_0^t D_h \theta_s$ and the map $t \mapsto D_h \theta_t(\omega)$ is in $L^2[0, T]$ for ω a.s.. Hence we can write the last equality as

$$(D_h T_\theta F)(\omega) = (D_h F)(\omega + \Theta(\omega)) + (D_{D_h \theta(\omega)} F)(\omega + \Theta(\omega)).$$

Using this equation we obtain

$$\langle h, DT_\theta F(\omega) \rangle = \langle h, DF(\omega + \Theta(\omega)) \rangle + \langle D_h \theta(\omega), DF(\omega + \Theta(\omega)) \rangle.$$

Furthermore, considered as a function in t we have $D_h \theta_t(\omega) = \langle h, D\theta_t(\omega) \rangle$ and therefore the second summand above gives $\langle h, \int_0^T D\theta_t(\omega) \cdot D_t F(\omega + \Theta(\omega)) dt \rangle$. Hence we have established the following equality :

$$\langle h, DT_\theta F(\omega) \rangle = \langle h, DF(\omega + \Theta(\omega)) \rangle + \langle h, \int_0^T D\theta_t(\omega) \cdot D_t F(\omega + \Theta(\omega)) dt \rangle.$$

Since this relationship holds for all $h \in L^2[0, T]$ we get

$$DT_\theta F = T_\theta DF + \int_0^T (D\theta_t \cdot T_\theta D_t F) dt.$$

This now holds for all cylindrical functionals as above. For general $F \in \mathbb{D}_{1,p}$ we can find a sequence (F_n) of cylindrical functionals s.t.

$$\lim_n \|F_n - F\|_{L^p(\Omega)} = 0 \quad (2)$$

$$\lim_n \|DF_n - DF\|_{L^p(\Omega, L^2[0, T])} = 0. \quad (3)$$

By the continuity properties of T_θ we have that $T_\theta F_n \rightarrow T_\theta F$ in $L^1(\Omega)$. If we can show that $DT_\theta F_n \rightarrow G$ in $L^1(\Omega, L^2[0, T])$ then by definition of D and completeness of $\mathbb{D}_{1,1}$ we have $T_\theta F \in \mathbb{D}_{1,1}$ and $G = DT_\theta F$. From the first part we know that for all $n \in \mathbb{N}$.

$$DT_\theta F_n = T_\theta DF_n + \int_0^T (D\theta_t \cdot T_\theta D_t F_n) dt. \quad (4)$$

So in order to proof our general result it suffices to show that

$$\left\| DT_\theta F_n - \left(T_\theta DF + \int_0^T D\theta_t \cdot T_\theta D_t F dt \right) \right\|_{L^1(\Omega, L^2[0, T])} \rightarrow 0.$$

Setting $G_n = F_n - F$ we find that the expression on the left side above is equal to

$$\left\| T_\theta DG_n + \int_0^T D\theta_t \cdot T_\theta D_t G_n dt \right\|_{L^1(\Omega, L^2[0, T])}$$

and using the triangle inequality we see that it is dominated by

$$\|T_\theta DG_n\|_{L^1(\Omega, L^2[0, T])} + \left\| \int_0^T D\theta_t \cdot T_\theta D_t G_n dt \right\|_{L^1(\Omega, L^2[0, T])}.$$

Denoting with $\|T_\theta\|$ the mapping norm of T_θ on L^p we can see that the first summand is bounded by

$$\|T_\theta DG_n\|_{L^1(\Omega, L^2[0, T])} \leq \|T_\theta DG_n\|_{L^p(\Omega, L^2[0, T])} \leq \|T_\theta\| \cdot \|DG_n\|_{L^p(\Omega, L^2[0, T])}.$$

We know that $\lim_n \|DG_n\|_{L^p(\Omega, L^2[0, T])} = 0$ and since $\|T_\theta\| < \infty$ we are left to show that

$$\lim_n \left\| \int_0^T D\theta_t \cdot T_\theta D_t G_n dt \right\|_{L^1(\Omega, L^2[0, T])} = 0$$

This expression is by definition of the norm in $L^1(\Omega, L^2[0, T])$ equal to $\lim_n C_n$ with C_n

$$C_n := \left\| \left\| \int_0^T D\theta_t \cdot T_\theta D_t G_n dt \right\|_{L^2[0, T]} \right\|_{L^1(\Omega)}.$$

We have

$$\left\| \int_0^T D\theta_t \cdot T_\theta D_t G_n dt \right\|_{L^2[0, T]}^2 = \int_0^T \left(\int_0^T D_s \theta_t \cdot T_\theta D_t G_n dt \right)^2 ds. \quad (5)$$

For fixed s (and ω which we omit in the notation) the maps $t \mapsto D_s \theta_t$ and $t \mapsto T_\theta D_t G_n$ are in $L^2[0, T]$ and by applying the Cauchy-Schwartz inequality we get

$$\left| \int_0^T D_s \theta_t \cdot T_\theta D_t G_n dt \right| \leq \|D_s \theta\|_{L^2[0, T]} \cdot \|T_\theta D G_n\|_{L^2[0, T]}.$$

Therefore equation (5) implies that

$$\left\| \left\| \int_0^T D \theta_t \cdot T_\theta D_t G_n dt \right\|_{L^2[0, T]} \right\|_{L^1(\Omega)} \leq \left\| \int_0^T \|D_s \theta\|_{L^2[0, T]}^2 \cdot \|T_\theta D G_n\|_{L^2[0, T]}^2 ds \right\|_{L^1(\Omega)}^{\frac{1}{2}}.$$

Since the second factor in the integral does not depend on s we can put it in front of the integral and get

$$\begin{aligned} C_n &\leq \left\| \left\{ \|T_\theta D G_n\|_{L^2[0, T]} \cdot \left(\int_0^T \|D_s \theta\|_{L^2[0, T]}^2 ds \right)^{\frac{1}{2}} \right\} \right\|_{L^1(\Omega)} \\ &\leq \left\| \left\{ \|T_\theta D G_n\|_{L^2[0, T]} \cdot \|D \theta\|_{L^2([0, T] \times [0, T])} \right\} \right\|_{L^1(\Omega)} \end{aligned}$$

Using our assumptions we have $\|T_\theta D G_n\|_{L^2[0, T]} \in L^p(\Omega)$ and $\|D \theta\|_{L^2([0, T] \times [0, T])} \in L^q(\Omega)$ and therefore by applying the Hölder inequality we get

$$C_n \leq \left\| \left\{ \|T_\theta D G_n\|_{L^2[0, T]} \right\} \right\|_{L^p(\Omega)} \cdot \left\| \left\{ \|D \theta\|_{L^2([0, T] \times [0, T])} \right\} \right\|_{L^q(\Omega)}$$

Using again continuity of T_θ and setting $C := \left\| \left\{ \|D \theta\|_{L^2([0, T] \times [0, T])} \right\} \right\|_{L^q(\Omega)}$ we get

$$C_n \leq C \cdot \|T_\theta\| \|D G_n\|_{L^p(\Omega, L^2[0, T])}$$

and the result follows from equation (3) and $G_n = F_n - F$. \square

Remark 3.1. With the assumption $\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$ a slightly modified proof shows that $T_\theta F \in \mathbb{D}_{1, r}$.

Remark 3.2. The continuity condition on T_θ is a trivial one in the case that $p = \infty$. In this case we always have $\|T_\theta F\|_\infty \leq \|F\|_\infty$.

4 The Malliavin Derivative and Girsanov Transformations

Let us assume that the process (θ_t) from the previous section is adapted. If

$$\mathcal{E} \left(\int_0^\cdot -\theta_s dW_s \right) = \exp \left(\int_0^\cdot -\theta_s dW_s - \frac{1}{2} \theta_s^2 ds \right)$$

is a martingale under the standard Wiener measure we can define a new probability measure \mathbb{P}^θ on $C_0[0, T]$ via

$$\frac{d\mathbb{P}^\theta}{d\mathbb{P}} = \mathcal{E} \left(\int_0^T -\theta_s dW_s \right).$$

Then by the Girsanov Theorem W^θ defined by $W_t^\theta := W_t + \int_0^t \theta_s ds$ is a Brownian motion on $C_0[0, T]$ under \mathbb{P}^θ . Now, for a functional $F : C_0[0, T] \rightarrow \mathbb{R}$ we can consider both the Malliavin

derivative with respect to W and the Malliavin derivative with respect to W^θ . To simplify the notation we denote the Malliavin derivative $D^{\mathbb{P}^\theta}$ with respect to W^θ as D^θ and write \mathbb{D}^θ for $\mathbb{D}^{\mathbb{P}^\theta}$. By our assumption F factorizes as $F = \tilde{F} \circ W^\theta = T_\theta \tilde{F}$. Then $F \in \mathbb{D}_{1,1}^\theta$ if and only if $\tilde{F} \in \mathbb{D}_{1,1}$. If T_θ is invertible then we have $\tilde{F} = T_\theta^{-1}F$. We call a pair (θ, ψ) of adapted stochastic processes which satisfy

$$\psi_s(\omega) = -\theta_s \left(\omega + \int_0^s \psi_u(\omega) du \right), \quad \theta_s(\omega) = -\psi_s \left(\omega + \int_0^s \theta_u(\omega) du \right)$$

an admissible Girsanov pair if (θ_s) satisfies the Novikov condition $\mathbb{E} \left(\exp \left(\frac{1}{2} \int_0^T \theta_s^2 ds \right) \right) < \infty$. It is very easy to show that the existence of an admissible Girsanov pair (θ, ψ) is a sufficient criterion for the invertibility of T_θ and that in this case $(T_\theta)^{-1} = T_\psi$.

Proposition 4.1. *Let $p, q \geq 1$ s.t. $\frac{1}{p} + \frac{1}{q} = 1$. Furthermore let (θ, ψ) be an admissible Girsanov pair such that $(\psi_s) \in \mathbb{L}_{1,q}$ and there exists a constant $C > 0$ s.t. $\mathbb{E}_\theta(|F|^p) \leq C \cdot \mathbb{E}(|F|^p)$ for all $F \in L^1(C_0[0, T])$. Then if $F \in \mathbb{D}_{1,p}$ we have $F \in \mathbb{D}_{1,1}^\theta$ and*

$$D^\theta F = DF + \int_0^T T_\theta D\psi_t \cdot D_t F dt.$$

Proof. The assumption that $\mathbb{E}_\theta(|F|^p) \leq C \cdot \mathbb{E}(|F|^p)$ guarantees that the operator T_θ is continuous as an operator on L^p . By the open mapping theorem we can conclude that T_ψ is also continuous and furthermore (ψ_s) satisfies the assumptions in Theorem 3.1 (where (θ_s) is replaced by (ψ_s)). We can then write $F = T_\theta(T_\psi F) = T_\psi F \circ W^\theta$. By definition of D^θ and application of Theorem 3.1 we have

$$\begin{aligned} D^\theta F &= DT_\psi F \circ W^\theta = \left(T_\psi F DF + \int_0^T D\psi_t \cdot T_\psi D_t F dt \right) \circ W^\theta \\ &= T_\theta T_\psi DF + T_\theta \left(\int_0^T D\psi_t \cdot T_\psi D_t F dt \right) = DF + \int_0^T (T_\theta D\psi_t) \cdot D_t F dt \end{aligned}$$

where we used that the integral with respect to dt can be computed point-wise. \square

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