Relativistic smooth particle hydrodynamics on a given background spacetime

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Relativistic smooth particle hydrodynamics on a given background space-time

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Abstract. We review the derivation of fixed-metric, relativistic smooth particle hydrodynamics (SPH) from the Lagrangian of an ideal fluid. Combining the Euler-Lagrange equations with the first law of thermodynamics, we explicitely derive evolution equations for the canonical momentum and energy. This new set of SPH equations also accounts for corrective terms that result from derivatives of the SPH smoothing kernel and that are called "grad-h" terms in non-relativistic SPH. The new equations differ from earlier formulations with respect to these corrective terms and the symmetries in the SPH particle indices while being identical in the gravitational terms.

1. Introduction

Relativity is a crucial ingredient in a variety of astrophysical phenomena, both due to velocities approaching the speed of light, as for example in AGN jets, and due to strong gravity shaping space-time geometry, say, near a black hole. In the recent past, substantial progress has been made in the development of special- and general relativistic numerical tools [1, 2, 3] that can tackle a variety of pressing astrophysical problems. While most work on numerical relativistic gas dynamics has been performed in a Eulerian framework, a couple of Lagrangian smooth particle hydrodynamics (SPH) approaches exist.

The first relativistic SPH formulations were developed by Kheyfets et al. [4] and Mann [5, 6]. Shortly after, Laguna et al. [7] developed a 3D, general-relativistic SPH code that was subsequently applied to the tidal disruption of stars by massive black holes [8]. Their SPH formulation is complicated by several issues: the continuity equation contains a gravitational source term that requires SPH kernels for curved space-times. Moreover, owing to their choice of variables, the equations contain time derivatives of Lorentz factors that are treated by finite difference approximations and restrict the ability to handle shocks to only moderate Lorentz factors. The Laguna et al. formulation has recently been extended by Rantsiou et al. [9] and applied to neutron star black hole binaries. In a separate approach, Chow and Monaghan [10] obtained accurate special-relativistic test results even for flow problems with large Lorentz factors. They evolve the total energy rather than the thermal energy and by applying an artificial viscosity scheme that borrows concepts from Riemann solvers [11]. More recently, Siegler and Riffert [12] and Siegler [13] have presented a set of equations for both the special- and general-relativistic case. Based on the conservative form of Lagrangian hydrodynamics their choice of variables bypassed many of the complications that have plagued earlier relativistic SPH formulations. They were able to simulate accurately some test problems with very large Lorentz factors.

An elegant approach that is based on the discretized Lagrangian of a perfect fluid was suggested in Monaghan and Price [14]. Provided that the discretized Lagrangian posseses the correct symmetries, in such an approach nature's conservation laws are hard-wired into the resulting SPH equations. Moreover, the Euler-Lagrange equations determine the evolution of the fluid and do not leave much room for arbitrariness in the derivation. We are not aware of existing numerical implementations of these equations, but a different derivation that lead to a similar equation set has been tested in [15].

All of the above approaches used a fixed background metric and it was only recently that SPH has been applied to the study of flows with strong self-gravity. In these approaches Post-Newtonian approximations were used[16, 17, 18, 19], and, more recently, the conformal flatness approximation [20, 21] has been implemented [22, 23, 24, 25].

In this paper we will review the derivation of the general-relativistic SPH equations from a Lagrangian, similar to the work presented in [14]. Extending this work, we also account for corrective extra terms due to kernel derivatives, that are called "grad-h"

terms in non-relativistic SPH [26, 27].

2. Relativistic SPH from a variational principle

Several authors [28, 29, 26, 27] have derived non-relativistic SPH equations from a variational principle. Recently, such a variational approach has also been applied to the general-relativistic case assuming a given background space-time metric [14]. In the following review we will adhere to a similar strategy, but also account for the general-relativistic "grad-h"-terms.

We assume that a prescribed metric $g_{\mu\nu}$ is known as a function of the coordinates and that the perturbations that the fluid induces to the space-time geometry can be safely neglected. We further use units in which the speed of light is equal to unity, $c = 1$, we adopt a metric with signature (-,+,+,+). We reserve greek letters for space-time indices from 0...3 with 0 being the temporal component, while i and j refer to spatial components and SPH particles are labeled by a, b and k. Contravariant spatial indices of a vector quantity w at particle a are denoted as w_a^i, while covariant ones will be written as $(w_i)_a$. The line element and proper time are given by $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ and $d\tau^2 = -ds^2$ and the proper time is related to a coordinate time t by

$$\Theta d\tau = dt, \tag{1}$$

where we have introduced a generalization of the Lorentz-factor

$$\Theta \equiv \frac{1}{\sqrt{-g_{\mu\nu}v^\mu v^\nu}} \quad \text{with} \quad v^\alpha = \frac{dx^\alpha}{dt}. \tag{2}$$

This relates to the four-velocity U^ν by

$$v^\mu = \frac{dx^\mu}{dt} = \frac{dx^\mu}{d\tau}\frac{d\tau}{dt} = \frac{U^\mu}{\Theta} = \frac{U^\mu}{U^0}, \tag{3}$$

which is normalized to $U^\mu U_\mu = -1$.

2.1. The relativistic fluid Lagrangian

The Lagrangian of a relativistic fluid is given by [30]

$$L = -\int T^{\mu\nu}U_\mu U_\nu \sqrt{-g}dV, \tag{4}$$

where $g = \det(g_{\mu\nu})$ and $T^{\mu\nu}$ denotes the energy-momentum tensor of an ideal fluid without viscosity and conductivity

$$T^{\mu\nu} = (\rho + P)U^\mu U^\nu + Pg^{\mu\nu}. \tag{5}$$

Here ρ is the energy density of the fluid as measured in the local rest frame and P is the fluid pressure. For clarity, we will write out explicity factors of c in the following lines. The energy density can be split up in a term associated with the rest mass and one given by the thermal energy contribution:

$$\rho = \rho_{\rm rest} + u\rho_{\rm rest}/c^2 = nm_0c^2(1 + u/c^2) \tag{6}$$

Here n is the baryon number density in the local fluid rest frame, m_0 is the baryon mass‡ and $u = u(n, s)$ the specific energy, with s being the specific entropy. From now on, we will measure all energies in units of m_0c^2 (and use again $c = 1$). With this convention the energy momentum tensor reads

$$T^{\mu\nu} = \{n(1 + u) + P\} U^\mu U^\nu + Pg^{\mu\nu}. \tag{7}$$

With the normalization of the four-velocity the Lagrangian can be written as

$$L = -\int n(1 + u)\sqrt{-g}dV. \tag{8}$$

2.2. The SPH discretization

In the following we apply the reasoning and principles behind modern SPH formulations to the general-relativistic, fixed-metric case. For a detailed account on modern SPH we refer the interested reader to a recent review [31]. To perform practical simulations we give up general covariance and choose a particular frame ("computing frame") in which the computations are carried out. This requires a suitable transformation between computing frame quantities and quantities that are evaluated in the local rest frame of a fluid particle.

To find a SPH discretization in terms of a suitable density variable, we follow an approach similar to [12]. Local baryon number conservation, $(U^\mu n)_{;\mu} = 0$, can be expressed as

$$\partial_\mu(\sqrt{-g}U^\mu n) = 0, \tag{9}$$

or, more explicitely, as

$$\partial_t(N) + \partial_i(Nv^i) = 0, \tag{10}$$

where we have made use of Eq. (3) and have introduced the computing frame number density

$$N = \sqrt{-g}\Theta n. \tag{11}$$

The total conserved baryon number can then be expressed as a sum over fluid parcels with volume ΔV_b located at $\vec{r}_b$, where each parcel carries a baryon number ν_b

$$\mathcal{N} = \int NdV \simeq \sum_b N_b \Delta V_b = \sum_b \nu_b. \tag{12}$$

Eq. (10) looks like the Newtonian continuity equation and we will use it for the SPH discretization process. Similar to standard SPH, one can approximate the continuum by fluid parcels ("particles"), so that a quantity f can be approximated as by

$$\tilde{f}(\vec{r}) \simeq \sum_b f_b \frac{\nu_b}{N_b} W(\vec{r} - \vec{r}_b, h), \tag{13}$$

where the subscript b indicates that a quantity is evaluated at a position $\vec{r}_b$ and W is a suitable smoothing kernel whose width is determined by the so-called smoothing length

‡ The appropriate baryon mass depends on the neutron to proton ratio, i.e. on the nuclear composition of the considered fluid.

h. If we keep all ν_b constant in time, exact baryon number conservation is guaranteed and no continuity equation needs to be solved (this can be done, though, if desired). For the kernel W, we assume that it has a compact support, so that sums only contain a local set of particles, and that it is radial, i.e. $W(\vec{r}-\vec{r}_b,h)=W(|\vec{r}-\vec{r}_b|,h)$, so that $\partial W(|\vec{r}-\vec{r}_b|,h)/\partial\vec{r}=-\partial W(|\vec{r}-\vec{r}_b|,h)/\partial\vec{r}_b$.
If Eq. (13) is applied to the baryon number density N, one finds §

$$N_a = N(\vec{r}_a) = \sum_b \nu_b W(\vec{r}_a - \vec{r}_b, h_a), \tag{14}$$

where we have chosen to evaluate the density estimate at $\vec{r}_a$ with the local smoothing length h_a. Note that we are evaluating locally smoothed quantities with flat-space kernels which assumes that the local space-time curvature radius is large in comparison to the local fluid resolution length. Such an approach is very convenient, but (more involved) alternatives to this approach also exist in the literature [4, 7]. The Newtonian mass density estimate can be recovered by the replacements $N_b \to \rho_b$ and $\nu_b \to m_b$.
It is usually desirable to adjust the smoothing length locally to fully exploit the natural adaptivity of a particle method. One convenient way to do so by using the local density is

$$h_a = \frac{\eta}{N_a^{1/3}}, \tag{15}$$

where the parameter η controls how many neighbor particles are taken into account for density estimates. Since Eqs. (15) and (14) mutually depend on each other, an iteration is required at each time step to obtain consistent values for both.
For later use in the evolution equations we also provide the derivatives of the computing frame number density (with the notation $\vec{r}_{bk}=\vec{r}_b-\vec{r}_k$, $r_{bk}=|\vec{r}_{bk}|$ and $W_{bk}(h)=W(|\vec{r}_b-\vec{r}_k|,h)$):

$$\frac{\partial N_b}{\partial x_a^i} = \sum_k \nu_k \left\{ \frac{\partial W_{bk}(h_b)}{\partial r_{bk}} \frac{\partial r_{bk}}{\partial x_a^i} + \frac{\partial W_{bk}(h_b)}{\partial h_b} \frac{\partial h_b}{\partial N_b} \frac{\partial N_b}{\partial x_a^i} \right\}. \tag{16}$$

Collecting the $\partial N_b/\partial x_a^i$ terms on both sides one obtains

$$\frac{\partial N_b}{\partial x_a^i} = \frac{1}{\Omega_b} \sum_k \nu_k \frac{\partial W_{bk}(h_b)}{\partial r_{bk}} \frac{\partial r_{bk}}{\partial x_a^i} = \frac{1}{\Omega_b} \sum_k \nu_k \frac{\partial W_{kb}(h_b)}{\partial x_b^i} (\delta_{ba} - \delta_{ka}), \tag{17}$$

where

$$\Omega_b = 1 - \frac{\partial h_b}{\partial N_b} \sum_k \nu_k \frac{\partial W_{bk}(h_b)}{\partial h_b} \tag{18}$$

is a corrective term of order unity, often called "grad-h"-term in a non-relativistic context. In a similar way, one finds the time derivative

$$\frac{dN_b}{dt} = \frac{1}{\Omega_b} \sum_k \nu_k v_{bk}^i \frac{\partial W_{bk}(h_b)}{\partial x_b^i}, \tag{19}$$

§ From now on we are dropping the distinction between "original" and approximated quantities.

where $\vec{v}_{bk} = \vec{v}_b - \vec{v}_k$.
Motivated by Eqs. (11) and (12), we can re-write the fluid Lagrangian, Eq. (8), in terms of our computing frame number density N,

$$L = -\int \frac{1+u}{\Theta} N dV, \tag{20}$$

which leads to the general-relativistic, discretized SPH-Lagrangian

$$L_{\rm SPH} = -\sum_b \nu_b \left(\frac{1+u}{\Theta}\right)_b. \tag{21}$$

2.3. The momentum equation

In the following we use the Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial v_a^i} - \frac{\partial L}{\partial x_a^i} = 0, \tag{22}$$

to derive evolution equations for the canonical momentum and the canonical energy per baryon. The canonical momentum is

$$(p_i)_a \equiv \frac{\partial L}{\partial v_a^i} = -\frac{\partial}{\partial v_a^i}\sum_b \nu_b \left(\frac{1+u}{\Theta}\right)_b, \tag{23}$$

where the velocity dependence enters directly via the generalized Lorentz-factor Θ and via

$$\frac{\partial u_b}{\partial v_a^i} = \frac{\partial u_b}{\partial n_b}\frac{\partial n_b}{\partial v_a^i} = \frac{P_b}{n_b^2}\frac{N_b}{\sqrt{-g}_b}\frac{\partial}{\partial v_a^i}\left(\frac{1}{\Theta}\right)_b, \tag{24}$$

where we have used the first law of thermodynamics, $\partial u_b/\partial n_b = P_b/n_b^2$, and the relation between computing frame and local rest-frame density, Eq. (11). With

$$\frac{\partial}{\partial v_a^i}\left(\frac{1}{\Theta_b}\right) = -\Theta_b (g_{i\mu}v^\mu)_a \delta_{ab} \tag{25}$$

one finds the canonical momentum per baryon

$$(S_i)_a \equiv \frac{1}{\nu_a}\frac{\partial L}{\partial v_a^i} = \Theta_a\left(1 + u_a + \frac{P_a}{n_a}\right)(g_{i\mu}v^\mu)_a = \left(1 + u_a + \frac{P_a}{n_a}\right)(U_i)_a. \tag{26}$$

We now need $\partial L/\partial x_a^i$ to obtain, via Eq. (22), an evolution equation for $(S_i)_a$. The resulting terms can be split up into "hydrodynamics terms" which contain density gradients and "gravitational terms" which are proportional to gradients of the metric. To evaluate $\partial[(1+u_b)/\Theta_b]/\partial x_a^i$ one uses

$$\frac{\partial}{\partial x_a^i}\left(\frac{1}{\Theta_b}\right) = -\left(\frac{U^\mu U^\nu}{2\Theta}\frac{\partial g_{\mu\nu}}{\partial x^i}\right)_b \delta_{ab} \tag{27}$$

and applies once more the first law of thermodynamics together with Eq. (11). The hydrodynamic contribution then becomes

$$\begin{aligned}\left(\frac{\partial L}{\partial x_a^i}\right)_{\rm hydro} &= -\sum_b \nu_b \frac{P_b\sqrt{-g}_b}{N_b^2}\frac{\partial N_b}{\partial x_a^i} \\ &= -\nu_a \sum_b \nu_b \left\{\frac{P_a\sqrt{-g}_a}{\Omega_a N_a^2}\frac{\partial W_{ab}(h_a)}{\partial x_a^i} + \frac{P_b\sqrt{-g}_b}{\Omega_b N_b^2}\frac{\partial W_{ab}(h_b)}{\partial x_a^i}\right\},\end{aligned} \tag{28}$$

where we have applied Eq. (17) and the kernel property $\partial W_{ba}(h_b)/\partial x_b^i = -\partial W_{ab}(h_b)/\partial x_a^i$. The remaining gravity terms simplify on use of Eqs. (11) and (7) to

$$\left(\frac{\partial L}{\partial x_a^i}\right)_{\text{gravity}} = \left(\frac{\nu\sqrt{-g}}{2N}T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial x^i}\right)_a . \tag{29}$$

Thus, the final, general-relativistic SPH momentum equation reads

$$\begin{aligned}\frac{d(S_i)_a}{dt} &= \frac{1}{\nu_a}\frac{\partial L}{\partial x_a^i} \\ &= -\sum_b \nu_b \left\{ \frac{P_a\sqrt{-g}_a}{\Omega_a N_a^2}\frac{\partial W_{ab}(h_a)}{\partial x_a^i} + \frac{P_b\sqrt{-g}_b}{\Omega_b N_b^2}\frac{\partial W_{ab}(h_b)}{\partial x_a^i}\right\} \\ &\quad + \left(\frac{\sqrt{-g}}{2N}T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial x^i}\right)_a .\end{aligned} \tag{30}$$

The gravity contribution to this equation is identical to the one in [12, 14], but the hydrodynamical terms differ both due to the presence of the grad-h terms and due to a different symmetrization in the particle indices.

2.4. The energy equation

For a suitable energy variable one can start from the canonical energy

$$E \equiv \sum_a \frac{\partial L}{\partial v_a^i}v_a^i - L = \sum_a \nu_a \left(v_a^i(S_i)_a + \frac{1+u_a}{\Theta_a}\right) \tag{31}$$

to identify

$$e_a \equiv v_a^i(S_i)_a + \frac{1+u_a}{\Theta_a}, \tag{32}$$

whose evolution equation follows from straight forward differentiation

$$\frac{de_a}{dt} = (S_i)_a\frac{dv_a^i}{dt} + v_a^i\frac{d(S_i)_a}{dt} + \frac{d}{dt}\left(\frac{1+u_a}{\Theta_a}\right). \tag{33}$$

As we will see below, the first term cancels with a corresponding term resulting from the third term. Similar to the derivation of the momentum equation, the term $d[(1+u_a)/\Theta_a]/dt$ splits up into a term involving derivatives of the density and a term which contains derivatives of the metric tensor. Applying the first law of thermodynamics, Eq.(11), once more and

$$\frac{d}{dt}\left(\frac{1}{\sqrt{-g}\Theta}\right)_a = -\left(\frac{1}{2\sqrt{-g}\Theta}g^{\mu\nu}\frac{dg_{\mu\nu}}{dt}\right)_a - \left(\frac{1}{\sqrt{-g}\Theta^2}\frac{d\Theta}{dt}\right)_a, \tag{34}$$

yields the evolution equation of the thermal energy

$$\frac{du_a}{dt} = \frac{P_a\sqrt{-g}_a\Theta_a}{N_a^2}\frac{dN_a}{dt} - \frac{P_a}{2n_a}g^{\mu\nu}\frac{dg_{\mu\nu}}{dt} - \frac{P_a}{n_a\Theta_a}\frac{d\Theta_a}{dt}. \tag{35}$$

If the time derivative of the generalized Lorentz factor Θ is expressed as

$$\frac{d\Theta_a}{dt} = \left(\frac{\Theta^3}{2}v^\mu v^\nu\frac{dg_{\mu\nu}}{dt} + \Theta^3 g_{\mu\nu}\frac{dv^\mu}{dt}v^\nu\right)_a \tag{36}$$

one finds

$$\frac{d}{dt}\left(\frac{1+u}{\Theta}\right)_a = \frac{P_a\sqrt{-g}_a}{N_a^2}\frac{dN_a}{dt} - (S_i)_a\frac{v_a^i}{dt} - \left(\frac{\sqrt{-g}}{2N}T^{\mu\nu}\frac{dg_{\mu\nu}}{dt}\right)_a \tag{37}$$

and on using Eqs. (30), (19), (37) and

$$\frac{dg_{\mu\nu}}{dt} = \frac{\partial g_{\mu\nu}}{\partial x^i}v^i + \partial_t g_{\mu\nu} \tag{38}$$

the final general-relativistic energy equation becomes

$$\begin{aligned}\frac{de_a}{dt} = & -\sum_b \nu_b \left\{\frac{P_a\sqrt{-g}_a v_b^i}{\Omega_a N_a^2}\,\frac{\partial W_{ab}(h_a)}{\partial x_a^i} + \frac{P_b\sqrt{-g}_b v_a^i}{\Omega_b N_b^2}\,\frac{\partial W_{ab}(h_b)}{\partial x_a^i}\right\} \\ & -\left(\frac{\sqrt{-g}}{2N}T^{\mu\nu}\partial_t g_{\mu\nu}\right)_a .\end{aligned} \tag{39}$$

Together with an equation of state, the equations (14), (30) and (39) represent our complete and self-consistently derived set of SPH equations. The gravitational terms are identical to those of [12, 14], but the hydrodynamic terms differ in both the particle symmetrization and the presence of the grad-h terms. Note that the only choices in our above derivation were the h-dependence in Eq. (14) and how to adapt the smoothing length. The subsequent calculation contained no arbitrariness concerning the symmetry in particle indices, everything followed stringently from the first law of thermodynamics and the Euler-Lagrange equations. Another important point to note is that the derived energy equation, Eq. (39), does not contain destabilizing [32] time derivatives of Lorentz factors on the RHS –in contrast to the thermal energy equation (35) and to earlier SPH formulations [7]. For a practical simulation involving shocks, Eqs. (30) and (39) need to be augmented by artificial viscosity terms similar to the special-relativistic case [10, 12, 33]. In order to couple to a generic space-time evolution code, the partial derivative $\partial_t g_{\mu\nu}$ in Eq.(39) would need to be re-written in terms of the extrinsic curvature, see e.g. [34].

2.5. The special-relativistic limit

In the special-relativistic limit we can neglect the gravitational terms in Eqs. (30) and (39). In addition in flat space-time with Cartesian coordinates one has $\sqrt{-g} \to 1$ and $\Theta \to \gamma$, and Eq. (11) becomes $N = \gamma n$, which simply expresses the increase in the computing frame number density N with respect to the local fluid rest frame density n due the Lorentz contraction. The momentum and energy equations reduce in this limit to

$$\left(\frac{d\vec{S}_a}{dt}\right)_{\rm SR} = -\sum_b \nu_b \left\{\frac{P_a}{\Omega_a N_a^2}\frac{\partial W_{ab}(h_a)}{\partial \vec{r}_a} + \frac{P_b}{\Omega_b N_b^2}\frac{\partial W_{ab}(h_b)}{\partial \vec{r}_a}\right\} \tag{40}$$

and

$$\left(\frac{de_a}{dt}\right)_{\rm SR} = -\sum_b \nu_b \left\{\frac{P_a\vec{v}_b}{\Omega_a N_a^2}\cdot\frac{\partial W_{ab}(h_a)}{\partial \vec{r}_a} + \frac{P_b\vec{v}_a}{\Omega_b N_b^2}\cdot\frac{\partial W_{ab}(h_b)}{\partial \vec{r}_a}\right\} \tag{41}$$

which are the equations recently derived and successfully tested in a special-relativistic context [33].

3. Summary

In this article we have reviewed in detail the derivation of the general-relativistic SPH equations form a variational principle for the case of a given background metric. We have used the computing frame baryon number density in the SPH discretization process and derived evolution equations for the canonical energy and momentum. If the latter are chosen as numerical variables, no derivatives of Lorentz factors occur in the evolution equations. As in non-relativistic SPH, one has the choice between calculating (here: computing frame baryon number) densities via a particle summation or by evolving the continuity equation. The derived equation set also contains corrective terms from smoothing kernel derivatives, so-called grad-h terms. The hydrodynamic parts of the equation set differ from earlier general-relativistic SPH formulations by the presence of these corrective terms and by the symmetry in the particle indices, the contributions from the gravitational field are identical to those found in earlier work. In the special-relativistic limit in Cartesian coordiates, the equations reduce to a recently derived equation set that has been successfully tested in large number of benchmark problems [33]. Future applications of this SPH formulation will include tidal disruptions of stars by massive black holes.

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