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# Interaction Between Ring-Shaped Permanent Magnets With Symbolic Gradients: Application to Magnetic Bearing System Optimization

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This paper presents a semi-analytical calculation of the interaction between magnetic bodies with ring shapes. It leads to fast and accurate evaluation of forces and torques but also to the symbolic expression of their gradients. Our method can be used to compute partial derivatives to address optimization needs. It has been implemented into an available software environment in order to be automatically available for end users. Efficient optimization can be done to size magnetic bearing systems.

**Index Terms**—Elliptic integrals, magnetostatic analytical modeling, optimization, ring-shaped magnet.

## I. INTRODUCTION

### A. Magnet Bearing Systems

APPLICATIONS which use magnetic devices like magnetic bearings and couplings, strong field generation, or micro actuators and sensors, can be modeled by analytical expressions. Ring-shaped permanent magnets are specially used in magnetic bearings like Fig. 1, and are used in several systems:

- flywheels: to store rotational energy;
- centrifuge: to collect Uranium-235;
- turbopump: to obtain and maintain high vacuum.

### B. Magnetic Interactions Modeling

The modeling of cylindrical magnets can be based on Coulombian method with equivalent surface charges  $\sigma_s = \vec{J} \cdot \vec{n}$  where “n” defines the unit vector normal to surface and “J” the magnetic polarization. Volume charges appear for radial polarization.

By symbolic integration of (1) in polar coordinates, axial and radial components are expressed with elliptic integral of the three kinds [2]–[4]

$$\vec{H}_P = \frac{\sigma}{4\pi\mu_0} \iint_s \frac{\vec{r}' - \vec{r}}{|\vec{r}' - \vec{r}|^3} dS. \quad (1)$$

Forces can then be found by numerical integration of these fields by an equivalent charge (surface  $\sigma_S$  and volume  $\sigma_V$ ) method applied on the other magnets

$$\vec{F} = \iint_s \sigma_s \cdot \vec{H}_{ext} dS + \iiint_v \sigma_v \cdot \vec{H}_{ext} dv. \quad (2)$$

### C. Sensitivity Calculation to Perform Optimization

More and more optimization procedures are performed over analytical model to obtain constraints satisfaction and optimal

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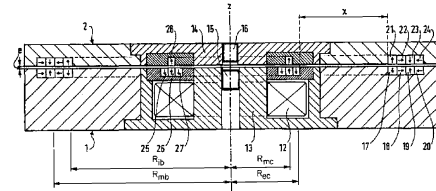


Fig. 1. Magnetic bearing patent [1].

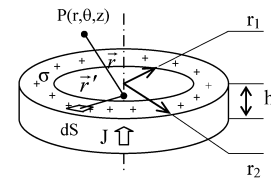


Fig. 2. Cylindrical magnet equivalent surface charges modeling.

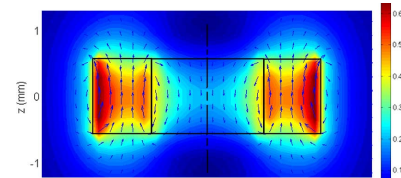


Fig. 3.  $|B(x, z)|$  and  $B(x, z)$  vectors for  $y = 0$ ,  $M_z = 1$  T.

design, even with the availability of numerical model. In our case, partial derivatives of force can be expressed using the general ((3)) ( $a, b, c \in \mathbb{R}$ )

$$\frac{\partial}{\partial c} \int_{a(c)}^{b(c)} f(c, x) dx = \int_{a(c)}^{b(c)} \frac{\partial f(c, x)}{\partial c} dx + \frac{\partial b(c)}{\partial c} \cdot f(c, b(c)) - \frac{\partial a(c)}{\partial c} \cdot f(c, a(c)). \quad (3)$$

Each term of (2) derivation is available expected partial derivatives of the elliptic evaluation of  $H_{ext}$ .

The aim of the paper is to present symbolic approach to compute magnetic forces and torques sensitivity, and to present automatic tool to combine partial derivatives.

## II. ELLIPTIC INTEGRALS

### A. Definitions

Elliptic integrals appear during the integration of magnetic field in polar coordinates. There are of three kinds, given here

((4), (5), (6)) in Legendre's notation for incomplete form (*the complete form is for  $\Phi = \pi/2$* )

$$F(k, \phi) = \int_0^\phi \frac{1}{\sqrt{1 - k^2 \cdot \sin(\theta)}} d\theta \quad \text{"first kind"} \quad (4)$$

$$E(k, \phi) = \int_0^\phi \sqrt{1 - k^2 \cdot \sin(\theta)^2} d\theta \quad \text{"second kind"} \quad (5)$$

$$\Pi(k, n, \phi) = \int_0^\phi \frac{1}{(1 + n \cdot \sin(\theta)^2) \cdot \sqrt{1 - k^2 \cdot \sin(\theta)^2}} d\theta \quad \text{"third kind"}. \quad (6)$$

The definition can be lightly different for several authors or mathematical environments ( $m = k^2$ , or " $n$ " is preceded by a minus sign).

### B. Numerical Evaluation

There are several algorithms for the computation of complete or incomplete elliptic integrals, mainly based on arithmetic-geometric mean (AGM) algorithm. They can differ for the range of parameter " $\Phi$ ," " $k$ ," and " $n$ " [6]. For example, the algorithm given in [5] for " $\Pi$ " will not work in the modeling of cylindrical magnets where " $k$ " range is out of  $[0, 1]$ . We are using a performing algorithm founded by Carlson [7] and available in Numerical Recipes [8].

We have compared Elliptic implementation with the one given in (7) where symbolic integration is done as far as possible (*1D numerical integration remains*). Computation is about 200 times faster, and has less singularity. Fig. 3 presents induction values obtained in few seconds and plotted in Matlab shown in

$$H_z = \frac{\sigma_s}{4\pi\mu_0} \cdot \int_{\theta_0}^{\theta_1} \sum_{i=0}^1 \sum_{j=0}^1 (-1)^{i+j} [(z - (-1)^j c) g(\theta)] d\theta$$

with

$$g(\theta) = \frac{-(z - (-1)^j c)^2 - (x^2 + y^2) + r_i (x \cos(\theta) + y \sin(\theta))}{((x \sin(\theta) - y \cos(\theta))^2 + (z - (-1)^j c)^2) \phi(x, y, z, i, j, \theta, c)}. \quad (7)$$

### C. Symbolic Sensitivity

For each elliptic function, a partial derivative regarding " $\Phi$ " parameter can be obtained

$$\frac{dF(k, \phi)}{d\phi} = \frac{1}{\sqrt{1 - k^2 \cdot \sin(\phi)^2}} \quad (8)$$

$$\frac{dE(k, \phi)}{d\phi} = \sqrt{1 - k^2 \cdot \sin(\phi)^2} \quad (9)$$

$$\frac{d\Pi(k, n, \phi)}{d\phi} = \frac{1}{(1 + n \cdot \sin(\phi)^2) \cdot \sqrt{1 - k^2 \cdot \sin(\phi)^2}}. \quad (10)$$

Another partial derivative can be computed regarding " $k$ " parameter ((12), (13), (14)). The demonstration ((11)) is done for second kind function  $E(k, \Phi)$

$$\begin{aligned} \frac{d}{dk} \sqrt{1 - k^2 \cdot \sin(\theta)^2} &= \frac{-k \cdot \sin(\theta)^2}{\sqrt{1 - k^2 \cdot \sin(\theta)^2}} = \frac{1}{k} \cdot \frac{1 - k^2 \cdot \sin(\theta)^2 - 1}{\sqrt{1 - k^2 \cdot \sin(\theta)^2}} \\ &= \frac{1}{k} \cdot \left( \sqrt{1 - k^2 \cdot \sin(\theta)^2} - \frac{1}{\sqrt{1 - k^2 \cdot \sin(\theta)^2}} \right). \end{aligned} \quad (11)$$

$$\text{Then,} \quad \frac{d}{dk} E = \frac{1}{k} \cdot (E - F). \quad (12)$$

The same can be done for " $F$ " and " $\Pi$ "

$$\begin{aligned} \frac{d}{dk} F &= \frac{1}{k} \cdot \left( \frac{E}{1 - k^2} - F \right) \\ &+ \frac{k \cdot \sin(2 \cdot \phi)}{2 \cdot (k^2 - 1) \cdot \sqrt{1 - k^2 \cdot \sin(\phi)^2}} \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{d}{dk} \Pi &= \frac{-k}{k^2 + n} \cdot \left[ \frac{E}{k^2 - 1} + \Pi \right. \\ &\left. - \frac{k^2 \cdot \sin(2 \cdot \phi)}{2 \cdot (k^2 - 1) \cdot \sqrt{1 - k^2 \cdot \sin(\phi)^2}} \right]. \end{aligned} \quad (14)$$

Last partial derivative for " $\Pi$ " regarding " $n$ "

$$\begin{aligned} \frac{d}{dn} \Pi &= \frac{1}{2 \cdot n(n+1) \cdot (k^2 + n)} \cdot [n \cdot E - (k^2 + n) \cdot F \\ &+ (k^2 - n^2) \cdot \Pi + \frac{n^2 \cdot \sqrt{1 - k^2 \cdot \sin(\phi)^2} \cdot \sin(2 \cdot \phi)}{2 \cdot (1 + n \cdot \sin(\phi))}] \end{aligned} \quad (15)$$

## III. SOFTWARE IMPLEMENTATION

### A. CADES Framework

A framework detailed in [9] and available at [10], has been developed for optimization requirements. The modeling language is based on the analytical model description made of algebraic equations and explicitly defined functions. However, this simple description may be limited in electromagnetic, and semi-analytical modeling is also available thanks to numerically computed external functions with analytical model call back possibilities.

1) *Semi-Analytical Modeling*: A standard interface has been defined to integrate numerical code into CADES Framework called "external functions." Many experiences have been done to compose heterogeneous solvers like FLUX2D, SABER, MATLAB, ... using this functionality. In our study two numerical code are required, the Elliptic functions library to compute magnetic fields and a numerical integration routine to evaluate forces. The challenge in modeling for optimization is to evaluate the model sensitivity.

2) *Global Sensitivity Modeling*: The external function interface can also provide a Jacobian matrix for each numerical routine. First of all, partial derivative of magnetic fields is given by partial derivatives of elliptic functions composed with standard functions depending on the magnetic field formulas. Then, partial derivative of forces is expressed from integral derivation and

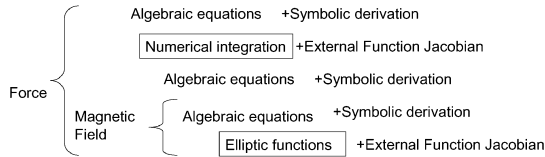


Fig. 4. Global sensitivity composition.

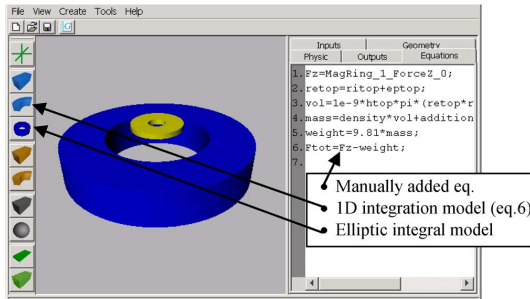


Fig. 5. MacMMems: semi-analytical modeling of magnetic components.

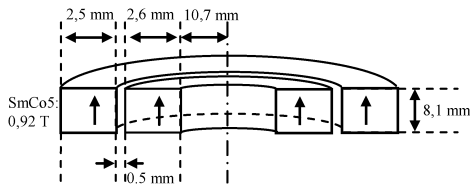


Fig. 6. Simple magnetic bearing geometry description.

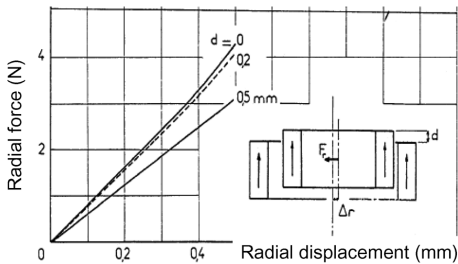


Fig. 7. Radial force vs. radial displacement for several axial positions.

field derivatives (see Fig. 4). The composition is done automatically in the CADES Framework to produce the global Jacobian matrix.

P. Enciu describes CADES semi-analytical modelling and Jacobian compositions with A.D. (automatic differentiation) technique. Compared with our symbolic approach, the A.D. technique is general but slower during computation [11].

In order to reach high performances, CADES framework is also able to select a path in the partial derivative composition tree. Indeed, during optimization only objective function and constraints have to be derivate regarding design parameters.

**B. MacMMems: Macro Modeler for Magnetic Memes**

Software has been developed in order to capitalize analytical and semi-analytical models for simple shape magnetic bodies (magnets, coils, diamagnetic and analytical ferromagnetic) and make them interact (forces and torques computation). Equations can also easily be added.

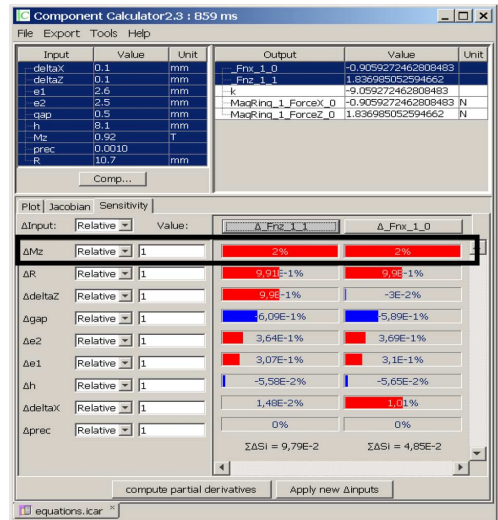


Fig. 8. Sensitivity calculation (859 ms) of axial (Fnx) and radial (Fnz) regarding all parameters, sorted by Fnx most influenced parameters.

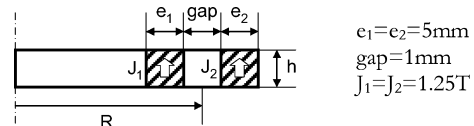


Fig. 9. Magnets parameters to be designed.

When system description is done, MacMMems generates equations, which are passed through CADES to produce an autonomous computation code (software component) with symbolic derivatives.

**IV. EXAMPLE OF ELEMENTARY MAGNETIC BEARING**

**A. Geometry Description**

Fig. 6 presents the geometry of a simple magnetic bearing based on axially magnetized ring shaped magnets.

**B. Comparison With Measures**

Fig. 7 presents radial force measurements [12], [13]. Axial and radial forces have been retrieved using the modeling formulas.

**C. Sensitivity Analysis**

Fig. 8 presents CADES calculator on this model and details sensitivity values. These sensitivities are computed very quickly using model Jacobian matrix and are very accurate to evaluate stiffness. The most influencing parameter is magnetization value (Mz) which increase Fnx and Fnz of 2% when it increase of 1%.

**V. STIFFNESS OPTIMIZATION**

**A. Stiffness Regarding Magnets Height**

Radial stiffness  $(Fr(\Delta r))/\Delta r$  can be optimized regarding geometrical and physical parameters. In our basic example, only height parameter (“h” in Fig. 9) can reach an optimum.

Fig. 10 shows that an optimal value of height exists to obtain a maximal value of the stiffness.

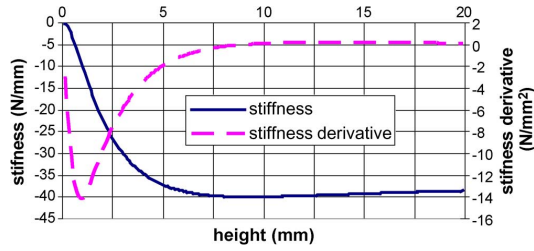


Fig. 10. Radial stiffness and its derivative regarding magnets height.

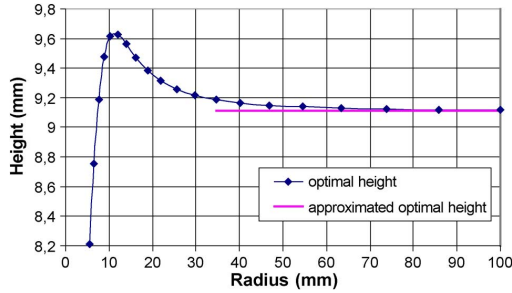


Fig. 11. Optimal height values regarding magnet radius.

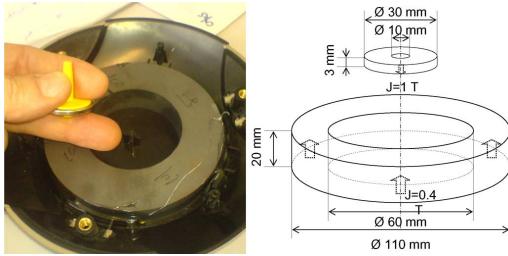


Fig. 12. Levitron photography and geometrical parameterization.

### B. Optimal Height Depending on Radius

In the approximation of infinite radius, and for geometry with  $e_1 = 5$ .gap (see parameters name in Fig. 9), it has been proved that optimal height =  $\sqrt{83} = 9.11$  mm [12], [13]. This result has been obtained by a model where rings are approximated by 2D rectangular section magnets. With our new model and its derivatives it is possible to find more accurate optimization results for low radius.

Fig. 11 has been obtained by successive optimization using quasi-Newton algorithm (*SQP: sequential quadratic programming*) allowing finding optimal height very quickly (*all points are obtained in less than 15 seconds*) due to noiseless model derivatives.

## VI. APPLICATION TO LEVITRON IDENTIFICATION

Levitron is based on axial magnetized cylindrical magnets, a fixed one and a smaller mobile one on the top. Magnetizations are opposed, creating an unstable repulsing force which is stabilized by gyroscopic effect.

We have measured such a system (*geometry and magnetic induction values in several points*) to identify magnets magnetization. A least square procedure has been done with our model using sensitivities to ensure quick and accurate identification convergence.

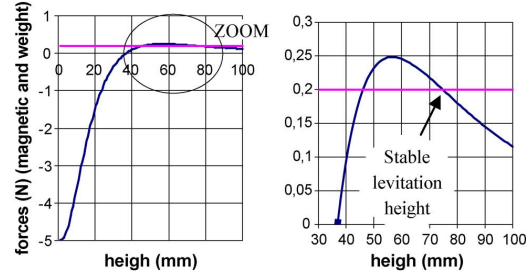


Fig. 13. Levitron magnetic force and weight giving levitation height.

Simulated levitation height is founded at 7.5 cm Fig. 13 (*intersection with weight*), which correspond well to our measurements.

## VII. CONCLUSIONS

Partial derivatives of elliptic functions are given in order to be composed and to produce the partial derivative of magnetic field produced by circular shaped magnets. Using integration symbolic derivation, partial derivatives of forces are also available. This modeling technique has been implemented in our modeling and optimization framework and has shown very good performances regarding other methods. This original and generic approach has been demonstrated in three cases:

- Magnetic bearing sensitivity study;
- Magnetic bearing stiffness optimization;
- Levitron identification and simulation.

Using symbolic gradients and dedicated tools to generate it automatically, brings significant improves regarding classical method (*manual analytical modeling, or numerical with generic software*).

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