Fuzzy vs probability uncertainty analysis of seismic displacement measurements issued from D-InSAR and SAR image correlation measurement: Application to the Kashmir earthquake (2005)

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Abstract—**An emerging way to reduce the geodetic parameter uncertainty is to combine the large numbers of data provided by satellite radar images. However, the measurements by radar images are subjected both to random and epistemic uncertainties. Thus, mathematical theories which are adequate for each type of uncertainty representation and handling have to be selected. Probability theory is known as the adequate theory for uncertainties corresponding to random variables, but questionable for epistemic uncertainties, arising from information incompleteness. Fuzzy theory being a generalization of interval mathematics, it is more adapted to such uncertainty. Moreover it provides a bridge with probability theory by its ability to represent a family of probability distributions. Therefore, we consider here the conventional probability and the fuzzy approaches for handling the random and epistemic uncertainties of D-InSAR and SAR correlation measurements. The applications are performed on the measurement of displacement field due to the Kashmir earthquake (2005).**

Keywords: measurement uncertainty, fuzzy theory, remote sensing, SAR image, ground displacement, seismic observations

I. INTRODUCTION

Geodetic data, such as satellite images (radar and optic), are very important remote sensing sources of information for ground displacement measurement with great accuracy over large area. So far, with the increasing number of operational sensors, large volumes of SAR images acquired in different modes, ascending and descending passes at various incident angles and frequencies, are available. Moreover, the launching in the following years of the future satellite generation Sentinel will provide a large number of free SAR data [1]. Consequently, using large number of geodetic measurements in order to accurately determine the displacement is becoming more and more frequent in geophysics, especially to better constraint the geophysical modeling [2][3][4][5]. In this context, one important purpose of geodetic data processing is to reduce parameter uncertainty by an adequate combination of all the available measurements. The classical causes for uncertainty are generally divided in three classes: gross errors, epistemic errors, and random errors. Gross errors are due to incorrect use of instrument and they lead to outliers that have to be avoided or detected by control methods. Epistemic errors are due to a lack of knowledge about the phenomenon, and they have to be reduced by correction methods and additional knowledge. The remaining error is considered as random, and is due to phenomenon variability. Concerning the measurements by radar (SAR) image, the uncertainties arise from noise sources of radar instrument, on the path of radar wave propagation, at the reflecting surface, as well as uncertainty sources introduced by data processing [4]. On one hand, random uncertainty exists due to uncorrelated noise, since there are usually some backscattering property changes on the ground between two subsequent SAR acquisitions. On the other hand, epistemic uncertainties can be induced by atmospheric disturbances depending on the state of atmosphere and the ground surface at the time of the two SAR acquisitions. Also, it can result from the imprecision of orbit auxiliary information, Digital Elevation Model errors, as well as from the imperfect corrections during data processing, which deviate the data by a constant or a ramp from the true value.

To model such epistemic uncertainties coming from limited knowledge, probability theory is questionable, and thus fuzzy theory has been has been proposed in [6][7][8] and further developed by a few authors in a general measurement context [9][10][11][12], and also by a few authors in geosciences [13][14][15]. Thus, it is worthwhile to study the combination of the most suitable uncertainty theories. This paper is a first contribution to such issues for the measurement of displacement field by a joint inversion of D-Insar and SAR image correlation measurements by a least squares adjustment.

This paper is organized as follows. In section II, the conventional probability approach and the fuzzy approach for uncertainty representation and propagation are detailed. The available data in the considered application and their associated uncertainties are described in section III. Then, the two approaches are applied to the 3D displacement field measurement at the Earth's surface due to the Kashmir earthquake (M_w=7, 2005). The behaviours of each uncertainty approach are highlighted through inter-comparisons of results. Finally, some conclusions and perspectives are drawn.

II. UNCERTAINTY ANALYSIS

In geodetic practice, there are always multiples sources of uncertainty in the considered measurement, which leads to complex characteristics for the associated uncertainties. Thus, mathematical theories which are adequate for at least some parts of uncertainty representation and handling have to be considered in a competitive way. Probability theory is the adequate theory for uncertainties corresponding to random variables, the latter being described by one probability distribution (often a Gaussian one) or more simply by the first two moments, i.e. the mean and the variance. Epistemic uncertainties, arising from information incompleteness, are often described by an interval and thus cannot be associated to one single probability distribution. Fuzzy theory can be understood as a generalization of interval mathematics and provides a bridge with probability theory by its ability to represent a family of probability distributions [16]. Therefore, hereafter, we consider the conventional probability and fuzzy approaches for handling the random and epistemic uncertainties.

A. Conventional probability approach

The standard reference in uncertainty modelling is the "Guide to the Expression of Uncertainty in Measurement (GUM)" edited by an international consortium of legal and professional organizations [17]. GUM groups the occurring uncertain quantities into "Type A" and "Type B". Uncertainties of "Type A" are determined with the classical statistical methods, while "Type B" is subject to other uncertainties like experience with and knowledge about an instrument. Both types of uncertainty can have random and epistemic components. In fact, the GUM proposes to treat both uncertainties (random and epistemic) in a stochastic framework and introduces variances to describe their uncertainties and treats them with the law of propagation of variance, generally assuming independence. Applying this approach to linear inversion by a least squares adjustment, the uncertainties are propagated as follows. Let us consider $f_k(x_1, x_2, \ldots, x_n)$ a set of *m* functions which are linear combinations of *n* variables with coefficients $a_{1,k}, a_{2,k}, \ldots, a_{n,k}, (k = 1, \ldots, m)$. Thus:

$$f_k = \sum_{i=1}^{n} a_{i,k} x_i : \quad f = A^T x$$

If the variance-covariance matrix of x is denoted by Σ_x:

$$\Sigma_x = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & . \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} & . \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 & . \\ . & . & . & . \end{pmatrix}$$

Then the variance-covariance matrix Σ_f of f is given by:

$$\Sigma_{f_{i,j}} = \sum_{k}^{n} \sum_{l1}^{n} a_{i,k} \Sigma x_{k,l} a_{l,j} : \quad \Sigma_f = A^T \Sigma_x A$$

When the data vector R of dimension n is related to the model vector u of dimension n by a nxm matrix P: $R=Pu$, then the inversion by the generalized least squares method leads to:

$u = \left(P^T \Sigma_R^{-1} P\right) P^T \Sigma_R^{-1} R$ with Σ_R the variance-covariance matrix of R.

For determining confidence intervals for the parameter, the GUM suggest to use a Gaussian distribution (justified by the central limit theorem), and for other distributions to apply Monte Carlo simulations.

This well known approach is fully justified in cases of a lot of data having independent random uncertainties but questionable for epistemic uncertainties often dependent and far from a Gaussian representation, and then it generally leads to an over-optimistic assessment of the uncertainties.

B. Fuzzy approach

The possibility theory, was first introduced by L. Zadeh in 1978 [18], it is associated with the theory of fuzzy sets by the semantics of uncertainty that it gives the membership function. A possibility distribution π is a mapping from a set to the unit interval such that $\pi(x) = 1$ for some x belonging to the set of reals. A possibility distribution π_1 is called more specific (i.e. more thinner in a broad sense) than π_2 as soon as $\forall x \in R, \pi_1(x) \le \pi_2(x)$ (fuzzy set inclusion). The more specific π, the more informative it is. If $\pi(x) = 1$ for some x and $\pi(y) = 0$ for all $y \ne x$, then π is totally specific (fully precise and certain knowledge), if $\pi(x) = 1$ for all x then π is totally non specific (complete ignorance).

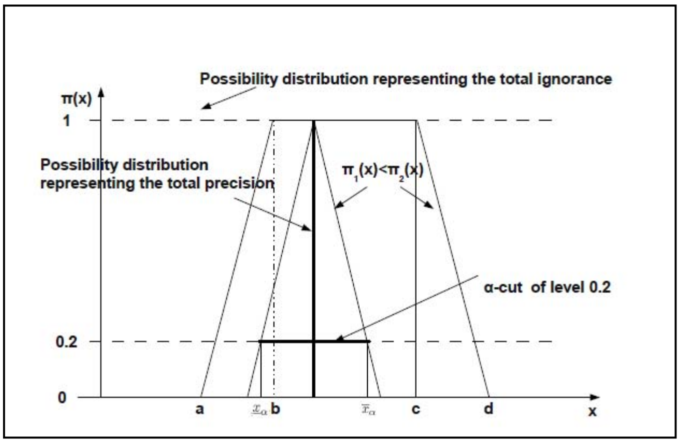


Figure 1. Examples of possiblity distributions

In fact, a numerical degree of possibility can be viewed as an upper bound to a probability degree [16]. Namely, with every possibility distribution π one can associate a non-empty family of probability measures dominated by the possibility measure: $\mathcal{P}(\pi) = \{P, \forall A \subset R, P(A) \le \Pi(A)\}$. This provides a bridge between probability and possibility, and there is also a bridge with interval calculus. Indeed, a unimodal numerical possibility distribution may also be viewed as a nested set of coverage intervals, which are the α-cuts of π : $[\underline{x}_\alpha, \overline{x}_\alpha] = \{x, \pi(x) \ge \alpha\}$. Obviously, the confidence intervals built around the same point x_0 are nested. It has been proven in [7] that stacking the coverage intervals of a probability distribution F on top of one another leads to a possibility distribution (denoted π^{x_0} having x_0 as modal value). In fact, in this way, the α-cuts of π^{x_0} are identified with the confidence interval I_β^* of probability level $\beta = 1-\alpha$ around the nominal value x_0 . In this way a probability distribution can be represented by an equivalent possibility distribution. Moreover, a possibility distribution can be used for representing a family of probability distributions by taking the largest $1-\alpha$ confidence intervals obtained from each probability distributions of the family. This is useful to represent uncertainty when only partial probability knowledge is available. For example if the variable is known to be bounded and unimodal (with mode R) and with σ_R as standard deviation, then the maximum specific possibility distribution is a triangular possibility distribution with the mode R as vertex and with $\left[R-\sqrt{3}\sigma_R, R+\sqrt{3}\sigma_R\right]$ as support. In addition, due to the confidence interval interpretation, a possibility distribution is a convenient way to express expert knowledge. To represent the uncertainty in the fuzzy approach by a single parameter (in a way similar as the variance in the probability approach) the commonly used parameter is the full width at half maximum of the possibility distribution which corresponds to a α-cut level of *0.5*. In summary a possibility distribution can both model random and epistemic uncertainties in a unified modeling. Afterwards, the possibility distributions are propagated in the least square adjustment using fuzzy arithmetic based on Zadeh's extension principle [18]:

$$u = \left(P^T \Sigma_R^{-1} P\right) P^T \Sigma_R^{-1} \otimes R \text{ with } \otimes \text{ the fuzzy multiplication.}$$

In this principle, the variables are considered as non interactive variables; this corresponds somehow to consider a total dependence between variables. Consequently, uncertainty propagation by a fuzzy approach leads to an over-pessimistic assessment of the uncertainties.

C. Displacement uncertainty analysis by the two approaches

Here, to combine the available data, we apply the Generalized Least Squares (GLS) method which gives the nominal displacement value and its corresponding variance from the variances of the measurements provided by the sources. For the fuzzy approach, as no fully equivalent fuzzy least square method is yet available, we use the GLS method to obtain the forward model. Then we build the possibility distributions of the displacement from the value R and their associated uncertainty σ_R, considering σ_R contains both random and epistemic components. Moreover, the measurements are considered as bounded (this is the case in the considered context), thus we represent them by a symmetric triangular fuzzy distribution with support $\left[R-\sqrt{3}\sigma_R, R+\sqrt{3}\sigma_R\right]$. Therefore, the full width at half maximum is $2\sqrt{3}\sigma_R$. Let us remark that with a Gaussian assumption of standard deviation σ_R, the value corresponding to the 0.5 α-cuts the width of the equivalent possibility distribution is equal to 1.35 σ_R .

III. Application to the Kashmir earthquake (2005)

A. Description of the available data

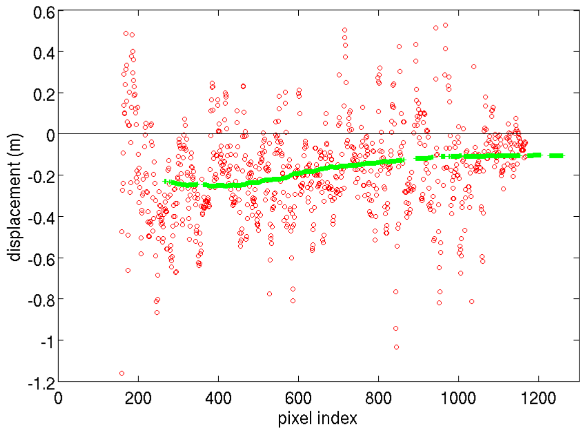
The sub-pixel image correlation and the differential interferometry (D-InSAR) are two conventional techniques used to extract displacement measures from SAR data. The sub-pixel image correlation computes the offsets in range (line of sight) and azimuth (along the trajectory of satellite) directions on amplitude images, with a sub-pixel accuracy. It is widely used to measure the displacement of great magnitude [19] [20] [21]. The D-InSAR provides the phase information, which means the displacement, in range direction with an uncertainty of order of the cm, even mm [22][23]. This technique is usually applied to measure the displacement of small magnitude. In case of a strong earthquake induced by a rupture of a fault, in near field of the fault, the measures from sub-pixel image correlation can provide reliable displacement information. While in far field of the fault, the measures from D-InSAR are taken as accurate sources [19].

In this paper, a series of coseismic ENVISAT images from October 2004 to June 2006 is used to map the deformation due to the Kashmir (2005) earthquake. 22 measures from subpixel image correlation and 5 measures from D-InSAR, are available respectively. In near field of the fault, because of coherence loss, phase information cannot be extracted by D-InSAR, thus there is no D-InSAR measurements available in this area. These measurements can be classified in four families according to their acquisition geometry: ascending range (Asc. Rg), ascending azimuth (Asc. Az), descending range (Des. Rg) and descending azimuth (Des. Az). In each family, in first approximation, all the measurements are considered as corresponding to the same displacement (in the same direction) because the incident angle is the same for all the measurements.

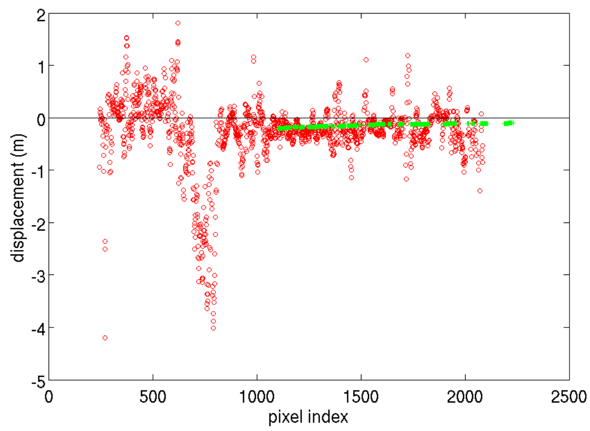
For measurements from sub-pixel image correlation, the uncertainty parameter is the pseudo-variance associated with the displacement value obtained from the correlation technique in ROIPAC software [24], which defines the quality of the cross-correlation between two amplitude images. On one hand, random uncertainty exists in these measurements because of the uncorrelated noise present in the data. On the other hand, epistemic uncertainty may be present due to the DEM errors or the defect of correlation method, which is difficult to quantify. As a result, the pseudo-variance used as uncertainty parameter is considered to include both random and epistemic components. Note that the possible epistemic uncertainty due to the imperfect data processing is not included in the pseudo-variance.

For measurements from D-InSAR, the uncertainty parameter corresponds to the variance of the phase value estimated from the coherence [25]. It characterizes mainly random variations in the phase value. However, epistemic uncertainty due to phase unwrapping errors, atmospheric impact, etc., is probably present in the measurement, which should be taken into account in the uncertainty management approaches.

Fig. 2 shows two examples of measurement uncertainty values issued both from sub-pixel image correlation and D-InSAR. The two profiles are issued from displacement images.



(a)



(b)

Figure 2. Examples of uncertain measurements

The first profile is located in an area far from the fault and the second profile passes across the fault. In theory, the same displacement values should be found by these two sources where both measurements are available, as they measure exactly the same quantity of displacement. However, with the presence of uncertainty in both measurements, a discrepancy of displacement values is observed between both profiles. On one hand, a more or less significant fluctuation of displacement value is observed in the sub-pixel image correlation measurements, which complies with the presence of random uncertainty. Near the fault, the fluctuation is small. While in the area far from the fault where the displacement magnitude is small, the fluctuation becomes significant. Consequently, it is probable that epistemic uncertainty is also present in one or the other measurement. However, it seems that the random uncertainty is more important than the epistemic uncertainty in our data sets.

B. 3D Displacement measurement field

The different measurements from sub-pixel image correlation and DInSAR are the different projections of the 3D displacement at the Earth's surface (E, N, Up) in range and azimuth directions. Consequently, the 3D displacement field can be constructed from at least 3 different projections by a linear inversion. In this case, R corresponds to the different measures from sub-pixel image correlation and D-InSAR. P corresponds to the projection vectors matrix. u denotes the 3D displacement with 3 components E, N, Up. To resolve this linear inverse problem, the generalized least square methods are used. In order to highlight the behaviours of probability and fuzzy approaches, three levels of comparisons are considered: between displacement values, between uncertainties and between distributions. Moreover, the effect of uncertainty reduction due to adding D-InSAR measures is analyzed.

C. Probability and fuzzy uncertainty parameter assessment

Regarding the nominal displacement value, the results in different cases are globally consistent, with an average difference in order of mm. Regarding the uncertainty parameter value, the evolution varies from one case to another. The uncertainties in fuzzy approach are always larger than those in probability approach. Adding data sets from D-InSAR reduces the uncertainties in both conventional and fuzzy approaches.

In order to understand the spatial evolution of uncertainty in both approaches, the ratio of conventional uncertainty on fuzzy uncertainty is performed. A geographic effect is observed (Fig. 3).

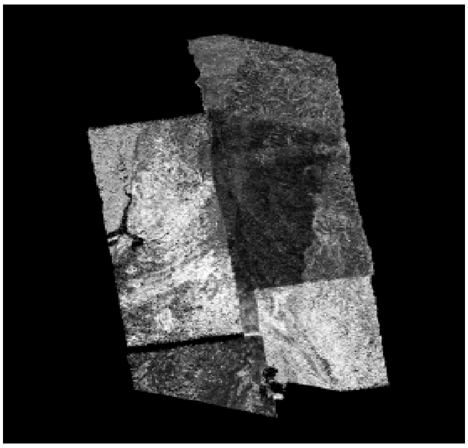


Figure 3. Examples of ratio of probability and fuzzy uncertainty parameters for the E component

In the darker area, the difference between the two uncertainty parameter is large, because in this area, there are more data sets available. In conventional approach, the output uncertainty is reduced. In fuzzy approach, on the contrary, the output uncertainty remains constant or increases slightly. Consequently, the difference between the two uncertainty parameters increases.

D. Discussion and work in progress

According to the foundations of probability and fuzzy theories, the uncertainty in the considered probability approach is under estimated, while the uncertainty in the proposed fuzzy approach is over estimated. Therefore, in the context of our data sets, the actual uncertainty should be situated between these two uncertainties. In the final paper the comparison will be further detailed by making the comparison of the probability and possibility distributions. Results with a pre-fusion scheme which consists in fusing some data before entering the joint inversion (made by a least square adjustment in our case) will be presented.

Furthermore, the application of the fuzzy approach to reduce uncertainty for seismic fault parameter estimation by nonlinear inversion of a mechanical deformation model will be investigated.

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REFERENCES

[1] E. Attema. Mission requirements document for the european radar observatory sentinel-1. ES-RS-ESA-SY-0007, 1, 2005.

[2] R. Pedersen, S. Jhonsson, T. Arnadhottir, F. Sigmundsson, and K. L. Feigl. Fault slip distribution of two June 2000 Mw6:5 earthquake in South Iceland estimated from joint inversion of InSAR and GPS measurements.Earth and Planetary Science Letters, (213), 2003.

[3] D. A. Schmidt, R. Burgmann, R. M. Nadeau, and M. d'Alessio. Distribution of aseismic slip rate on the Hayward fault inferred from seismic and geodetic data. Journal of Geophysical Research, 110(B08406), 2005.

[4] H. Sudhaus and S. Jhonsson. Improved source modelling through combined use of InSAR and GPS under consideration of correlated data errors: application to the June 2000 Kleifarvatn earthquake, Iceland. Geophysical Journal International, (176), 2009.

[5] R. Grandin, A. Socquet, R. Binet, Y. Klinger, E. Jacques, J.B. de Chabalier, G.C.P. King, C. Lasserre, S. Tait, P. Tapponnier, A. Delorme, and P. Pinzuti. September 2005 Manda Hararo-Dabbahu rifting event, Afar (Ethiopia): Constraints provided by geodetic data. Journal of Geophysical Research, 114(B08404), 2009.

[6] Mauris G., Lasserre V., Foulloy L., Fuzzy modeling of measurement data acquired from physical sensors, *IEEE Trans. on Measurement and Instrumentation*, Vol. 49, No. 6, December 2000, pp. 1201-1205.

[7] Mauris G., Lasserre V., Foulloy L., A fuzzy approach for the expression of uncertainty in measurement, *Int. Journal of Measurement*, Vol. 29, No 3, March 2001, pp. 165-177.

[8] Mauris G., Expression of measurement uncertainty in a very limited knowledge context: a possibility theory-based approach, IEEE Trans. on Instrumentation and Measurement, Vol. 56, No. 3, 2007, pp. 731-735.

[9] Urbanski M., Wasowki J., Fuzzy approach to the theory of measurement inexactness, *Int. Journal of Measurement*, Vol. 34, 2003, pp. 67-74

[10] Ferrero A., Salicone S., The random-fuzzy variables: a new approach to the expression of uncertainty, *IEEE Trans. On Instrumentation and Measurement*, Vol. 53, No 5, 2004, pp. 1370-1377.

[11] Ferrero A., Salicone S., A comparative aanlysis of the statistical and random-fuzzy approaches in the expression of uncertaitny in measurement, *IEEE Trans. On Instrumentation and Measurement*, Vol. 54, No 4, 2005, pp. 1475-1481.

[12] Mencattini A., Salicone S., How to process the random part of RFVs: comparison of available methods and new proposals , *IEEE Trans. On Instrumentation and Measurement*, Vol. 59, No 1, 2010, pp. 15-26.

[13] N.E.C. Verhoest, B. De Baets, F. Mattia, G. Satalino, C. Lucau, and P. Defourny. A possibilistic approach to soil moisture retrieval from ers synthetic aperture radar backscattering under soil roughness uncertainty. Water Resources Research, 43(W07435), 2007.

[14] P.P. Mujumdar and S. Ghosh. Modeling gcm and scenario uncertainty using a possibilistic approach: Application to the mahanadi river, india. Water Resources Research, 44(W06407), 2008.

[15] A.P. Jacquin. Possibilistic uncertainty analysis of a conceptual model of snowmelt runoff. Hydrology and Earth System Sciences, 14:1681–1695, 2010.

[16] Dubois D., Foulloy L., Mauris G., Prade H., Probability-possibility transformations, triangular fuzzy sets and probabilistic inequalities, Reliable Computing, Vol. 10, No 4, 2004, pp. 273-297.

[17] Guide for the expression of uncertainty in measurement, ISO, second edition, 1995.

[18] L. A. Zadeh. Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems, 1(1):3–28, 1978.

[19] E. Pathier, E. J. Fielding, T. J. Wright, R. Walker, B. E. Parsons, and S. Hensley. Displacement field and slip distribution of the 2005 Kashmir earthquake from SAR imagery. Geophysical Research Letters, 33(L20310), 2006.

[20] S. H. Yun, H. Zebker, P. Segall, A. Hooper, and M. Poland. Interferogram formation in the presence of complex and large deformation. Geophysical Research Letters, 34(L12305), 2007.

[21] T. J. Wright, C. Ebinger, J. Biggs, A. Ayele, G. Yirgu, D. Keir, andA. Stork. Magma-maintained rift segmentation at continental rupture in the 2005 Afar dyking episode. Nature, 442(doi:10.1038/nature04978), 2006

[22] Davidson, Russell, Mackinnon, and James G. Estimation and inference in econometrics. Oxford University Press, isbn 978-0-19-506011-9 edition, 1993.

[23] P.A. Cornillon and E. Matzner-Lober. Rgression, Theorie et applications. Springer Paris edition, 2007.

[24] P.A. Rosen, S. Hensley, G. Peltzer, and M. Simons. Updated Repeat Orbit Interferometry Package released. Transactions, American Geophysical Union, 85(47), 2004.

[25] E. Trouvé, J.M. Nicolas, and H. Maitre, Improving Phase Unwrapping Techniques by the Use of Local Frequency Estimates, IEEE Transaction on Geoscience and Remote Sensing, 36(6), 1998.