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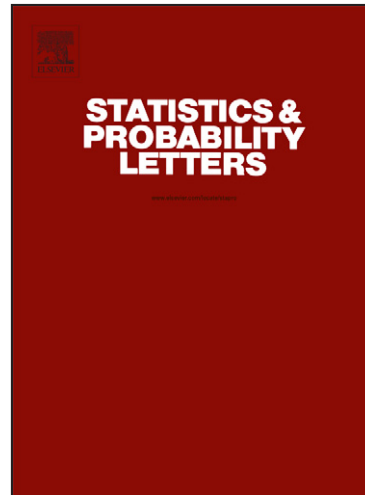
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A game version of the Cowan-Zabczyk-Bruss' problem

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Abstract

The paper deal with the continuous-time two person non-zero sum game extension of the no information secretary problem. The objects appear according to the compound Poisson process and each player can choose only one applicant. If both players would like to select the same one, then the priority is assigned randomly. The aim of the players is to choose the best candidate. A construction of Nash equilibria for such game is presented. The extension of the game with randomized stopping times is taken into account. The Nash values for such extension is obtained. Analysis of the solutions for different priority defining lotteries are given.

Key words: stopping time, stopping game, Markov process, compound Poisson process, non-zero sum game, random priority, randomize stopping time

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1 Introduction

The main topic of the paper is a game version of a continuous-time generalization of the secretary problem (SP). The part of long history of SP and its generalization has been presented in survey papers by Ferguson (1989) and Samuels (1991). The game versions of the problem has been reviewed by Sakaguchi (1995) and Nowak and Szajowski (1998). A continuous-time version of the SP with a random number of object in a finite time interval was investigated by Cowan and Zabczyk (1978). Bruss (1987) extended this model by admitting a compound Poisson stream of options: a man has been allowed a fixed time T in which he has to find an apartment. Opportunities to inspect apartments occur at the epochs of a homogeneous Poisson process of unknown intensity λ with exponential prior. He inspects each apartment immediately

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when the opportunity arises and decides instantly whether to accept or not it. At any epoch he is able to rank a given apartment amongst all those inspected to date, where all permutations of ranks are equally likely and independent of the Poisson process. The objective is to maximize the probability of selecting the best apartment from those (if any) available in the interval $[0, T]$. The problem considered in this paper should be seen as research toward modeling environmental details, the relation between players (the decision makers) and circumstances of decisions. Let us be more specific.

Modeling relation between decision makers is important when there is only one stream of options (in continuous time modeled by some counting process). Two decision maker model of stopping the Markov process can be applied to investigate the competitive SP. A non-zero sum discrete time game approach considered by Szajowski (1994) gives model for the following situation. At each moment $n = 1, 2, \dots$ the decision makers (henceforth called Player 1 and Player 2) are able to observe the Markov chain sequentially. Each player has his utility function $g_i : \mathbb{E} \rightarrow \mathfrak{R}$, $i = 1, 2$, and at each moment n each decides separately if he accepts or rejects the realization x_n of X_n . If it happens that both players have selected the same moment n to accept x_n , then a lottery chooses Player 1 with a probability α_n to give him the right (priority) of the acceptance while Player 2 is chosen with the probability $1 - \alpha_n$. The player which has been rejected by the lottery may select any other realization x_n in the later moments. Once accepted realization cannot be rejected, once rejected cannot be reconsidered. The aim of each player is to choose a realization which maximizes his expected utility. The problem considered by Fushimi (1981) was a trigger for the consideration of Szajowski (1994). In fact, the problem will be formulated as a two person non-zero sum game with the concept of the Nash equilibrium as the solution. The problem with permanent priority for Player 1 (i.e. $\alpha_n = 1$, $n = 1, 2, \dots$) has been solved by Ferenstein (1992). The continuous time full-information two person SP with imperfect observation has been solved by Porosinski and Szajowski (1996).

In this paper the game considered in Szajowski (1994) is generalized to the continuous-time version of the SP problem investigated in Bruss (1987). The mathematical model will be presented and equilibria for each α defining priority will be derived in Section 4 where also interesting properties of some solutions are point out (see also conclusion in Section 5). The description of the stream of option is presented in Section 2 and the definition of the strategies and the solution in the game version is given in Section 3.

2 The optimal stopping of compound Poisson stream of options

2.1 Formulation of the best choice problem

Let S_1, S_2, \dots denote the arrival times of the Poisson process $\{N_t\}_{t \geq 0}$. For unknown intensity λ an exponential prior density $g(\lambda) = ae^{-a\lambda} \mathbb{I}_{\{\lambda > 0\}}(\lambda)$ is

assumed, where a is known, positive parameter. By Bayes' theorem, the conditional posterior density is of the form

$$\begin{aligned} f(\lambda|S_j = s) &= f(\lambda|S_j = s, S_{j-1} = s_{j-1}, \dots, S_1 = s_1) \\ &= \frac{\lambda^j}{j!} (s+a)^{j+1} e^{-(s+1)\lambda} \mathbb{I}_{\{\lambda>0\}}(\lambda), s \in [0, T] \end{aligned}$$

and

$$\begin{aligned} \mathbf{P}(N(T) = n|S_1 = t_1, \dots, S_{j-1} = t_{j-1}, S_j = s) \\ = \mathbf{P}(N(T) = n|S_j = s) = \binom{n}{j} \left(\frac{s+a}{T+a}\right)^{j+1} \left(1 - \frac{s+a}{T+a}\right)^{n-j}. \end{aligned} \quad (1)$$

Let (j, s) denote the state of the process, when the option number j arrives at time s . Define the relative rank of the j -th option by Y_j and its absolute rank by X_j (for the details see Suchwałko and Szajowski (2002)). Based on observation of the relative ranks and the moments of arrivals of the candidates the aim is to stop on the best option.

Let $\mathcal{F}_t = \sigma\{N_t, Y_1, Y_2, \dots, Y_{N_t}\}$ and let \mathfrak{M} be the set of all stopping times with respect to σ -fields $\{\mathcal{F}_t\}_{t \geq 0}$.

$$\mathbf{P}(X_{\tau^*} = 1) = \sup_{\tau \leq T} \mathbf{P}(X_\tau = 1)$$

One can consider the arrival times only and $\mathcal{F}_n = \sigma\{S_1, \dots, S_n, Y_1, \dots, Y_n\}$, because \mathcal{F}_t for $S_n \leq t < S_{n+1}$ is equivalent with \mathcal{F}_n . We can consider equivalently

$$\mathbf{P}(X_{\sigma^*} = 1) = \sup_{\sigma} \mathbf{P}(X_\sigma = 1).$$

2.2 Solution of the problem of stopping on the best

For further consideration we have $\xi_j = (j, S_j)$. Let us define

$$W_j(s) = \sup_{\tau \geq j} \mathbf{P}(X_\tau = 1|S_j = s, Y_j = 1)$$

and $U_j(s) = \sum_{n=j}^{\infty} \mathbf{P}(X_j = 1, N(T) = n|S_j = s, Y_j = 1)$.

We have (see Gilbert and Mosteller (1966))

$$\mathbf{P}(X_j = 1, N(T) = n|Y_j = 1) = \frac{j}{n} \quad (2)$$

We calculate $U_j(s)$ using (2) and (1) (see Bruss (1987)):

$$U_j(s) = \sum_{n=j}^{\infty} \frac{j}{n} \binom{n}{j} \left(\frac{s+a}{T+a}\right)^{j+1} \left(1 - \frac{s+a}{T+a}\right)^{n-j} = \frac{s+a}{T+a}.$$

Define the probability of realizing the goal doing one step more starting from (j, s)

$$V_j(s) = \int_0^{T-s} \sum_{k=1}^{\infty} p_{(j,s)}^{(k,u)} W_{j+k}^{r_{j+k}}(s+u) du$$

where

$$p_{(j,s)}^{(k,u)} = \int_0^{\infty} \mathbf{P}(S_{j+k} = s+u | S_j = s, \lambda) \\ \times \mathbf{P}(Y_{j+k} = 1 | Y_j = 1, S_j = s, S_{j+k} = s+u, \lambda) \cdot f(\lambda | S_j = s) d\lambda$$

and

$$q(s, u, \lambda) = \mathbf{P}(Y_{j+k} = 1 | Y_j = 1, S_j = s, S_{j+k} = s+u, \lambda) \quad (3) \\ = \frac{j}{(j+k)(j+k-1)}.$$

By the theory of optimal stopping we have $W_j(s) = \max\{U_j(s), V_j(s)\}$ for $j = 1, 2, \dots, s \in [0, T]$.

We have (see Bruss (1987)) that

$$p_{(j,s)}^{(k,u)} = \int_0^{\infty} \frac{\lambda e^{-\lambda u} (\lambda u)^{k-1}}{(k-1)!} q(s, u, \lambda) \frac{e^{-\lambda(s+a)} \lambda^j (s+a)^{j+1}}{j!} d\lambda \quad (4) \\ = \frac{s+a}{(s+a+u)^2} \binom{j+k-2}{k-1} \left(\frac{s+a}{s+a+u}\right)^j \left(\frac{u}{s+a+u}\right)^{k-1}.$$

Let B be the one-step look-ahead stopping region. It means that B is the set of states (j, s) for which selecting the current relatively best option is at least as good as waiting for the next relatively best option to appear and then selecting it. Define additionally the average payoff for doing one step more by

$$R_j(s) = \int_0^{T-s} \sum_{k=1}^{\infty} p_{(j,s)}^{(k,u)} U_{j+k}(s+u) du \quad (5)$$

Therefore the set B is given by formula

$$B = \{(j, s) : U_j(s) - R_j(s) \geq 0\} \quad (6)$$

In order to find the set B we are solving the inequality from (6). Let us define

$$h_j(s) = U_j(s) - R_j(s) = \frac{s+a}{T+a} + \frac{s+a}{T+a} \log\left(\frac{s+a}{T+a}\right)$$

Then $B = \{(j, s) : s \geq s^*\}$, where $s^* = \frac{T+a}{e} - a$ and

$$V_j(s) = -\frac{s+a}{T+a} \log\left(\frac{s+a}{T+a}\right) \mathbb{I}_{\{s \geq s^*\}} + e^{-1} \mathbb{I}_{\{s < s^*\}}.$$

3 The game with random priority

In the problem of optimal stopping the basic class of strategies \mathfrak{M} are Markov times with respect to σ -fields $\{\mathcal{F}_n\}_{n=1}^{\infty}$. This class of strategies is not sufficient in the stopping game (see Yasuda (1985)). A strategy for Player 1 (2) is a random sequence $p = (p_n) \in \mathcal{P}$ ($q = (q_n) \in \mathcal{Q}$) such that, for each n : (i) p_n, q_n are adapted to \mathcal{F}_n ; (ii) $0 \leq p_n, q_n \leq 1$ a.s..

Let $\{A_i\}_{i=1}^{\infty}$ and $\{B_i\}_{i=1}^{\infty}$ be i.i.d.r.v. of the uniform distribution on $[0, 1]$ and independent of Markov process $(\xi_n, \mathcal{F}_n, \mathbf{P}_x)_{n=0}^{\infty}$ with the state space $\mathbb{E} = \mathbb{N} \times \mathfrak{R}^+$. Let \mathcal{H}_n be the σ -field generated by $\mathcal{F}_n, \{A_i\}_{i=1}^n$ and $\{B_i\}_{i=1}^n$. A randomized Markov time $\lambda(p)$ for strategy $p = (p_n) \in \mathcal{P}$ and $\mu(q)$ for strategy $q = (q_n) \in \mathcal{Q}$ are defined by $\lambda(p) = \inf\{n \geq 1 : A_n \leq p_n\}$ and $\mu(q) = \inf\{n \geq 1 : B_n \leq q_n\}$, respectively. We denote by $\tilde{\Lambda}$ and $\tilde{\mathbf{M}}$ the sets of all randomized strategies of Player 1 and Player 2.

The random assignment of the priority to the player requires to consider the modified strategies. Denote $\mathcal{T}_k = \{\tau \in \mathcal{T} : \tau \geq k\}$. One can define the set of strategies $\tilde{\Lambda} = \{(p, \{\sigma_n^1\}) : p \in \mathcal{P}, \{\sigma_n^1\} \in \mathcal{T}_{n+1} \text{ for every } n\}$ and let $\tilde{\mathbf{M}} = \{(q, \{\sigma_n^2\}) : q \in \mathcal{Q}, \{\sigma_n^2\} \in \mathcal{T}_{n+1} \text{ for every } n\}$ for Player 1 and Player 2, respectively.

Let $\{\varsigma_i\}_{i=1}^{\infty}$ be i.i.d.r.v. uniformly distributed on $[0, 1]$, independent of $\bigvee_{n=1}^{\infty} \mathcal{H}_n$, and the lottery is given by $\bar{\alpha} = (\alpha_1, \alpha_2, \dots)$. Denote $\tilde{\mathcal{H}}_n = \sigma\{\mathcal{H}_n, \varsigma_1, \dots, \varsigma_n\}$ and let $\tilde{\mathcal{T}}$ be the set of Markov times with respect to $(\tilde{\mathcal{H}}_n)_{n=0}^{\infty}$. For every pair (\mathbf{s}, \mathbf{t}) such that $\mathbf{s} \in \tilde{\Lambda}, \mathbf{t} \in \tilde{\mathbf{M}}$ we define $\tau_1(\mathbf{s}, \mathbf{t}) = \lambda(p)\mathbb{I}_{\{\lambda(p) < \mu(q)\}} + (\lambda(p)\mathbb{I}_{\{\varsigma_{\lambda(p)} \leq \alpha_{\lambda(p)}\}} + \sigma_{\mu(q)}^1\mathbb{I}_{\{\varsigma_{\lambda(p)} > \alpha_{\lambda(p)}\}})\mathbb{I}_{\{\lambda(p) = \mu(q)\}} + \sigma_{\mu(q)}^1\mathbb{I}_{\{\lambda(p) > \mu(q)\}}$ and $\tau_2(\mathbf{s}, \mathbf{t}) = \mu(q)\mathbb{I}_{\{\lambda(p) > \mu(q)\}} + (\mu(q)\mathbb{I}_{\{\varsigma_{\mu(q)} > \alpha_{\mu(q)}\}} + \sigma_{\lambda(p)}^2\mathbb{I}_{\{\varsigma_{\mu(q)} \leq \alpha_{\mu(q)}\}})\mathbb{I}_{\{\lambda(p) = \mu(q)\}} + \sigma_{\lambda(p)}^2\mathbb{I}_{\{\lambda(p) < \mu(q)\}}$. The Markov times $\tau_1(\mathbf{s}, \mathbf{t})$ and $\tau_2(\mathbf{s}, \mathbf{t})$ are selection times of Player 1 and Player 2.

For each $(\mathbf{s}, \mathbf{t}) \in \tilde{\Lambda} \times \tilde{\mathbf{M}}$ and given $\bar{\alpha}$ the payoff function for the i -th player is defined as $f_i(\mathbf{s}, \mathbf{t}) = g_i(X_{\tau_i(\mathbf{s}, \mathbf{t})})$. Let $\tilde{R}_i(j, s, \mathbf{s}, \mathbf{t}) = \mathbf{E}_{(j,s)} f_i(\mathbf{s}, \mathbf{t}) = \mathbf{E}_{(j,s)} g_i(\xi_{\tau_i(\mathbf{s}, \mathbf{t})})$ be the expected gain of i -th player if the players use (\mathbf{s}, \mathbf{t}) . We have defined the game in normal form $(\tilde{\Lambda}, \tilde{\mathbf{M}}, \tilde{R}_1, \tilde{R}_2)$. This random priority game will be denoted \mathcal{G}_{rp} .

Definition 3.1 A pair $(\mathbf{s}^*, \mathbf{t}^*)$ of strategies such that $\mathbf{s}^* \in \tilde{\Lambda}$ and $\mathbf{t}^* \in \tilde{\mathbf{M}}$ is called a Nash equilibrium in \mathcal{G}_{rp} if for all $(j, s) \in \mathbb{E}$

$$v_1(j, s) = \tilde{R}_1(j, s, \mathbf{s}^*, \mathbf{t}^*) \geq \tilde{R}_1(j, s, \mathbf{s}, \mathbf{t}^*) \quad \text{for every } \mathbf{s} \in \tilde{\Lambda}, \quad (7)$$

$$v_2(j, s) = \tilde{R}_2(j, s, \mathbf{s}^*, \mathbf{t}^*) \geq \tilde{R}_2(j, s, \mathbf{s}^*, \mathbf{t}) \quad \text{for every } \mathbf{t} \in \tilde{\mathbf{M}}. \quad (8)$$

The pair $(v_1(j, s), v_2(j, s))$ will be called the Nash value.

Assume $\mathbf{E}_{(j,s)}|g_i(\xi_n)| < \infty$, for $(j, s) \in \mathbb{E}$. Denote $h_i(j, s) = \sup_{\tau \in \mathcal{T}} \mathbf{E}_{(j,s)} g_i(\xi_\tau)$ and σ^{*i} a stopping time such that $h_i(j, s) = \mathbf{E}_{(j,s)} g_i(\xi_{\sigma^{*i}})$ for every $(j, s) \in \mathbb{E}$, $i = 1, 2$. Let $\Gamma^i = \{(j, s) \in \mathbb{E} : h_i(j, s) = g_i(j, s)\}$. We have $\sigma^{*i} = \inf\{n : \xi_n \in \Gamma^i\}$ (see Shiryaev (1978)). Denote $\sigma_k^{*i} = \inf\{n > k : \xi_n \in \Gamma^i\}$. Taking into account the above definition of \mathcal{G}_{rp} one can conclude that the Nash values of this game are the same as in the auxiliary game \mathcal{G}_{wp} with the payoff functions

$$\begin{aligned} \varphi_1(p, q) &= g_1(\xi_{\lambda(p)})\mathbb{I}_{\{\lambda(p) < \mu(q)\}} + \tilde{h}_1(\xi_{\mu(q)})\mathbb{I}_{\{\lambda(p) > \mu(q)\}} \\ &\quad + \left[g_1(\xi_{\lambda(p)})\alpha_{\lambda(p)} + \tilde{h}_1(\xi_{\lambda(p)})(1 - \alpha_{\lambda(p)}) \right] \mathbb{I}_{\{\lambda(p) = \mu(q)\}}, \end{aligned} \quad (9)$$

$$\begin{aligned} \varphi_2(p, q) &= g_2(\xi_{\mu(q)})\mathbb{I}_{\{\mu(q) < \lambda(p)\}} + \tilde{h}_2(\xi_{\lambda(p)})\mathbb{I}_{\{\mu(q) > \lambda(p)\}} \\ &\quad + \left[g_2(\xi_{\lambda(p)})(1 - \alpha_{\lambda(p)}) + \tilde{h}_2(\xi_{\lambda(p)})\alpha_{\lambda(p)} \right] \mathbb{I}_{\{\lambda(p) = \mu(q)\}}, \end{aligned} \quad (10)$$

for each $p \in \mathcal{P}$, $q \in \mathcal{Q}$, where $\tilde{h}_i(j, s) = \mathbf{E}_{(j,s)} h_i(\xi_1)$. Denote $R_i(j, s, p, q) = \mathbf{E}_{(j,s)} \varphi_i(p, q)$ for every $(j, s) \in \mathbb{E}$, $i = 1, 2$.

Let $\mathcal{P}_n = \{p = (p_n) \in \mathcal{P} : p_1 = \dots = p_{n-1} = 0\}$ and $\mathcal{Q}_n = \{q = (q_n) \in \mathcal{Q} : q_1 = \dots = q_{n-1} = 0\}$. We will use the following convention: if $p \in \mathcal{P}$ then (p_n, p) is the strategy belonging to \mathcal{P} in which the n -th coordinate is changed to p_n .

Definition 3.2 A pair $(p^*, q^*) \in \mathcal{P}_n \times \mathcal{Q}_n$ is called an equilibrium point of \mathcal{G}_{wp} at n if

$$\begin{aligned} v_1(j, s) &= \mathbf{E}_{(j,s)} \varphi_1(p^*, q^*) \geq \mathbf{E}_{(j,s)} \varphi_1(p, q^*) \quad \text{for every } p \in \mathcal{P}_n, \mathbf{P}_x\text{-a.s.}, \\ v_2(j, s) &= \mathbf{E}_{(j,s)} \varphi_2(p^*, q^*) \geq \mathbf{E}_{(j,s)} \varphi_2(p^*, q) \quad \text{for every } q \in \mathcal{Q}_n, \mathbf{P}_x\text{-a.s.} \end{aligned}$$

A Nash equilibrium point is a solution of \mathcal{G}_{wp} . The pair $(v_1(0, 0), v_2(0, 0))$ of values is a Nash value corresponding to $(p^*, q^*) \in \mathcal{P} \times \mathcal{Q}$.

Theorem 3.3 There exists a Nash equilibrium (p^*, q^*) in the game \mathcal{G}_{wp} . The Nash value is a solution of the equation:

$$(v_1(j, s), v_2(j, s)) = \begin{pmatrix} (\tilde{g}_1(j, s), \tilde{g}_2(j, s)) & (g_1(j, s), \tilde{h}_2(j, s)) \\ (\tilde{h}_1(j, s), g_2(j, s)) & (\tilde{v}_1(j, s), \tilde{v}_2(j, s)) \end{pmatrix} \quad (11)$$

where $\tilde{v}_j(j, s) = \mathbf{E}_{(j,s)} v_j(\xi_1)$, $j = 1, 2$.

The solution of the game \mathcal{G}_{rp} can be constructed based on the solution (p^*, q^*) of the corresponding game \mathcal{G}_{wp} .

Theorem 3.4 Game \mathcal{G}_{rp} has a solution. The pair $(\mathbf{s}^*, \mathbf{t}^*)$, where $\mathbf{s}^* = (p^*, \{\sigma_n^{*1}\}) \in \tilde{\mathbf{A}}$ and $\mathbf{t}^* = (q^*, \{\sigma_n^{*2}\}) \in \tilde{\mathbf{M}}$, is an equilibrium point. The value of the game is $(v_1(0, 0), v_2(0, 0))$.

4 Two person best choice problem with random priority

Let us consider the two person game with random priority described in Section 3 related to the SP when the options are arriving according the compound Poisson process. Based on definition of Section 2 and 3, when $\mathbb{E} = \mathbb{N} \times \mathfrak{R}^+$, define $g_i(j, s) = U_j(s)$, $i = 1, 2$, $(j, s) \in \mathbb{E}$. Let $\alpha_i = \alpha$ for $i = 1, 2, \dots$. We have $\tilde{g}_1(j, s) = \alpha U_j(s) + (1 - \alpha)V_j(s)$, $\tilde{g}_2(j, s) = (1 - \alpha)U_j(s) + \alpha V_j(s)$. First of all we determine the equilibrium which give the highest value for Player 1. By analysis of the matrices (11) we have that for $(j, s) \in B$ the strategy $(1, 1)$ is an equilibrium point. We have then, $i = 1, 2$,

$$\tilde{v}_i(j, s) = \int_0^{T-s} \sum_{k=1}^{\infty} p_{(j,s)}^{(k,u)} \tilde{g}_i(j+k, s+u) du$$

For $s = s^*$ we have two pure equilibria in (11): $(1, 0)$ and $(0, 1)$ and one in randomized strategies. Since for $s < s^*$ we have $U_j(s) < V_j(s)$ for every $j \in \mathbb{N}$ henceforth we can choose $(1, 0)$ at $s \in (s^* - \epsilon, s^*)$. Under this assumption

$$\begin{aligned} \tilde{v}_1(j, s) &= \int_0^{s^*-s} \sum_{k=1}^{\infty} p_{(j,s)}^{(k,u)} U_{j+k}(s+u) du + \int_{s^*-s}^{T-s} \sum_{k=1}^{\infty} p_{(j,s)}^{(k,u)} \tilde{g}_1(j+k, s+u) du \\ \tilde{v}_2(j, s) &= \int_0^{s^*-s} \sum_{k=1}^{\infty} p_{(j,s)}^{(k,u)} V_{j+k}(s+u) du + \int_{s^*-s}^{T-s} \sum_{k=1}^{\infty} p_{(j,s)}^{(k,u)} \tilde{g}_2(j+k, s+u) du. \end{aligned}$$

Since $U_j(s)$ is increasing and $V_j(s)$ is constant for $s < s^*$ and the strategy $(1, 0)$ can be used as equilibrium in $s_b \leq s \leq s^*$, where $s_b = \inf\{s < s^* : \tilde{v}_1(j, s) \leq g_1(j, s)\}$. Denote $s_b' = \inf\{s < s^* : \tilde{v}_2(j, s) \leq g_2(j, s)\}$. We have $s_b < s_b'$ if $\alpha < \alpha_0 = \min\{\alpha \in [0, 1] : \frac{2}{2+\alpha} \geq e^{-\frac{1-\alpha}{2}}\} \cong 0.5299$. Denote

$$\begin{aligned} w_1(j, r, s, \alpha) &= \int_0^{s-r} \sum_{k=1}^{\infty} p_{(j,r)}^{(k,u)} U_{j+k}(r+u) du + \int_{s-r}^{T-r} \sum_{k=1}^{\infty} p_{(j,r)}^{(k,u)} \tilde{g}_1(j+k, r+u) du \\ w_2(j, r, s, \alpha) &= \int_0^{s-r} \sum_{k=1}^{\infty} p_{(j,r)}^{(k,u)} V_{j+k}(r+u) du + \int_{s-r}^{T-r} \sum_{k=1}^{\infty} p_{(j,r)}^{(k,u)} \tilde{g}_2(j+k, r+u) du. \end{aligned}$$

For $\alpha < \alpha_0$ we have

$$(p_r^*, q_r^*) = (1, 1)\mathbb{I}_{\{s \geq s^*\}}(s) + (1, 0)\mathbb{I}_{\{s_b \leq s < s^*\}}(s) + (0, 0)\mathbb{I}_{\{0 \leq s < s_b\}}(s), \quad (12)$$

and

$$\begin{aligned} v_i(j, s) &= w_i(j, s, s, \alpha)\mathbb{I}_{\{s \geq s^*\}}(s) + w_i(j, s, s^*, \alpha)\mathbb{I}_{\{s_b \leq s < s^*\}}(s) \\ &\quad + w_i(j, s_b, s^*, \alpha)\mathbb{I}_{\{0 \leq s < s_b\}}(s), \end{aligned} \quad (13)$$

for $i = 1, 2$, where

$$\begin{aligned} w_1(j, r, s, \alpha) &= -\frac{r+a}{T+a} \ln\left(\frac{r+a}{T+a}\right) + \frac{1-\alpha}{2} \left(\frac{r+a}{T+a}\right) \ln\left(\frac{s+a}{T+a}\right) + \ln^2\left(\frac{s+a}{T+a}\right), \\ w_2(j, r, s, \alpha) &= \frac{s+a}{T+a} - \frac{r+a}{T+a} - (1-\alpha)\frac{r+a}{T+a} \left(\ln\left(\frac{s+a}{T+a}\right) + \frac{\alpha}{2} \left(\frac{r+a}{T+a}\right) \ln^2\left(\frac{s+a}{T+a}\right)\right). \end{aligned}$$

The value of the game is $(v_1, v_2) = (v_1(0, 0), v_2(0, 0))$ where $b = e^{-\frac{3-\alpha}{2}}$ and

$$(v_1, v_2) = (e^{-\frac{3-\alpha}{2}}, e^{-1} - \frac{\alpha}{2}e^{-\frac{3-\alpha}{2}}). \quad (14)$$

Let $\alpha \geq \alpha_0$. Denote

$$\begin{aligned} u_1(j, r, s, t, \alpha) &= \int_0^{s-r} \sum_{k=1}^{\infty} p_{(j,r)}^{(k,u)} V_{j+k}(r+u) du + \int_{s-r}^{t-r} \sum_{k=1}^{\infty} p_{(j,r)}^{(k,u)} U_{j+k}(r+u) du \\ &\quad + \int_{t-r}^{T-r} \sum_{k=1}^{\infty} p_{(j,s)}^{(k,u)} \tilde{g}_1(j+k, r+u) du \\ u_2(j, r, s, t, \alpha) &= \int_0^{s-r} \sum_{k=1}^{\infty} p_{(j,r)}^{(k,u)} U_{j+k}(r+u) du + \int_{s-r}^{t-r} \sum_{k=1}^{\infty} p_{(j,r)}^{(k,u)} V_{j+k}(r+u) du \\ &\quad + \int_{t-r}^{T-r} \sum_{k=1}^{\infty} p_{(j,s)}^{(k,u)} \tilde{g}_2(j+k, r+u) du. \end{aligned}$$

Similar analysis as above leads to conclusion that

$$\begin{aligned} (p_r^*, q_r^*) &= (1, 1)\mathbb{I}_{\{s \geq s^*\}}(s) + (1, 0)\mathbb{I}_{\{s_b \leq s < s^*\}}(s) \\ &\quad + (0, 1)\mathbb{I}_{\{s_c \leq s < s_b\}}(s) + (0, 0)\mathbb{I}_{\{0 \leq s < r_c\}}(s), \end{aligned} \quad (15)$$

and

$$\begin{aligned} v_i(j, s) &= u_i(j, s, s, s, \alpha)\mathbb{I}_{\{s \geq s^*\}}(s) + u_i(j, s, s, s^*, \alpha)\mathbb{I}_{\{s_b \leq s < s^*\}}(s) \\ &\quad + u_i(j, s, s_b, s^*, \alpha)\mathbb{I}_{\{s_c \leq s < s_b\}}(s) + u_i(j, s_c, s_b, s^*, \alpha)\mathbb{I}_{\{0 \leq s < r_c\}}(s), \end{aligned} \quad (16)$$

$i = 1, 2$, where $s_c = \inf\{s < s_b : \tilde{v}_2(j, s) \leq U_j(s)\}$ and $u_1(j, r, s, t, \alpha) = z - x\frac{z}{y} + \frac{x}{y}\hat{w}_1(y, z, \alpha)$ and $u_2(j, r, s, t, \alpha) = x \ln \frac{y}{x} + \frac{x}{y}\hat{w}_2(y, z, \alpha)$, $x = \frac{r+a}{T+a}$, $y = \frac{s+a}{T+a}$, $z = x = \frac{t+a}{T+a}$. The value of the game for this equilibrium point is

$$(v_1, v_2) = (e^{-1} + e^{-\frac{5}{2}+e\frac{1-\alpha}{2}}(1 - e^{\frac{1-\alpha}{2}}), e^{-\frac{5}{2}+e\frac{1-\alpha}{2}}). \quad (17)$$

Theorem 4.1 *In the random priority two person non-zero sum game of choosing the best applicant when the stream of options appears according to the compound Poisson process the Nash equilibrium which gives the maximal probability of success for Player 1 is given by (12) for $\alpha < \alpha_0$ and by (15) for $\alpha \geq \alpha_0$. The Nash value for the equilibrium is (14) and (17), respectively.*

5 Conclusion

Similarly as in consideration by Szajowski (1994) and Neumann et al. (2002) the other Nash equilibria can be constructed. There are similarities between the considered model and the asymptotic behavior of Nash equilibria for the non-zero sum game version of the SP with number of objects tending to infinity. It allows to use the results of Neumann et al. (2002) to get the set

of all Nash solutions for the game \mathcal{G}_{rp} according to the definition 3.1. The optimal stopping problems for choosing non-extremal candidates show similar relations between the asymptotic solution of the finite horizon case and the solution for the poissonian stream of option (see Suchwałko and Szajowski (2003)).

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