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# Design of fault tolerant control for nonlinear systems subject to time varying faults

T. Bouarar, B. Marx, D. Maquin & J. Ragot

*Centre de Recherche en Automatique de Nancy, UMR 7039 - Nancy-Université, CNRS,  
2, Avenue de la Forêt de Haye, 54516 Vandoeuvre-Lès-Nancy, France*

**ABSTRACT:** In this paper, a Fault Tolerant Control (FTC) problem for discrete time nonlinear systems represented by Takagi-Sugeno (T-S) models is investigated. The goal is to design a fault tolerant controller taking into account the faults affecting the overall system behavior in order to ensure the system stability. The principal idea is to introduce a Proportional Integral (PI) observer to detect and to estimate an eventual fault occurring in the system. Based on Lyapunov theory, two new approaches are proposed in term of Linear Matrix Inequalities (LMI) leading to synthesize an FTC laws ensuring the tracking between the reference model states and the faulty system ones. These results concern the case of time varying faults modeled by exponential function and first order polynomial. To illustrate the effectiveness of the proposed approaches, an academic example is considered.

## 1 INTRODUCTION

Generally speaking, there exists two strategies for faulty systems control: the passive strategy and the active one. In the case of the passive strategy, also called robust control, the controller design problem has been widely studied in the literature and many approaches have been proposed for linear and nonlinear systems. The objective is to ensure simultaneously the stability of the system and the insensitivity to certain faults. Nevertheless, robust control methodology concerns a specific class of faults characterized by a bounded norm. The active control or Fault Tolerant Control (FTC) has been introduced to overcome the passive control drawbacks. Indeed, the FTC method allows improving the system performances for a large class of faults. The principal idea of this strategy is to reconfigure the control law according to the fault detection and estimation performed by an observer to allow the faulty system to accomplish its mission. Since the introduction of FTC techniques, several works have been developed. In linear system frameworks, a FTC approach based on pseudo-inverse technique has been proposed by (Gao & Antsaklis 1992). The main idea of this technique is to minimize a Frobenius norm leading to determine the controller gains. Thereafter, an extension of this approach has been proposed by (Staroswiecki 2005). In (Liu & Patton 1998), the FTC gains have been determined such that the eigenvalues of the controlled faulty system and those of a reference model are identical.

In the case of linear descriptor systems described by differential and algebraic equations, an approach has been proposed by (Marx & Georges 2004).

In the last decades, Takagi-Sugeno nonlinear systems (Takagi & Sugeno 1985) have attracted a great deal attention, since they allow extending the linear systems theory to nonlinear ones (Tanaka & Wang 2001, Feng 2006). Thus, many problems dealing with stability, stabilization, observer design and diagnosis have been widely studied. Nevertheless, the FTC problem based on this kind of model is not largely treated. Some works have been introduced in recent years, for instance, trajectory tracking FTC design approach for Takagi-Sugeno systems subject to actuator faults has been developed by (Ichalal 2009). Nevertheless, this approach may be conservative and some results should be improved by obtaining more relaxed conditions. More recently, new less conservative approach has been developed by (Bouarar et al. 2011). Note that these approaches concern the Takagi-Sugeno systems with measurable premise variables (i.e. premise variables depending on the the input or the output). In the other hand, when the premise variables are unmeasurable (depend on the states of the system), the FTC design problem has been studied by (Ichalal et al. 2010a, Ichalal et al. 2010b).

In the above studies, the considered faults affecting the system behavior are modeled by a constant function. However, in practice, the faults are often time variant.

Based on Lyapunov theory, two approaches dealing with FTC design for nonlinear systems represented by discrete Takagi-Sugeno systems with measurable premise variables are proposed. The objective is to ensure the tracking between a healthy reference nonlinear model and the eventually faulty nonlinear system. The proposed approaches are formulated in terms of Linear Matrix Inequalities (LMI) and they respectively concern the cases when fault dynamics is modelled by exponential function and first order polynomial. Moreover, the developed approaches does not require knowledge of the considered fault varying functions coefficients. To illustrate the applicability and the effectiveness of the proposed approaches, an academic example is considered.

To simplify the mathematical expressions and to improve the paper readability, we consider the following notations:

$$\Gamma_\mu = \sum_{i=1}^r \mu_i(\xi(k)) \Gamma_i,$$

$$\Gamma_{\mu\mu} = \sum_{i=1}^r \sum_{j=1}^r \mu_i(\xi(k)) \mu_j(\xi(k)) \Gamma_{ij},$$
 in a bloc

matrix, an asterisk  $*$  denotes the transposed element in the symmetric position, in the mathematical expressions  $\chi(k_+)$  is equivalent to  $\chi(k+1)$  and  $diag(\Lambda_1 \cdots \Lambda_r)$  represent a block diagonal matrix. The following lemmas are needed to provide LMI conditions.

**Lemma 1** (Zhou & Khargonekar 1988): Consider two real matrices  $\Phi$  and  $\Xi$  with appropriate dimensions, for any positive scalar  $\tau$  the following inequality holds:

$$\Phi^T \Xi^T + \Xi^T \Phi \leq \tau \Phi^T \Phi + \tau^{-1} \Xi^T \Xi^T \quad (1)$$

**Lemma 2** (Boyd et al. 1994): Consider the matrices  $T_i = T_i^T, i \in \{0, \dots, k\}$ . The following expressions are equivalent:

$$\forall \zeta, \zeta^T T_0 \zeta \geq 0 \text{ and } \zeta^T T_i \zeta \geq 0, \forall i \in \{1, \dots, k\} \quad (2)$$

$$\exists \rho_1 \geq 0, \dots, \rho_k \geq 0 \text{ such that } \forall \zeta, T_0 - \sum_{i=1}^k \rho_i T_i \geq 0 \quad (3)$$

## 2 TAKAGI-SUGENO MODEL

A Takagi-Sugeno (T-S) model allow the representation of a nonlinear system behavior by the interpolation of a set of linear submodels (Takagi and Sugeno 1985), (Tanaka and Wang 2001). Each submodel contributes to the global behavior of the nonlinear system through a weighting function  $\xi_l(k)$ . The T-S structure is given by:

$$\begin{cases} x(k_+) = \sum_{i=1}^r \mu_i(\xi(k)) (A_i x(k) + B_i u(k)) \\ y(k) = \sum_{i=1}^r \mu_i(\xi(k)) (C_i x(k) + D_i u(k)) \end{cases} \quad (4)$$

where  $r$  represent the number of local linear submodels,  $\xi(k) = (\xi_1(k) \cdots \xi_j(k))$  represent the vector of premise variables which can be measurable (input  $u(t)$  or/and output of the system  $y(t)$ ) or unmeasurable (the system state  $x(t)$ ).  $A_i \in \mathbb{R}^{n \times n}, B_i \in \mathbb{R}^{n \times m}, C_i \in \mathbb{R}^{p \times n}$  and  $D_i \in \mathbb{R}^{p \times m}$  are the matrices of the  $i^{th}$  linear submodel representing the plant behavior in the local region.

The weighting functions satisfy the convex sum property, i.e:

$$\begin{cases} 0 \leq \mu_i(\xi(k)) \leq 1 \\ \sum_{i=1}^r \mu_i(\xi(k)) = 1 \end{cases} \quad (5)$$

The T-S model defined in (4) is a nonlinear system since the weighting functions blinding together the  $r$  linear submodels are nonlinear.

The Takagi-Sugeno model shown its interest both in theoretical and practice fields. Indeed, the T-S model can represent exactly a nonlinear system in operating region of the state space. In other hand, thanks to convex sum property of the weighting functions (5), it is possible to extend the linear control theory to the nonlinear case.

## 3 PROBLEM STATEMENT

Let us consider (4) as a reference model corresponding to the healthy system. Let us also consider the faulty T-S model given by:

$$\begin{cases} x_f(k_+) = \sum_{i=1}^r \mu_i(\xi(k)) (A_i x_f(k) + B_i u_f(k) + G_i f(k)) \\ y_f(k) = \sum_{i=1}^r \mu_i(\xi(k)) (C_i x_f(k) + D_i u_f(k) + W_i f(k)) \end{cases} \quad (6)$$

where  $G_i \in \mathbb{R}^{n \times q}$  and  $W_i \in \mathbb{R}^{p \times q}$  describe the distribution matrices of the faults acting on the system.  $x_f(k) \in \mathbb{R}^n, y_f(k) \in \mathbb{R}^p, u_f(k) \in \mathbb{R}^m$  and  $f(k) \in \mathbb{R}^q$  represent respectively the faulty state, the faulty output, the FTC law and the faults affecting the T-S model.

To ensure the tracking between the faulty T-S model (6) and the healthy one (4), consider the following FTC law:

$$u_f(k) = u(k) + u_c(k) \quad (7)$$

where  $u_c(k) = \sum_{i=1}^r \mu_i(\xi(k)) (K_i (x(k) - \hat{x}_f(k)) - \hat{f}(k))$ , with  $K_i \in \mathbb{R}^{m \times n}$  are the state feedback gain matrices to be determined.

Note that this control law is an extension of a classical linear state feedback law. It is obtained by an interpolation using the same weighting functions as the model, and use an estimate of the fault.

The considered FTC law methodology is based on the scheme described in Figure 1. The conception of the FTC controller needs the knowledge of

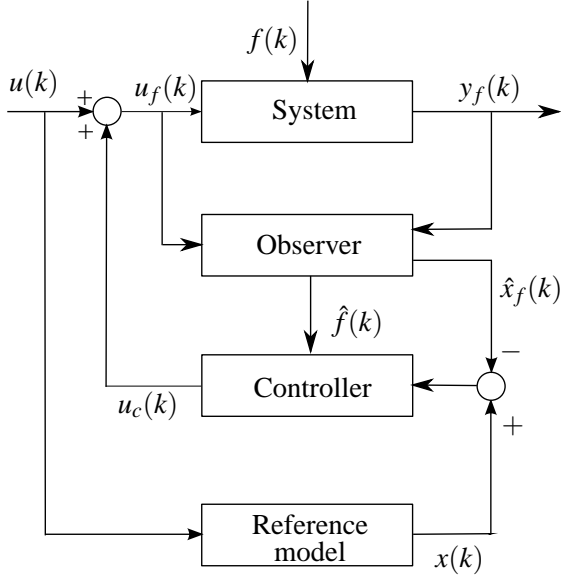


Figure 1: Tracking fault tolerant controller design scheme

both faults and state estimates. Thus, the following Proportional Integral (PI) observer is considered.

$$\begin{cases} \hat{x}_f(k_+) = \sum_{i=1}^r \mu_i(\xi(k)) (A_i \hat{x}_f(k) + B_i u_f(k) + \varphi_i(k)) \\ \hat{y}_f(k) = \sum_{i=1}^r \mu_i(\xi(k)) (C_i \hat{x}_f(k) + D_i u_f(k) + W_i \hat{f}(k)) \\ \hat{f}(k_+) = \sum_{i=1}^r \mu_i(\xi(k)) H_i^2 (y_f(k) - \hat{y}_f(k)) + \hat{f}(k) \end{cases} \quad (8)$$

where  $\varphi_i(k) = G_i \hat{f}(k) + H_i^1 (y_f(k) - \hat{y}_f(k))$ ,  $H_i^1 \in \mathbb{R}^{n \times p}$  and  $H_i^2 \in \mathbb{R}^{q \times p}$  are gain matrices of the PI observer to be determined.

Along of this work, we assume that:

- The system states and the faults are observable from the output.
- The nonlinear weighting functions depend only on measurable premise variables (i.e.  $y(t)$  and/or  $u(t)$ ).

#### 4 FTC DESIGN LMI CONDITIONS

In this work, two kind of time varying faults are considered. The first one concerns the faults modeled by exponential function. The second one concerns the case of faults modeled by a first order polynomial.

##### 4.1 Case of faults approximated by an exponential function

Let us consider that the system behavior is affected by several faults in the form:

$$f_i(k) = e^{a_i k + b_i} \quad (9)$$

where  $a_i, b_i \in \mathbb{R}$ , for  $i = 1, \dots, q$ . The variation of the faults is given by:

$$f_i(k_+) = e^{a_i} f_i(k) \quad (10)$$

Let us consider that  $a_i = a_{0,i} + \Delta a_i$ , with  $a_{0,i}$  and  $\Delta a_i$  representing respectively the nominal and the uncertain parts of the parameter  $a_i$ . This structure leads defining a set of exponential functions describing the faults affecting the T-S model behavior.

Let us also define:

$$a = \text{diag} ( e^{a_1} \quad \dots \quad e^{a_q} ) \quad (11)$$

$$a_0 = \text{diag} ( e^{a_{0,1}} \quad \dots \quad e^{a_{0,q}} ) \quad (12)$$

$$\Delta a = \text{diag} ( e^{\Delta a_1} \quad \dots \quad e^{\Delta a_q} ) \quad (13)$$

The uncertain part can be bounded as:

$$(\Delta a)^T \Delta a \leq \lambda \quad (14)$$

where  $\lambda \in \mathbb{R}^{q \times q}$  is a known diagonal positive definite matrix.

Let us define the state tracking, the state and the fault estimation errors given respectively as:

$$\begin{cases} e_p(k) = x(k) - x_f(k) \\ e_s(k) = x_f(k) - \hat{x}_f(k) \\ e_d(k) = f(k) - \hat{f}(k) \end{cases} \quad (15)$$

The dynamics of  $e_p(k)$ ,  $e_s(k)$  and  $e_d(k)$  are respectively given by:

$$e_p(k_+) = M_{\mu\mu} e_p(k) - \chi_{\mu\mu} e_s(k) - B_{\mu} e_d(k) + \Omega_{\mu} f(k) \quad (16)$$

where  $M_{\mu\mu} = A_{\mu} - B_{\mu} K_{\mu}$ ,  $\Omega_{\mu} = B_{\mu} - G_{\mu}$  and  $\chi_{\mu\mu} = B_{\mu} K_{\mu}$ .

$$e_s(k_+) = \Pi_{\mu\mu} e_s(k) + \Theta_{\mu\mu} e_d(k) \quad (17)$$

with  $\Pi_{\mu\mu} = A_{\mu} - H_{\mu}^1 C_{\mu}$  and  $\Theta_{\mu\mu} = G_{\mu} - H_{\mu}^1 W_{\mu}$ .

$$e_d(k_+) = -H_{\mu}^2 C_{\mu} e_s(k) + \Sigma_{\mu\mu} e_d(k) + \alpha f(k) \quad (18)$$

where  $\Sigma_{\mu\mu} = I - H_{\mu}^2 W_{\mu}$  and  $\alpha = a - I$ . The combination of (16), (17) and (18) leads to the following expression:

$$e(k_+) = \bar{A}_{\mu\mu} e(k) + Z_{\mu} f(k) \quad (19)$$

where  $Z_{\mu} = \begin{pmatrix} \Omega_{\mu} \\ 0 \\ \alpha \end{pmatrix}$ ,  $e(k) = \begin{pmatrix} e_p(k) \\ e_s(k) \\ e_d(k) \end{pmatrix}$  and

$$\bar{A}_{\mu\mu} = \begin{pmatrix} M_{\mu\mu} & -\chi_{\mu\mu} & -B_{\mu} \\ 0 & \Pi_{\mu\mu} & \Theta_{\mu\mu} \\ 0 & -H_{\mu}^2 C_{\mu} & \Sigma_{\mu\mu} \end{pmatrix}.$$

**Remark 1:** Along this work, we consider that the matrices  $B_i$  and  $G_i$  of the system (6) and the observer (8), have the same dimensions.

The conditions leading to FTC design controller are proposed in the following theorem 1.

**Theorem 1:** The state tracking error  $e_p(k)$ , the state and fault estimation errors  $e_s(k)$  and  $e_d(k)$  converge asymptotically to zero and the  $\mathcal{L}_2$ -gain from the faults to the errors  $e_p(k)$ ,  $e_s(k)$  and  $e_d(k)$  is bounded by  $\sqrt{\bar{\gamma}}$ , if there exists matrices  $X_1 = X_1^T \geq 0$ ,  $X_2 = X_2^T \geq 0$ ,  $X_3 = X_3^T \geq 0$ ,  $K_i$ ,  $L_i^1$  and  $L_i^2$  and positive scalars  $\bar{\gamma}$  and  $\tau$  such that the following LMI, for  $i = 1, 2, \dots, r$  hold

$$\Upsilon_{ij} < 0 \quad (20)$$

The matrix  $\Upsilon_{ij}$  is defined in the next page.

*Proof.* To study the asymptotic convergence to zero of the above errors  $e_p(k)$ ,  $e_s(k)$  and  $e_d(k)$ , we consider the following Lyapunov function candidate:

$$V(k) = e^T(k) X e(k) \quad (21)$$

with

$$X = X^T \geq 0 \quad (22)$$

Let us consider the following  $\mathcal{L}_2$  constraint minimizing the fault effect on  $e_p(k)$ ,  $e_s(k)$  and  $e_d(k)$ .

$$\sum_{k=0}^N e^T(k) Q e(k) \leq \gamma^2 \sum_{k=0}^N f^T(k) f(k) \quad (23)$$

where  $N$  denotes the final step time,  $\gamma$  represents the attenuation level and  $Q$  is a known symmetric positive definite weighting matrix. The error dynamics expressed in (19) is stable under the  $\mathcal{L}_2$  constraint (23) if:

$$\begin{pmatrix} Q - X & 0 \\ 0 & -\gamma^2 I \end{pmatrix} + \begin{pmatrix} \bar{A}_{\mu\mu}^T \\ Z_{\mu}^T \end{pmatrix} X \begin{pmatrix} \bar{A}_{\mu\mu} & Z_{\mu} \end{pmatrix} < 0 \quad (24)$$

To provide easily LMI stability conditions for (19), we choose the matrix structure  $X$  as:

$$X = \text{diag}( X_1 \quad X_2 \quad X_3 ) \quad (25)$$

According to (22), matrices  $X_1$ ,  $X_2$  and  $X_3$  are symmetric positive definite matrices.

By applying the Schur complement on (24), one can obtain:

$$\begin{pmatrix} Q - X & 0 & * \\ 0 & -\gamma^2 I & * \\ X \bar{A}_{\mu\mu} & X Z_{\mu} & -X \end{pmatrix} < 0 \quad (26)$$

Considering the matrices defined in (19), the mathematical development of (26) leads to:

$$\begin{aligned} & \Psi_{\mu\mu} + (\phi^1)^T \phi_{\mu\mu}^2 + (\phi_{\mu\mu}^2)^T \phi^1 + (\phi^1)^T \phi_{\mu\mu}^3 + \\ & (\phi_{\mu\mu}^3)^T \phi^1 + (\phi^4)^T \phi^5 + (\phi^5)^T \phi^4 < 0 \end{aligned} \quad (27)$$

where  $\phi^1 = ( 0 \quad 0 \quad 0 \quad 0 \quad X_1 \quad 0 \quad 0 )$ ,  
 $\phi_{\mu\mu}^2 = ( -B_{\mu} K_{\mu} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 )$ ,  
 $\phi_{\mu\mu}^3 = ( 0 \quad -B_{\mu} K_{\mu} \quad 0 \quad 0 \quad 0 \quad 0 )$ ,  
 $\phi^4 = ( 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad X_3 a_0 )$ ,  
 $\phi^5 = ( 0 \quad 0 \quad 0 \quad \Delta a \quad 0 \quad 0 \quad 0 )$  and  $\Psi_{\mu\mu}$  is given in the next page

To provide LMI conditions, consider the following bijective change of variables:  $\bar{\gamma} = \gamma^2$ ,  $L_{\mu}^1 = X_2 H_{\mu}^1$  and  $L_{\mu}^2 = X_3 H_{\mu}^2$ .

By applying lemma 1, (27) can be rewritten as:

$$\begin{aligned} & \Psi_{\mu\mu} + \tau_1 (\phi^1)^T \phi^1 + \tau_1^{-1} (\phi_{\mu\mu}^2)^T \phi_{\mu\mu}^2 + \tau_2 (\phi^1)^T \phi^1 + \\ & \tau_2^{-1} (\phi_{\mu\mu}^3)^T \phi_{\mu\mu}^3 + \tau (\phi^4)^T \phi^4 + \tau^{-1} (\phi^5)^T \phi^5 < 0 \end{aligned} \quad (28)$$

Considering  $\tau_1 = \tau_2 = 1$  and applying the Schur complement on (28), thus the sufficient LMI conditions proposed in theorem 1 hold.  $\square$

#### 4.2 Case of faults approximated by a first order polynomial

Let us consider the faults occurring in the system are modeled by a first order polynomial as:

$$f_i(k) = a_i k + b_i \quad (29)$$

where  $a_i, b_i \in \mathbb{R}$ , for  $i = 1, \dots, q$ .

In the same way as the first case, we define  $a$ ,  $a_i$  and  $\Delta a_i$  as follows:

$$\begin{cases} a = \text{diag}( a_1 \quad \dots \quad a_q ) \\ a_0 = \text{diag}( a_{0,1} \quad \dots \quad a_{0,q} ) \\ \Delta a = \text{diag}( \Delta a_1 \quad \dots \quad \Delta a_q ) \end{cases} \quad (30)$$

Let us consider that the uncertain part is bounded as:

$$(\Delta a)^T \Delta a \leq \delta \quad (31)$$

where  $\delta \in \mathbb{R}^{q \times q}$  is a known diagonal positive definite matrix.

In this case, the fault estimation dynamics is given by:

$$e_d(k_+) = -H_{\mu}^2 C_{\mu} e_s(k) + \Sigma_{\mu\mu} e_d(k) + a \quad (32)$$

The combination of (16), (17) and (32) leads to:

$$e(k_+) = \bar{A}_{\mu\mu} e(k) + E_{\mu} f(k) + P \quad (33)$$

where  $e(k)$  and  $\bar{A}_{\mu\mu}$  are defined in equation (19),

$$E = \begin{pmatrix} \Omega_{\mu} \\ 0 \\ 0 \end{pmatrix} \text{ and } P = \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix}.$$

$$\Upsilon_{ij} = \begin{pmatrix} Q_1 - X_1 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & * & 0 \\ 0 & Q_2 - X_2 & 0 & 0 & 0 & * & * & 0 & 0 & 0 & * \\ 0 & 0 & Q_3 - X_3 & 0 & * & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau^{-1}\lambda - \bar{\gamma}I & * & 0 & * & 0 & * & 0 & 0 \\ X_1 A_i & 0 & -X_1 B_i & X_1 (B_i - G_i) & -X_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & X_2 A_i - L_j^1 C_i & X_2 G_i - L_j^1 W_i & 0 & 0 & -X_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_j^2 C_i & X_3 - L_j^2 W_i & -X_3 & 0 & 0 & -X_3 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_0 X_3 & -\tau^{-1}I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & X_1 & 0 & 0 & 0 & -2I & 0 & 0 \\ B_i K_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ 0 & B_i K_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{pmatrix}$$

$$\Psi_{\mu\mu} = \begin{pmatrix} Q_1 - X_1 & 0 & 0 & 0 & * & 0 & 0 \\ 0 & Q_2 - X_2 & 0 & 0 & 0 & * & * \\ 0 & 0 & Q_3 - X_3 & 0 & 0 & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & 0 & * \\ X_1 A_\mu & 0 & -X_1 B_\mu & X_1 (B_\mu - G_\mu) & -X_1 & 0 & 0 \\ 0 & X_2 (A_\mu - H_\mu^1 C_\mu) & X_2 (G_\mu - H_\mu^1 W_\mu) & 0 & 0 & -X_2 & 0 \\ 0 & -X_3 H_\mu^2 C_\mu & X_3 (I - H_\mu^2 W_\mu) & -X_3 & 0 & 0 & -X_3 \end{pmatrix}$$

The main provided results are given in the following theorem 2.

**Theorem 2:** The state tracking error  $e_p(k)$ , the state and fault estimation errors  $e_s(k)$  and  $e_d(k)$  converge asymptotically to zero and the  $\mathcal{L}_2$ -gain from the faults to the errors  $e_p(k)$ ,  $e_s(k)$  and  $e_d(k)$  is bounded by  $\sqrt{\bar{\gamma}}$ , if there exists matrices  $X_1 = X_1^T \geq 0$ ,  $X_2 = X_2^T \geq 0$ ,  $X_3 = X_3^T \geq 0$ ,  $K_i$ ,  $L_i^1$  and  $L_i^2$  and positive scalars  $\bar{\gamma}$ ,  $\tau$  and  $\rho$  such that the following LMI are verified, for  $i = 1, 2, \dots, r$

$$\Phi_{ij} < 0 \quad (34)$$

where  $\Phi_{ij}$  is given in the next page, with:

$$\begin{aligned} \Phi^{1,1} &= \rho I + Q_1 - X_1, & \Phi^{2,2} &= \rho I + Q_2 - X_2 \\ \Phi^{3,3} &= \rho I + Q_3 - X_3, & \Phi^{5,5} &= -\rho \varepsilon I + \tau^{-1} \delta I \\ \Phi_i^{6,4} &= X_1 (B_i - G_i), & \Phi_{ij}^{7,2} &= X_2 A_i - L_j^1 C_i \\ \Phi_{ij}^{7,3} &= X_2 G_i - L_j^1 W_i & \text{and } \Phi_{ij}^{8,3} &= X_3 - L_j^2 W_i \end{aligned}$$

*Proof.* Considering (21), (22), (23) and (33), then following the same steps of the theorem 1 from (21) to (26), one can obtain:

$$\begin{pmatrix} \bar{A}_{\mu\mu}^T X \bar{A}_{\mu\mu} + Q - X & * & * \\ E^T X \bar{A}_{\mu\mu} & E^T X E - \gamma^2 I & * \\ P^T X \bar{A}_{\mu\mu} & P^T X E & P^T X P \end{pmatrix} < 0 \quad (35)$$

To transform (35) to a feasible problem, we consider the following inequality ensuring the asymptotic convergence of the error dynamics to a ball of radius  $\varepsilon$ .

$$\|e(k)\|_2^2 \geq \varepsilon I \quad (36)$$

where  $\varepsilon$  is a known small positive scalar.

The mathematical development of (36) leads to:

$$\begin{pmatrix} e^T(k) \\ f^T(k) \\ I \end{pmatrix}^T \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\varepsilon I \end{pmatrix} \begin{pmatrix} e(k) \\ f(k) \\ I \end{pmatrix} \geq 0 \quad (37)$$

By applying S-procedure lemma 2 on (35) and (37), one can obtain:

$$\begin{pmatrix} \rho I + Q - X & 0 & 0 \\ 0 & -\gamma^2 I & 0 \\ 0 & 0 & -\rho \varepsilon I \end{pmatrix} + \begin{pmatrix} \bar{A}_{\mu\mu}^T \\ E^T \\ P^T \end{pmatrix} X \begin{pmatrix} \bar{A}_{\mu\mu} & E & P \end{pmatrix} < 0 \quad (38)$$

By applying Schur complement on (38), this latter becomes

$$\begin{pmatrix} \rho I + Q - X & * & * & * \\ 0 & -\gamma^2 I & * & * \\ 0 & 0 & -\rho \varepsilon I & * \\ X \bar{A}_{\mu\mu} & X E & X P & -X \end{pmatrix} < 0 \quad (39)$$

Following the same path as for the proof of theorem 1 from (27) to the end, thus the sufficient LMI conditions proposed in theorem 2 hold.  $\square$

## 5 SIMULATION EXAMPLE

Let us consider the nonlinear T-S model (6) described by the following matrices and weighting nonlinear functions:

$$\Phi_{ij} = \begin{pmatrix} \Phi^{1,1} & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & * & 0 & 0 \\ 0 & \Phi^{2,2} & 0 & 0 & 0 & 0 & * & * & 0 & 0 & * & 0 \\ 0 & 0 & \Phi^{3,3} & 0 & 0 & * & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\bar{\gamma}I & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Phi^{5,5} & 0 & 0 & * & 0 & 0 & 0 & 0 \\ X_1 A_i & 0 & -X_1 B_i & \Phi_i^{6,4} & 0 & -X_1 & 0 & 0 & * & 0 & 0 & 0 \\ 0 & \Phi_{ij}^{7,2} & \Phi_{ij}^{7,3} & 0 & 0 & 0 & -X_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -L_j^2 C_i & \Phi_{ij}^{8,3} & 0 & X_3 a_0 & 0 & 0 & -X_3 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & X_1 & 0 & 0 & -2I & 0 & 0 & 0 \\ B_i K_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 & 0 \\ 0 & B_i K_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & X_3 & 0 & 0 & 0 & -\tau^{-1}I \end{pmatrix}$$

$$A_1 = \begin{pmatrix} -0.5 & 0.1 \\ -1 & -1 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 0.2 \\ -0.45 & -0.7 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 0.4 \\ 0.5 \end{pmatrix}, B_2 = \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix},$$

$$G_1 = \begin{pmatrix} 0.2 \\ 0.4 \end{pmatrix}, G_2 = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}, C_1 = (0.2 \ 0),$$

$$C_2 = (0.4 \ 0.1), W_1 = -0.3, W_2 = -0.4, \text{ the nominal input signal } u(k) = 0.5 \cos(\sin(0.1k)0.1k).$$

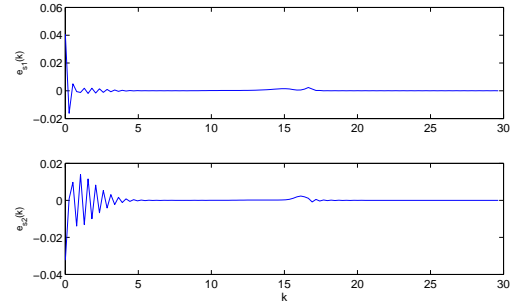


Figure 3: State estimation errors

The activation nonlinear functions are depending on the known nominal input  $u(k)$ , they are given by:  $\mu_1(u(k)) = 1 - \tanh(0.5 - u(k))$  and  $\mu_2(u(k)) = 1 - \mu_1(u(k))$ .

Let us consider that the fault affecting the system at  $9 \leq k \leq 17$  is given by:

$$f(k) = e^{0.11k-10} \quad (40)$$

The observer and the controller are designed for  $a_0 = e^{0.1}$  leading to  $\Delta a_0 = e^{0.01}$ . For simulation, the parameter  $\lambda$  defined in (14) is chosen equal to 1.3.

**Remark 2:** To show the robustness of the synthesized FTC controller and observer, the parameter value of the fault acting in the system are augmented. indeed, the following results given by Figures 2 to 6 are obtained for  $f(k) = e^{0.5k-10}$ .

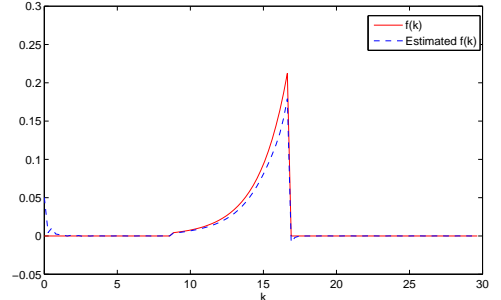


Figure 4: Fault and its estimation

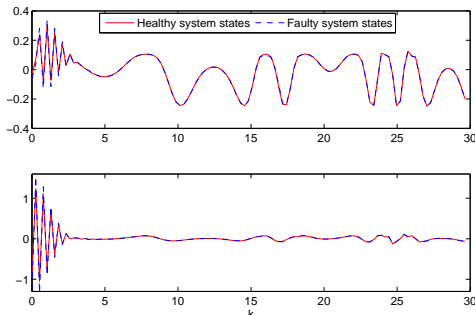


Figure 2: Reference model states vs. faulty system ones with FTC

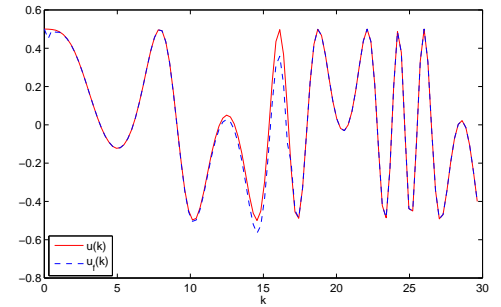


Figure 5: Nominal and FTC control input signals

## 6 CONCLUSION

In this paper, a new approach dealing with fault tolerant controller design problem for nonlinear systems represented by Takagi-Sugeno model has been investigated. The proposed results, obtained by using Lyapunov method, are formulated in terms of LMI which can be easily solved by using Matlab software. The effectiveness of the provided trajectory tracking ap-

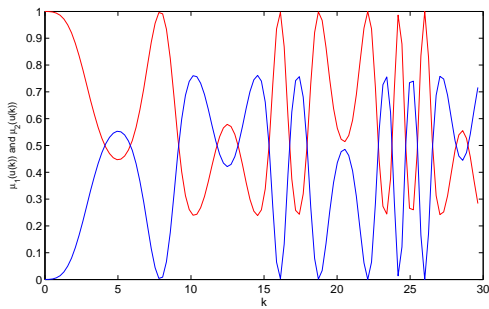


Figure 6: Weighting nonlinear functions

proaches has been illustrated by considering a numerical example. Indeed, the fault occurring in the system has been taken into account by the synthesized FTC controller allowing to ensure the tracking between the healthy system states and the faulty ones.

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