



**HAL**  
open science

# Temperature and thermal stresses in a pad/disc during braking

Aleksander Yevtushenko, Michal Kuciej

► **To cite this version:**

Aleksander Yevtushenko, Michal Kuciej. Temperature and thermal stresses in a pad/disc during braking. Applied Thermal Engineering, 2009, 30 (4), pp.354. 10.1016/j.applthermaleng.2009.09.015 . hal-00593336

**HAL Id: hal-00593336**

**<https://hal.science/hal-00593336>**

Submitted on 14 May 2011

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## Accepted Manuscript

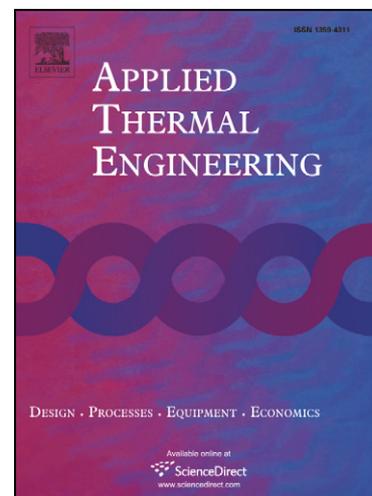
Temperature and thermal stresses in a pad/disc during braking

Aleksander Yevtushenko, Michal Kuciej

PII: S1359-4311(09)00289-0  
DOI: [10.1016/j.applthermaleng.2009.09.015](https://doi.org/10.1016/j.applthermaleng.2009.09.015)  
Reference: ATE 2894

To appear in: *Applied Thermal Engineering*

Received Date: 19 May 2009  
Accepted Date: 23 September 2009



Please cite this article as: A. Yevtushenko, M. Kuciej, Temperature and thermal stresses in a pad/disc during braking, *Applied Thermal Engineering* (2009), doi: [10.1016/j.applthermaleng.2009.09.015](https://doi.org/10.1016/j.applthermaleng.2009.09.015)

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

## Temperature and thermal stresses in a pad/disc during braking

Aleksander Yevtushenko, Michal Kuciej

*Faculty of Mechanical Engineering, Technical University of Bialystok (BTU),  
45C Wiejska Street, Bialystok, 15-351, Poland*

Correspondence to: A. Yevtushenko

e-mail: ayevt@pb.edu.pl,

phone number: (48) 085 746 92 00,

fax number: (48) 085 746 92 10.

ACCEPTED MANUSCRIPT

**ABSTRACT**

A two-element model of braking process for a tribosystem consisting of the pad (the strip) sliding with the time-dependent velocity (braking at uniform retardation) on a surface of the disc (the semi-space) is studied. The dependences of temperature and thermal stresses on the boundary conditions on upper surface of the ceramic-metal strip were investigated. It was proved that there is a possibility of applying the obtained results to modelling of a thermal cracking of the frictional elements during braking.

**Keywords:** Braking, Frictional heating, Heat conduction, Temperature, Thermal stresses, Contact problem.

**NOMENCLATURE**

$c$  – thickness of the strip in which the thermal stresses are calculated;  
 $c^* = c/d$ ;  
 $d$  – thickness of the pad (strip), m;  
 $E$  – Young's modulus;  
 $\text{erf}(x)$  – Gauss error function;  
 $\text{erfc}(x) = 1 - \text{erf}(x)$  – complementary error function;  
 $\text{ierfc}(x) = \pi^{-1/2} \exp(-x^2) - x \text{erfc}(x)$  – integral of the error function  $\text{erfc}(x)$ ;  
 $H(\cdot)$  – Heaviside's step function;  
 $f$  – frictional coefficient;  
 $K$  – thermal conductivity, W/(mK);  
 $k$  – thermal diffusivity,  $\text{m}^2/\text{s}$ ;  
 $p_0$  – pressure, Pa;  
 $T$  – temperature, °C;  
 $T^*$  – dimensionless temperature;  
 $t$  – time, s;  
 $t_c$  – time of the sign change of lateral stress;  
 $t_s$  – braking time;  
 $V$  – sliding speed, m/s;  
 $V_0$  – initial sliding speed, m/s;  
 $z$  – spatial coordinate, m.

**Greek symbols**

$\alpha_t$  – linear thermal expansion coefficient;  
 $\nu$  – Poisson's ratio;  
 $\sigma_0 = \alpha_t E T_0 / (1 - \nu)$  – stress scaling factor;  
 $\sigma_x$  – normal stress;  
 $\sigma^* = \sigma_x / \sigma_0$ ;  
 $\tau = k_s t / d^2$ ;  
 $\tau_s = k_s t_s / d^2$ ;  
 $\zeta = z / d$ .

**Indexes**

$f$  – foundation,  
 $s$  – strip.

**1. Introduction**

As a result of friction on the contact surface of the pad and disc the kinetic energy transforms into heat. Elements of brakes are heated and, hence, the conditions of operation of the friction patches become less favorable: their wear intensifies and the friction coefficient decreases, which may lead to emergency situations [1]. Thus, the problem of calculation of temperature and thermal stresses is one of the most important problems in the design of brakes [2].

The axi-symmetric and three-dimensional transient temperature field models of brake shoe with using of the integral-transform method have been proposed in articles [3-8], and with using of the finite elements method in article [9].

However, most often temperatures and stresses are obtained from a solution of a one-dimensional contact problem with transient frictional heat generation [10-15]. The one-dimensional models correspond to those cases when the Peclet number is large and, consequently, the frictional heat flux is normal to the contact surface. The verification of many analytical solutions with the results from the experimental data, which refers to the work of the braking devices, shows that the one-dimensional models are the sufficiently good approximation for the computation of the temperature during braking [16-18].

All the above mentioned solutions obtained allowed to define the temperature only during braking at  $0 \leq t \leq t_s$ , where  $t_s$  is the time of a stop. The solutions, allowing to calculate the temperature both at heating at a stage of braking, and at cooling after a stop, are proposed in articles [19-21].

The heating on a surface of friction during braking leads to temperature shock that generates surface cracks [22, 23]. Cleavage of the material in the process of thermal cracking results from tensile stresses when the friction elements are heated by the moving heat flux. When the stresses value exceeds the tensile strength of material, then cracks arise on the contact surface. Low thermal conductivity of friction patch materials is the reason why considerable thermal stresses are generated in a thin subsurface layer during braking. As a result, destruction of the friction surface can take place both during heating at braking, and during cooling after a stop. The mathematical model of thermal splitting of homogeneous and piece-wise homogeneous bodies at assumption of laser or frictional heating of their surface by a thermal flow of known intensity was proposed in articles [24, 25].

The aim of this paper is to analyse the temperature fields and thermal stresses in the tribosystem consisting of two frictional elements: the pad (the strip) sliding with the velocity  $V(t) = V_0(1 - t/t_s)$ ,  $0 \leq t \leq t_s$  (braking with constant retardation) on a surface of the disc (the semi-space). The obtained solution determines the temperature and thermal stresses in the tribosystem both in the heating phase during braking and in the cooling phase after a stop.

The numerical results for the temperature and thermal stresses are obtained for the metal-ceramic pad and the cast-iron disc. The metal-ceramic frictional materials are now extensively used in brake systems because of their high thermal stability and wear resistance [26, 27].

## 2. Statement of the problem

The problem of contact interaction of a plane-parallel strip (the pad) and semi-space (the disk) is under consideration. The scheme of contacting bodies is shown in Fig. 1. It is assumed, that at the initial time moment  $t = 0$  the outer surface of the strip and the foundation in infinity are subjected to the action of a constant pressures  $p_0$ . The strip slides over the surface of semi-space along the  $y$ -axis of the Cartesian coordinate system  $Oxyz$  with the centre at the plane of contact. The velocity of sliding  $V$  decreases linearly in time  $t$  from initial value  $V_0$  at  $t = 0$  to zero at the stop time moment  $t_s$ , so that the deceleration is constant (uniform braking):

$$V(t) = V_0 \left(1 - \frac{t}{t_s}\right) H(t_s - t), \quad t \geq 0. \quad (1)$$

The sliding is accompanied by frictional heat generation on a contact plane  $z = 0$ . The sum of the intensities of the frictional heat fluxes directed into each component of friction pair is equal to the specific friction power [28]:

$$q(t) = q_0 q^*(t), \quad t \geq 0, \quad (2)$$

where, taking the equation (1) into account, we have

$$q_0 = f V_0 p_0, \quad q^*(t) = \left(1 - \frac{t}{t_s}\right) H(t_s - t), \quad t \geq 0. \quad (3)$$

Generally, the contact surfaces of the strip and the semi-space are rough and the contact is imperfect. In this paper we consider the limiting case of the perfect contact between these surfaces. We also assume, that the temperature on the external surface of the strip is equal zero (the coefficient of heat transfer tends to infinity).

Further, all values and the parameters concerning the strip and the foundation will have bottom indexes “s” and “f”, respectively.

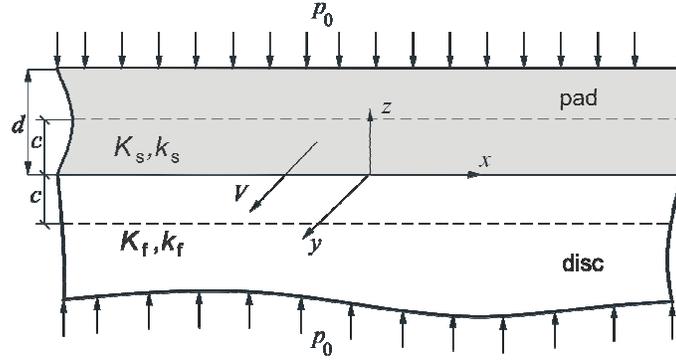


Fig. 1. Scheme of the problem.

In such statement, the transient temperature fields in the strip and in the foundation can be found from the solution of the one-dimensional heat conduction problem of friction during braking:

$$\frac{\partial^2 T^*(\zeta, \tau)}{\partial \zeta^2} = \frac{\partial T^*(\zeta, \tau)}{\partial \tau}, \quad 0 < \zeta < 1, \tau > 0, \quad (4)$$

$$\frac{\partial^2 T^*(\zeta, \tau)}{\partial \zeta^2} = \frac{1}{k^*} \frac{\partial T^*(\zeta, \tau)}{\partial \tau}, \quad -\infty < \zeta < 0, \tau > 0, \quad (5)$$

$$T^*(0+, \tau) = T^*(0-, \tau), \quad \tau > 0, \quad (6)$$

$$K^* \frac{\partial T^*}{\partial \zeta} \Big|_{\zeta=0-} - \frac{\partial T^*}{\partial \zeta} \Big|_{\zeta=0+} = q^*(\tau), \quad \tau > 0, \quad (7)$$

$$T^*(1, \tau) = 0, \quad \tau > 0, \quad (8)$$

$$T^*(\zeta, \tau) \rightarrow 0, \quad \zeta \rightarrow -\infty, \tau > 0, \quad (9)$$

$$T^*(\zeta, 0) = 0, \quad -\infty < \zeta \leq 1, \quad (10)$$

where

$$q^*(\tau) = \left(1 - \frac{\tau}{\tau_s}\right) H(\tau_s - \tau), \quad \tau \geq 0, \quad (11)$$

$$\zeta = \frac{z}{d}, \quad \tau = \frac{k_s t}{d^2}, \quad \tau_s = \frac{k_s t_s}{d^2}, \quad K^* = \frac{K_f}{K_s}, \quad k^* = \frac{k_f}{k_s}, \quad T_0 = \frac{q_0 d}{K_s}, \quad T^* = \frac{T}{T_0}. \quad (12)$$

### 3. Temperature

Taking the linearity of the boundary-value problem of heat conduction (4)–(10) and the form of function  $q^*(\tau)$  (11), the dimensionless transient temperature in the strip and in the foundation may be written in the form of superposition [21]:

$$T^*(\zeta, \tau) = [T^{(0)*}(\zeta, \tau) - T^{(1)*}(\zeta, \tau)]H(\tau) + T^{(1)*}(\zeta, \tau - \tau_s)H(\tau - \tau_s), \tau \geq 0, \quad (13)$$

where the upper indexes (0) and (1) correspond to solutions of the problem under consideration for the dimensionless temporary profile of the heat flux intensity in the boundary condition (7)

$$q^{(k)*}(\tau) = \left(\frac{\tau}{\tau_b}\right)^k, \tau > 0, k = 0, 1, \quad (14)$$

respectively.

We obtained the solutions of the parabolic boundary-value problem of heat conduction (4)–(10) with heat flux intensities (14) in the right-side of a boundary condition (7), by using the integral Laplace transform technique [29] with respect to the dimensionless time  $\tau$  in the form:

$$T^{(k)*}(\zeta, \tau) = \frac{1}{(1 + \varepsilon)} \sum_{n=0}^{\infty} \Lambda^n T_n^{(k)*}(\zeta, \tau), \quad -\infty < \zeta \leq 1, \tau \geq 0, k = 0, 1, \quad (15)$$

where

$$T_n^{(k)*}(\zeta, \tau) = 2\sqrt{\tau} \left(\frac{\tau}{\tau_s}\right)^k \times \begin{cases} F^{(k)}\left(\frac{2n + \zeta}{2\sqrt{\tau}}\right) - F^{(k)}\left(\frac{2n + 2 - \zeta}{2\sqrt{\tau}}\right), & 0 \leq \zeta \leq 1, \\ F^{(k)}\left[\frac{2n\sqrt{k^*} - \zeta}{2\sqrt{k^*}\tau}\right] - F^{(k)}\left[\frac{(2n + 2)\sqrt{k^*} - \zeta}{2\sqrt{k^*}\tau}\right], & -\infty < \zeta \leq 0, \end{cases} \quad (16)$$

$$\Lambda^n = \begin{cases} \lambda^n, & 0 \leq \lambda < 1, \\ (-1)^n |\lambda|^n, & -1 < \lambda \leq 0, \end{cases} \quad (17)$$

$$\lambda = \frac{1 - \varepsilon}{1 + \varepsilon}, \quad \varepsilon \equiv \frac{K^*}{\sqrt{k^*}} = \frac{K_f}{K_s} \sqrt{\frac{k_s}{k_f}}, \quad (18)$$

$$F^{(0)}(x) = \text{ierfc}(x), \quad F^{(1)}(x) = 3^{-1}[2(1 + x^2)\text{ierfc}(x) - x \text{erfc}(x)]. \quad (19)$$

The dimensionless parameter  $0 < \varepsilon < \infty$  (18) is known as “the coefficient of thermal activity of the material of the foundation relative to the material of the strip” [30].

The solutions (15)–(19) of heat problem of friction during braking (4)–(10) are obtained for zero temperature on the upper surface  $\zeta = 1$  of the strip (see boundary condition (8)). If this surface is thermally insulated, then

$$\left. \frac{\partial T_s^*}{\partial \zeta} \right|_{\zeta=1} = 0, \quad \tau > 0, \quad (20)$$

and the solutions of heat conduction equations (4) and (5) satisfying boundary and initial conditions (6), (7), (8), (10) and (20) are also found in the form (15) at:

$$T_n^{(k)*}(\zeta, \tau) = 2\sqrt{\tau} \left( \frac{\tau}{\tau_s} \right)^k \times \begin{cases} F^{(k)} \left( \frac{2n+\zeta}{2\sqrt{\tau}} \right) + F^{(k)} \left( \frac{2n+2-\zeta}{2\sqrt{\tau}} \right), & 0 \leq \zeta \leq 1, \\ F^{(k)} \left[ \frac{2n\sqrt{k^*} - \zeta}{2\sqrt{k^*}\tau} \right] + F^{(k)} \left[ \frac{(2n+2)\sqrt{k^*} - \zeta}{2\sqrt{k^*}\tau} \right], & -\infty < \zeta \leq 0. \end{cases} \quad (21)$$

The solutions (16) and (21) of the problem under consideration for the case of uniform sliding (when the upper index  $k = 0$ ) have been obtained in the article [31].

#### 4. Thermal stresses

Experimental examinations of the surface of the frictional elements of brakes proved that among the three normal components of the stress tensor – lateral  $\sigma_x$ , longitudinal  $\sigma_y$  and in the direction of heating  $\sigma_z$  – only the  $\sigma_x$  component exerts the most influence on thermal cracking [25]. It results in the material cracking into the direction of sliding (the heat flow motion). If the lateral component  $\sigma_x$  is greater enough than the tensile strength of materials, then the microcracks oriented at various angles to the direction of cracking are created, and divergence between the line of cracking and the direction of heat flow movement occurs. The normal component of stress tensor  $\sigma_z$  has no essential meaning in one-dimensional problem [32].

On the base of these data, the quasi-static longitudinal  $\sigma_x$  stress in the tribosystem, induced by the non-stationary temperature field (15)-(19) can be determined from the equations, which describe the thermal bending of thick plate of the thickness  $c$  with free ends (see Fig. 1) [33]:

$$\sigma_x(z, t) = \sigma_0 \sigma^*(\zeta, \tau), \quad |z| \leq c, \quad t \geq 0, \quad (22)$$

where the dimensionless stress  $\sigma^*(\zeta, \tau)$  can be found from the equation [25]:

$$\sigma^*(\zeta, \tau) = [\sigma^{(0)*}(\zeta, \tau) - \sigma^{(1)*}(\zeta, \tau)]H(\tau) + \sigma^{(1)*}(\zeta, \tau - \tau_s)H(\tau - \tau_s), \quad |\zeta| \leq c^*, \quad \tau \geq 0. \quad (23)$$

In the equation (23)  $\sigma^{(0)*}$  and  $\sigma^{(1)*}$  denote the dimensionless stresses corresponding to the dimensionless temperatures  $T^{(0)*}$  and  $T^{(1)*}$  (15), (16), respectively:

$$\sigma^{(k)*}(\zeta, \tau) = \varepsilon^{(k)*}(\zeta, \tau) - T^{(k)*}(\zeta, \tau), \quad |\zeta| \leq c^*, \quad \tau \geq 0, \quad k = 0, 1, \quad (24)$$

where

$$\varepsilon^{(k)*}(\zeta, \tau) = \frac{1}{(1 + \varepsilon)} \sum_{n=0}^{\infty} \Lambda^n \varepsilon_n^{(k)*}(\zeta, \tau), \quad (25)$$

$$\varepsilon_n^{(k)*}(\zeta, \tau) = Q_n^{(k)}(\tau) - \frac{|\zeta|}{c^*} \text{sign}(\zeta) R_n^{(k)}(\tau), \quad (26)$$

$$Q_n^{(k)}(\tau) = 4I_n^{(k)}(\tau) - 6J_n^{(k)}(\tau), \quad R_n^{(k)}(\tau) = 6I_n^{(k)}(\tau) - 12J_n^{(k)}(\tau), \quad (27)$$

$$I_n^{(k)}(\tau) = \frac{1}{c^*} \int_0^{c^*} T_n^{(k)*}(\pm\zeta, \tau) d\zeta, \quad J_n^{(k)}(\tau) = \frac{1}{(c^*)^2} \int_0^{c^*} \zeta T_n^{(k)*}(\pm\zeta, \tau) d\zeta. \quad (28)$$

In equations (28) and further, the top sign we shall choose for the strip and the bottom sign for a semi-space.

From formulas (26)-(28) it follows, that the dimensionless normal deformation  $\varepsilon^{(k)*}$  (25) is linearly dependent on the dimensionless distance  $\zeta$  from the surface of friction, and that normal stresses  $\sigma^{(k)*}$  (22)-(24) are proportional to the difference of this deformation and the dimensionless temperature  $T^*(\zeta, \tau)$  (13).

By substituting at the right side of formulas (28) the functions  $T_n^{(k)*}(\zeta, \tau)$  (16) and integrating, we obtain:

$$I_n^{(k)}(\tau) = \frac{4\sqrt{\kappa\tau}}{c^*} \left( \frac{\tau}{\tau_s} \right)^k [L_{2n}^{(k)}(\tau, \kappa, c^*) \pm L_{2n+2}^{(k)}(\tau, \kappa, \mp c^*)], \quad (29)$$

$$J_n^{(k)}(\tau) = \frac{4\kappa\tau}{(c^*)^2} \left( \frac{\tau}{\tau_s} \right)^k \{ [2\sqrt{\tau} M_{2n}^{(k)}(\tau, \kappa, c^*) - 2nL_{2n}^{(k)}(\tau, \kappa, c^*)] - [2\sqrt{\tau} M_{2n+2}^{(k)}(\tau, \kappa, \mp c^*) - (2n+2)L_{2n+2}^{(k)}(\tau, \kappa, \mp c^*)] \}, \quad (30)$$

where

$$L_n^{(k)}(\tau, \kappa, c^*) \equiv L^{(k)}\left(\frac{n\sqrt{\kappa} + c^*}{2\sqrt{\kappa\tau}}\right) - L^{(k)}\left(\frac{n}{2\sqrt{\tau}}\right), \quad (31)$$

$$M_n^{(k)}(\tau, \kappa, c^*) \equiv M^{(k)}\left(\frac{n\sqrt{\kappa} + c^*}{2\sqrt{\kappa\tau}}\right) - M^{(k)}\left(\frac{n}{2\sqrt{\tau}}\right), \quad (32)$$

$$L^{(0)}(x) \equiv \frac{1}{4} + \frac{x}{2\sqrt{\pi}} \exp(-x^2) - \frac{(1+2x^2)}{4} \text{erfc}(x), \quad (33)$$

$$M^{(0)}(x) \equiv \int_0^x t F^{(0)}(t) dt = \frac{1}{6\sqrt{\pi}} - \frac{(1-2x^2)}{6\sqrt{\pi}} \exp(-x^2) - \frac{x^3}{3} \text{erfc}(x), \quad (34)$$

$$L^{(1)}(x) \equiv \int_0^x F^{(1)}(t) dt = \frac{1}{8} + \frac{(5x+2x^2)}{12\sqrt{\pi}} \exp(-x^2) - \frac{(3+12x^2+4x^4)}{24} \text{erfc}(x), \quad (35)$$

$$M^{(1)}(x) \equiv \int_0^x t F^{(1)}(t) dt = \frac{1}{15\sqrt{\pi}} - \frac{(1-4x^2-2x^4)}{15\sqrt{\pi}} \exp(-x^2) - \frac{x^3(5+2x^2)}{15} \text{erfc}(x), \quad (36)$$

and the parameter  $\kappa = 1$  for the strip and  $\kappa \equiv k^*$  for the semi-space.

If the upper surface  $\zeta = 1$  of the strip is thermally insulated, then after substituting at the right side of formulas (28) the functions  $T_n^{(k)*}(\zeta, \tau)$  (21) and integrating, we find:

$$I_n^{(k)}(\tau) = \frac{4\sqrt{\kappa\tau}}{c^*} \left( \frac{\tau}{\tau_s} \right)^k [L_{2n}^{(k)}(\tau, \kappa, c^*) \mp L_{2n+2}^{(k)}(\tau, \kappa, \mp c^*)], \quad (37)$$

$$J_n^{(k)}(\tau) = \frac{4\kappa\tau}{(c^*)^2} \left( \frac{\tau}{\tau_s} \right)^k \{ [2\sqrt{\tau} M_{2n}^{(k)}(\tau, \kappa, c^*) - 2nL_{2n}^{(k)}(\tau, \kappa, c^*)] + \\ + [2\sqrt{\tau} M_{2n+2}^{(k)}(\tau, \kappa, \mp c^*) - (2n+2)L_{2n+2}^{(k)}(\tau, \kappa, \mp c^*)] \}. \quad (38)$$

## 5. Numerical analysis

The numerical results have been obtained for the friction couple a FMK-11 metal-ceramics pad (the strip) and a cast iron disk (the semi-space) for which [17]:  $K_s = 34.2 \text{ W m}^{-1} \text{ K}^{-1}$ ,  $k_s = 15.2 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}$ ,  $K_f = 51 \text{ W m}^{-1} \text{ K}^{-1}$ ,  $k_f = 14 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}$ . The ceramic-metal frictional material FMK-11 consists of 64% Fe, 15% Cu, 9% C, 3% SiO<sub>2</sub>, 3 % asbestos and 6% BaSO<sub>4</sub> [18]. The metal components of FMK-11 (Fe, Cu) provide to a material high heat conductivity and wear-in, and the non-metallic component (C, SiO<sub>2</sub>, *et al.*) increase the coefficient of friction and reduce propensity to jamming. The frictional material on the basis of FMK-11 is intended for work in the hard loaded wheel disk brakes of planes [18].

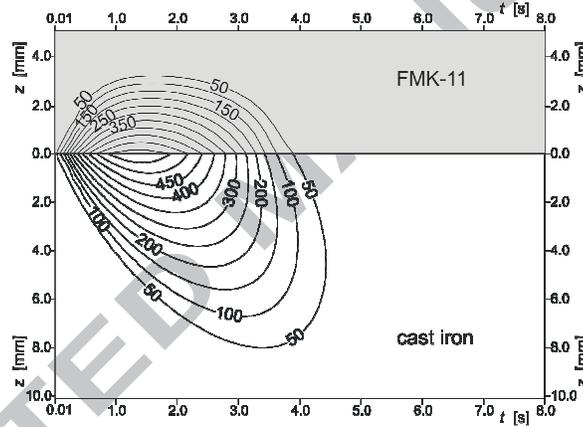
The friction conditions are: the pressure  $p_0 = 1 \text{ MPa}$ , the initial sliding speed  $V_0 = 30 \text{ m s}^{-1}$ , the coefficient of friction  $f = 0.7$  and the time of braking  $t_s = 3.44 \text{ s}$ . The initial temperature equals  $20 \text{ }^\circ\text{C}$ .

The dimensionless thermal stresses  $\sigma^*$  (23) were calculated at distance from the contact surface  $c = d = 5 \text{ mm}$  ( $c^* = 1$ ). All the results presented in Figs., were obtained for two limiting type of the boundary conditions on the upper surface  $z = d$  ( $\zeta = 1$ ) of the strip: a) at zero temperature (8); b) for thermal insulation (20).

Isotherms for the temperature constructed in the coordinate system  $(z, t)$  are shown in Fig.2a,b. Accordingly to the boundary condition (6), the temperatures of the strip and the foundation on the contact surface  $z = 0$  are equal. Provided that the braking starts, the temperature increases quickly, then it reaches its maximum and begins to decrease. The largest value of the temperature on the surface is reached in the case of thermally insulated

upper surface of the strip. In the case of the maintenance of zero temperature on the upper surface of the strip, the temperature on the surface of friction, having reached it's maximum value  $T_{\max} = 593^{\circ}\text{C}$  at the time moment  $t_{\max} = 1.6\text{s}$  (Fig. 2a), decreases quickly and reaches the initial value  $20^{\circ}\text{C}$  at the 4.1s time. Such a decrease of temperature in case of the thermally insulated upper surface of the strip is of other nature. Having reached the maximum value  $T_{\max} = 797^{\circ}\text{C}$  at the time moment  $t_{\max} = 2.6\text{s}$  (Fig. 2b), the temperature decreases much more slowly and the time for obtaining the initial temperature value is much longer, and is greater than 20s. The maximal temperature value  $T_{\max} = 797^{\circ}\text{C}$ , obtained as a result of numerical calculations, corresponds well with the experimental data  $T_{\max} = 760^{\circ}\text{C}$ , published in monograph [17, p.71].

a) the zero temperature on the upper surface of the strip:



b) thermally insulated upper surface of the strip:

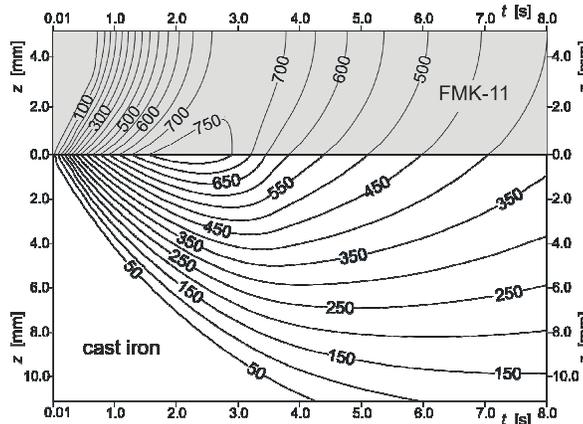
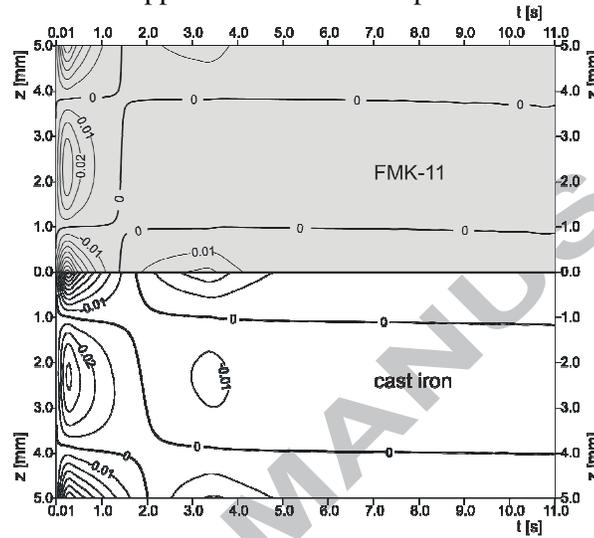


Fig. 2. Isotherms of the temperature  $T^{\circ}\text{C}$  in FMK-11 pad and cast iron disc.

The temperature in foundation for a time moment  $t = t_{\max}$  decreases with the increase in distance from the surface of friction. At the moment of time  $t_{\max} = 2.6\text{s}$  the temperature on the

upper surface of the strip ( $z = 5 \text{ mm}$ ) achieves nearly  $640^\circ\text{C}$ , when this surface is insulated (Fig. 2b). It is established, that the decrease of temperature for the time moment  $t_{\max} = 1.6 \text{ s}$  from maximum value  $T_{\max} = 593^\circ\text{C}$  on the contact surface to its initial value, may be described with the line function of spatial coordinate  $z$  at maintenance of zero temperature on the upper surface of the strip.

a) the zero temperature on the upper surface of the strip:



b) thermally insulated upper surface of the strip:

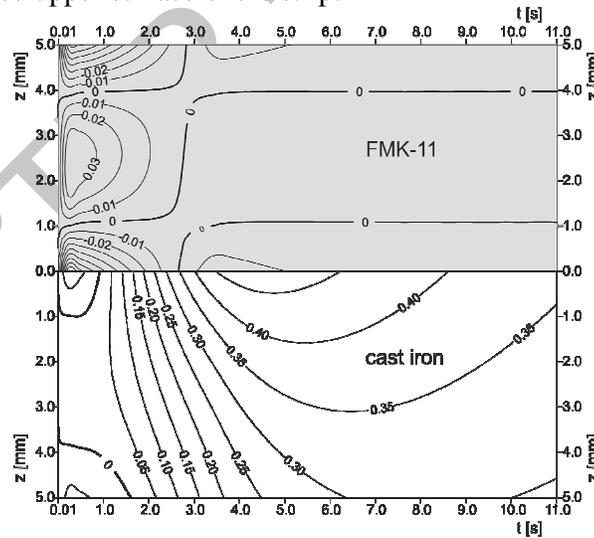


Fig. 3. Isolines of the dimensionless lateral stress  $\sigma^*$  in FMK-11 pad and cast iron disc.

Isolines of the dimensionless normal stresses  $\sigma^*$  (23) are presented in Figure 3. The distributions of these stresses in a FMK-11 pad and a cast iron disc are nearly identical, when temperature on the upper surface of the strip is equal zero (Fig. 3a). It is possible to explain such a symmetry by the fact, that the coefficient of thermal activity of materials slightly

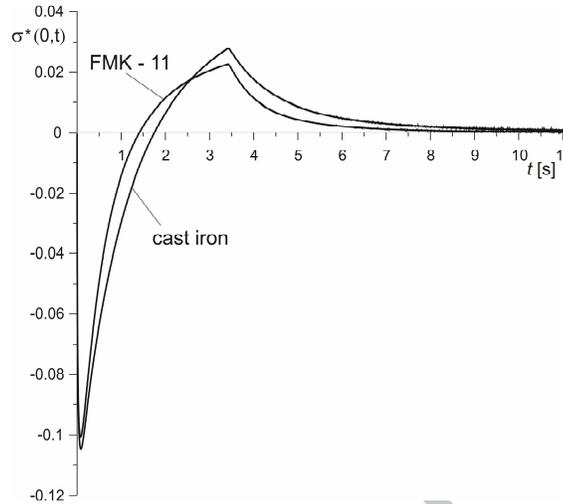
differs from unit ( $\varepsilon = 1.55$ ). It is observed, that in the time interval  $0 < t < 1.5$  s the regions of compressive stress occur near the surface of friction  $\zeta = 0$  (Fig. 3a). During the same time inside a pad and a disc the tensile stresses are generated. Between the regions with compressive and tensile stresses there are two isolines with zero stresses. The third line of zero stresses, which “descends” from the surface  $\zeta = 0$ , appears when the heating time is equal  $t \equiv t_c \approx 1.5$  s for the pad and  $t_c \approx 1.8$  s for the disc. These values of time are lower than time of braking  $t_s = 3.44$  s. It means, that during cooling phase at  $t > t_s$ , when there is no more heating, the sign of stresses does not change and the region of tensile stresses expands further from the heated surface – the line of zero stresses moves parallel to the surface of friction with the increase of time (Fig. 3a). A different distribution of dimensionless normal stress  $\sigma^*$  (23) is observed for thermal insulation of the upper surface of the strip (Fig. 3b). In the heating phase at the time interval  $0 < t < 2.8$  s for the pad and  $0 < t < 0.9$  s for the disc, the compressive superficial stresses occur. In this case, the level of tensile stresses in a disc is greater than in case of zero temperature on the upper surface of a strip.

The initiation of superficial cracks generation is accompanied with the monotonic increase of tensile normal stresses. In heating of frictional elements during braking, it is the change of sign of superficial stresses that plays key role in forecasting initiation and preventive maintenance of thermal splitting [25]. The dependencies of the thermal stresses of the pad and the disc on time are alike in case of maintenance of the initial temperature on the upper surface of the strip (Fig. 4a). It results from the fact, that the distribution of the temperature in these elements is alike, too (Fig. 3a). The distribution of the temperature in the pad and the disc differs a lot in case when the upper surface of the strip is insulated. The temperature reaches significant values on the whole depth of the pad, and its maximal value on the contact surface is  $200$  °C higher than in case of the initial temperature on the upper surface of the strip (Fig. 3b). It induces different values and evolution of the thermal stresses (Fig. 4b). In the case of thermal insulation of the upper surface of the strip, the magnitude of surface tensile stresses in the cast iron disc is much greater, than in case of zero temperature on this surface (Fig. 4). The most endangered element is the pad, since there are much bigger (than in the disc) tensile stresses occurring within it.

The  $t_c$  time of the sign change of lateral stress on the friction surface from compressive to tensile is an important parameter. It allows to estimate the moment in time, when the

beginning of the thermal cracking is possible. Naturally, this time needs to be extended. When the time of braking  $t_s$  gets longer, the  $t_c$  value rises (Fig. 5).

a) the zero temperature on the upper surface of the strip:



b) thermally insulated upper surface of the strip:

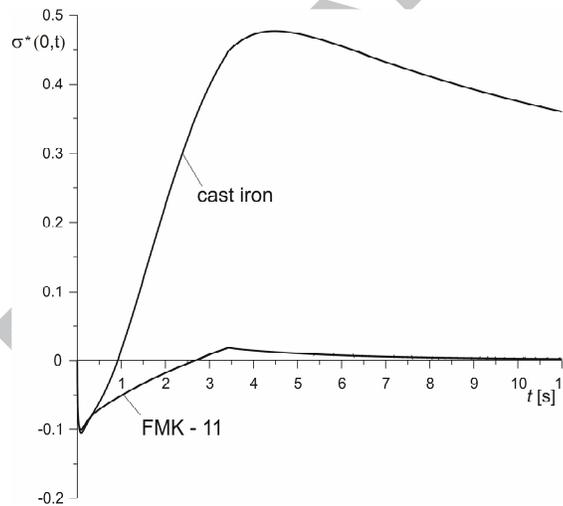
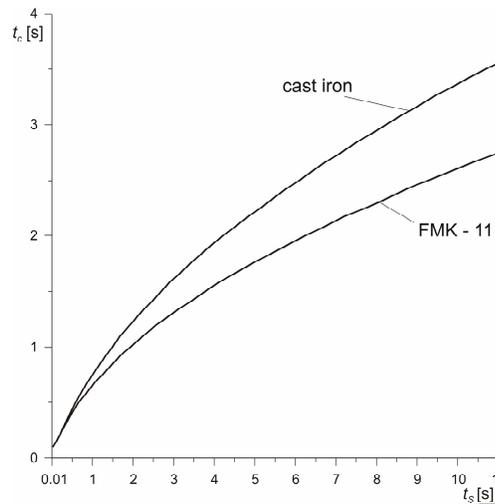


Fig. 4. Evolution of the dimensionless lateral stress  $\sigma^*$  on the contact surface.

With increase in time of braking  $t_s$ , the time  $t_c$  of change of sign on normal stress increases (Fig. 5). At fixed time of braking, whereas the upper surface of the strip is maintained at the zero temperature (is thermally insulated), the time of change of sign on normal stress for the pad is less (greater) than the same time for the cast-iron disc. The greatest difference between times of occurrence of tensile stresses on a surface of friction is observed in the case of the thermal insulation of the upper surface of the strip (Fig. 5b).

a) the zero temperature on the upper surface of the strip:



b) thermally insulated upper surface of the strip:

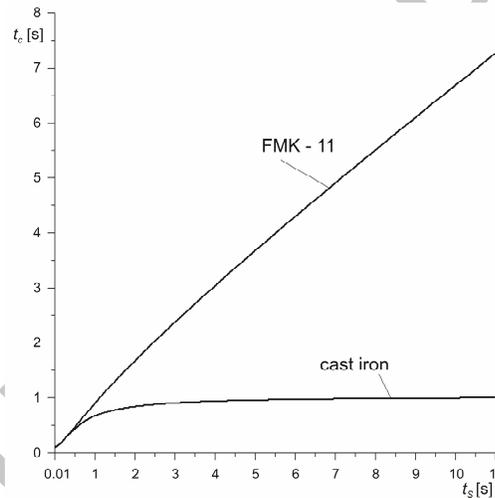


Fig. 5. The time  $t_c$  of the sign change of lateral stress  $\sigma_x$  on the surface of friction  $z = 0$  versus duration  $t_s$  of the braking.

## 6. Conclusions

The analytical solution of the transient heat conduction problem of friction during braking for the plane-parallel strip (the pad) sliding over the semi-infinity foundation (the disc) has been obtained. The temperature field and the thermal stresses for the friction couple metal-ceramic pad and cast iron disc have been studied. The influence of the boundary conditions on the upper surface of the pad on the distribution of temperature and thermal stresses has been investigated.

Analysis of the evolution of thermal stresses in the frictional elements during braking proves, that when it is heated considerable normal compressive stresses occur near the contact surface. The value of this stresses decreases with an increase of time and after some time moment  $t_c$  the sign changes – which means that the tensile stresses take place. The time when it happens increases monotonously with increase of a time braking  $t_s$ .

On the basis of achieved numerical data it is established, that the possible initiation of the superficial crack during braking can be described as the series of the following phases:

- due to local intensive frictional heating near a contact surface the field of compressive stresses is formed;
- after the beginning of braking in the some time moment  $t_c$  the tensile normal stresses occur near the subsurface region;
- when these stresses exceed the tensile strength of the material, the initiation of the surface cracks is possible.

## References

- [1] A.J. Day and A.B. Ahsi, Heat flow and temperatures in friction material during braking, Proceedings of the 17th Leeds-Lyon Symposium on Tribology Leeds, UK, September, (1990) 183–191.
- [2] A.V. Chichinadze, Calculation and Study of External Friction during Braking, Nauka, Moskow, 1967. (in Russian).
- [3] H.S. Qi, A.J. Day, Investigation of disc/pad interface temperatures in friction braking, *Wear* 262 (5–6) (2007) 505–513.
- [4] D.M. Rowson, The interfacial surface temperature of a disc brake, *Wear* 47 (2) (1978) 323–328.
- [5] J.R. Barber, T.W. Beamond, J.R. Waring, C. Pritchard, Implications of thermoelastic instability for the design of brakes, *Trans. ASME. J. Tribology* 107 (2) (1985) 206–210.
- [6] A.A. Yevtushenko, E.G. Ivanyk, Determination of heat and thermal distortion in braking systems, *Wear* 185 (1–2) (1995) 159-165.
- [7] A.A. Yevtushenko, E.G. Ivanyk, Determination of temperatures for sliding contact with applications for braking systems, *Wear*, 206 (1997) 53-59.
- [8] S.J. Matysiak, A.A. Yevtushenko, E.G. Ivanyk, Contact temperature and wear of composite friction elements during braking, *Int. J. Heat and Mass Transfer*, 45 (2002) 193-199.

- [9] Z. C. Zhu, Y. X. Peng, Z. Y. Shi, G.A. Chen, Three-dimensional transient temperature field of brake shoe during hoist's emergency braking, *Appl. Therm. Engng.* 29 (2009) 932–937.
- [10] G.A.G. Fasekas, Temperature gradients and heat stresses in brake drums. *SAE Trans.* 61 (1953) 279–284.
- [11] T.L. Ho, M.B. Peterson, F.F. Ling, Effect of frictional heating on braking materials, *Wear* 26 (1974) 73–79.
- [12] Z. Olesiak, Yu. Pyryev, A.A. Yevtushenko, Determination of Temperature and Wear during Braking, *Wear* 210 (1997) 120–126.
- [13] O.O. Yevtushenko, Yu.O. Pyr'ev, Calculation of the contact temperature of frictional elements of brakes, *Materials Science* 34 (2) (1998) 249–254.
- [14] A.A. Yevtushenko, E. G. Ivanyk, O.O. Yevtushenko, Exact formulae for determination of mean temperature and wear during braking, *Heat Mass Transfer* 35 (1999) 163–169.
- [15] Yu. Pyryev and A. Yevtushenko; The influence of the brakes friction elements thickness on the contact temperature and wear, *Heat Mass Transfer* 36 (4) (2000) 319–323.
- [16] J.J. Santini and F.E. Kennedy, Experimental investigation of surface temperatures and wear in disk brakes, *Lubr. Eng.* 31 (8) (1975) 402–404, 413–417.
- [17] A.V. Chichinadze, E. D. Brown, A. G. Ginsburg and E. V. Ignat'eva, Calculation, Testing, and Selection of Frictional Pairs, Nauka, Moscow, 1979. (in Russian).
- [18] V. A. Balakin, V. P. Sergienko, Heat Calculations of Brakes and Friction Joints, Gomel, MPRI NASB, 1999. (in Russian).
- [19] A. Evtushenko, S. Matysiak, M. Kutsei, Thermal problem of friction at braking of coated body, *J. Friction and Wear* 26 (2) (2005) 33–40.
- [20] S. Matysiak, O. Yevtushenko, M. Kutsei, Temperature field in the process of braking of a massive body with composite coating, *Materials Science* 43 (2007) 62–69.
- [21] A.A. Yevtushenko, M. Kuciej, Frictional heating during braking in a three-element tribosystem, *Int. J. Heat Mass Transfer* 52 (2009) 2942–2948.
- [22] T.J. Mackin, S.C.Noel, K.J. Ball, et al., Thermal cracking in disk brakes, *Eng. Failure Anal.* 9 (2002) 63–76.
- [23] J. Yamabe, M. Takagi, T. Matsui, Development of disc brake rotors for heavy- and medium-duty trucks with high thermal fatigue strength, *Technical Review* 15 (2003) 42–51.
- [24] A. Yevtushenko, M. Roźniakowska, M. Kuciej, Laser-induced thermal splitting in homogeneous body with coating, *Numerical Heat Transfer, Part A.* 52 (2007) 357–375.

- [25] A. Evtushenko, M. Kutsei, Initiating of thermal cracking of materials by frictional heating, *J. Friction and Wear* 27 (2) (2006) 9–16.
- [26] P.J. Blau, Compositions, Functions, and Testing of Friction Brake Materials and Their Additives, Oak Ridge National Laboratory Technical Report ORNL/TM 2001/64, Oak Ridge, Tennessee, 24 pp., 2001.
- [27] G. S. Daehn, M. C. Breslin, Co-continuous composite materials for friction and braking applications, *JOM Journal of the Minerals, Metals and Materials Society* 58 (11) (2006) 87–91.
- [28] F. F. Ling, *Surface Mechanics*, Wiley, New York, 1973.
- [29] I. N. Sneddon, *The Use of Integral Transforms*, McGraw-Hill, New York, 1972.
- [30] A. V. Luikov, *Analytical Heat Diffusion Theory*, Academic Press, New York, 1968.
- [31] A. A. Yevtushenko, M. Kuciej, Influence of convective cooling on the temperature in a frictionally heated strip and foundation, *Int. Comm. Heat Mass Transfer* 36 (2009) 129–136.
- [32] A. P. Dostanko, N. K. Toloczko, S. E. Karpowicz, Yu.S., Mezhinskii, A.M. Rusetskii, V.I. Semashko, A.A. Chmyl, N.N. Fedosenko, *Technology and technique of precise laser modification of solid-state structures*, Tiechnoprint, Minsk, 2002. (in Russian).
- [33] S. P. Timoshenko, J. N. Goodier, *Theory of elasticity*, McGraw-Hill, New York, 1970.