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ELICITING BELIEFS WITH A NARRATIVE ACTIVITY IN MATHEMATICS TEACHER EDUCATION

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In a teacher training course for mathematics teachers, a narrative activity was assigned to elicit participants' beliefs on proof. Proof is a very important topic in mathematics education, crucial to develop mathematical thinking, and therefore worth special attention. Individual narrations of one's relation with proof were collected, posted on the web and discussed. This allowed a number of different aspects and views to emerge and to be compared, providing a good opportunity for the trainees to become aware of their own beliefs and shortcomings. The positive outcomes of this experience suggest that narrative can be a suitable tool to address beliefs in teacher education.

INTRODUCTION AND BACKGROUND

Beliefs about mathematics and its teaching and learning are recognized to deeply influence teachers' instructional practice (Philipp, 2007; Thompson, 1992), as much as the social context and the teachers' level of thought and reflection (Ernest, 1989).

Teachers' beliefs are therefore an object of study of the research in mathematics education. Three different perspectives can be found in the literature, as emerges from Thompson's survey (1992) and the contributions collected by Leder, Pehkonen and Törner (2002): attention to teachers' beliefs, with particular reference to the construction of a suitable theoretical framework; focus on the dialectic relationship between teachers' beliefs and instructional practice; study of situations and experiences that may change teachers' beliefs. This last stream is directly linked to the emerging field of research in mathematics teacher education (Adler et al., 2005; Tirosh & Wood, 2008).

As widely reported in the literature (Llinares & Krainer, 2006), pre-service teachers join the teacher education program with beliefs about mathematics and mathematics teaching and learning which influence not only what kind of teachers they will become, but also how they attend the teacher education program. Two kinds of beliefs should be taken into account: about mathematics and about mathematics teaching and learning. Both of them are linked to content knowledge and to pedagogical content knowledge (Shulman, 1986).

Explicitly addressing prospective teachers' beliefs, therefore, aims to help them *“learn to do something different from - and better than- what they experienced in*

mathematics classes” (Ball, 1990, p. 11). To this end, educators have two responsibilities: “*to judge what prior learning can contribute to future growth and which may impede it*” as well as “*to construct the conditions for experiences which can foster future growth*” (ibid, p.12).

Transforming teachers’ beliefs is not an easy task because they are rooted in life experience and often unconscious, so much that Thompson (1992) underlines that “*teachers treat their beliefs as knowledge*” (p. 127). According to Thompson (ibid.), good results are obtained by making prospective teachers *doubt* about their own beliefs. Llinares & Krainer (2006) note that some pre-service education programs concentrate on making prospective teachers aware of beliefs by means of reflection on action, regarded as a way for teachers to construct the meaning and knowledge that guide their actions. In this perspective, the work on teachers’ beliefs is linked to the importance of preparing reflective teachers (Schön, 1983), able to challenge their own practice and beliefs.

In our opinion, an effective way to tackle this issue is to help beliefs emerge with a narrative activity and then address them by means of comparison and discussion with peers. This is the approach that we adopted in the described experience.

Narrative has been gaining increasing attention in the educational field in the past couple of decades, thanks to its cognitive and motivational potential (Dettori & Paiva 2009), and is increasingly used also in mathematics education, both as a teaching tool (Zazkis & Liljedahl, 2009), and as a methodology of study (Kaasila, 2007). Stories and narrations are an expressive form that is natural, informal and pleasant for people of any culture and age (Bruner, 1990; Schank, 2000). The narrative form suggests (possibly implicit) temporal and causal connections among the presented facts (Bruner, 1990; Herman, 2003). This transforms a sequence of events in what is called a configuration (Ricoeur 2005), i.e. a coherent whole in which each part contributes to the global meaning and at the same time takes meaning from its position within it (Bruner 2003). This leads the narrator, and those who read or listen a narrative, to become aware of more or less implicit relations, hence favouring thought organization and content understanding.

For these reasons, narrative can constitute an excellent starting point to foster self-awareness and reflection. Reflections are thoughts, mostly abstract and often mood-dependent, and beliefs are often unconscious and therefore difficult to retrieve. Narrations can aid to overcome both problems since they can help ground reflection in concrete, personal experience and let beliefs emerge from the narrated facts, events and feelings. It is not surprising, therefore, that narrative activities are increasingly used to support reflection on a variety of aspects, including beliefs, in the training of both pre-service and in-service teachers.

As pointed out by Zaslavsky, Chapman & Leikin (2003), the narrative approach leads mathematics teachers to elicit experiences they lived in the classroom, helping them to reflect on the complex domain of teaching practice and by this means fostering

professional growth. Dolk & den Hertog (2008), for instance, challenge small groups of teachers to collaboratively build stories of classroom situations presented by means of videos, with the aim to improve their ability to observe and correctly identify learning difficulties.

Oriented to the study of beliefs is the use of narrative made by Chapman (2005; 2008a; 2008b) to implement pre-service teachers' self-study and hence detect beliefs possibly conflicting with the views of mathematics education promoted in the training program. In her teacher education intervention, trainees write stories of both actual teaching (from their schooldays or the running practicum), and ideal, best practice teaching. These stories are shared with the other trainees and used as starting point for self-reflection, then revised and made once again object of reflection. Trainees are also given theoretical means to analyse narratives so that this approach can become part of their professional competence and also used in future work.

In the experience described in this paper, we relied on narrative to address trainees' beliefs on a particular aspect of mathematics learning, i.e., proof and proving. These are very important concepts in mathematics, at the very core of mathematical thinking, and should therefore be central in mathematics education.

The interest for this topic is underlined by the big amount of literature on the subject (see e.g. the international newsletter www.lettredelapreuve.it and the proceedings of the ICMI Study 19 "Proof and proving in Mathematics Education" (Lin et al., 2009)).

Among the authors who underline the importance of proof, we recall De Villiers (1990) and Hanna (1990), who advocate that the function played by mathematical proofs and theorems within mathematics should be stressed in the classroom in order to motivate students to get engaged in proving and to make them grasp the meaning and importance of proof. Hanna discusses proofs that (only) *prove* and proofs that (also) *explain*. De Villiers singles out different functions of proof in mathematics: verification/conviction, explanation, systematization, discovery, communication.

As concerns how to introduce it in school, Balacheff (1982) points out that the teaching of proofs and theorems should aim both to make students understand what is a proof, and learn to produce it. Weber (2005) dealing with the proving process of undergraduate students, argues that it may be seen as a problem solving activity and discusses what kind of learning is fostered by different ways of proving. Selden & Selden (1995) point out that many difficulties related to mathematical proof, especially at tertiary level, rely on the fact that students are used to see it as final product to be reproduced rather than as a process they should be able to work out.

All these considerations highlight the importance to include proof in mathematics teachers' preparation, considering it both as a product (that must meet some standard requirements) and as a process (intentionally aimed at the construction of such products). The different roles that proofs can play in mathematics learning and the need to see proofs not only as products but also as processes underline the importance

of explicitly addressing proof-related beliefs in teacher training. This was the focus of the described experience.

A NARRATIVE ACTIVITY TO ELICIT BELIEFS

In this paper we discuss a narrative activity that was planned and carried out during the academic year 2007-2008, in the context of a course in Mathematics Education for pre-service teachers of secondary school (PSTs). This 49-hour course, which took place in the first year of a two-year training program of the University of Genoa (Italy) and was taught by one of the authors (FM), had a special focus on the teaching and learning of proof.

We planned a narrative activity at two different points in the course, with different aims. The first of them, which is reported in this paper, aimed to make the PSTs become aware of their own beliefs, connecting them with their previous experience and comparing them with their colleagues', so as to boost reflection on the teaching and learning of mathematical proof. This is in line with our idea of working on pre-service teachers' beliefs by capitalizing on PST's different backgrounds as a source of shared reflection. At the same time, we aimed to deepen the use of narrative as a tool to elicit beliefs and contribute to explore the learning potential of narrative.

Methodology

In total, 16 PSTs took part in the course.

During the first class, an open narrative task was proposed:

Write down the story of your relationship with proof (it is important that you write a story, rather than considerations; your story will be the starting point for a collective reflection).

The individual stories were collected and posted, in anonymous form, in the course's online space. The PSTs were then asked to read all the stories and comment them in a forum created on purpose:

In the virtual space of the course, you find a word document that contains in anonymous form all the stories of your relationships with proof. Reading this document, you will read again your own story, together with the stories written by your colleagues. After reading all the stories, do you feel you would like to add something to your own story? Are there other episodes that would like to narrate? Other stories?

For our analysis, therefore, we have at disposal the written individual narrations of the PSTs' experiences with proof, as well as the comments posted in the forum.

ANALYSIS

Individual stories about proof

The short stories written by the trainees report difficulties and strengths, inclinations and dispositions, from which it is possible to spot beliefs and other affective factors. Looking at individual stories, we can single out two main types: "evolution stories"

(6 in total) and “permanence stories” (5 in total). The other contributions do not contain a clear temporal dimension and relate single episodes or general opinions.

Evolution stories narrate the positive evolution of the relationship with proof, or the shift from difficulty in understanding and building proofs to succeeding in both of them. We may note that negative evolution stories (from a positive to a negative attitude, from success to difficulty) are not present. This is not surprising, considering which special population was involved in this experience.

An example of “evolution story” is given by Anna’s narration:

At the beginning I had difficulties in understanding how to proceed, especially in proofs by contradiction, but once I understood the logical-deductive method, over the time I learned, thanks to a good teacher I had.

Permanence stories, on the other hand, show the stability over time of a relationship with proof. Here, we find not only positive but also negative situations, as some PSTs report feelings of uneasiness and anxiety, or even difficulties, when proving. Such difficulties and negative dispositions should not be considered as indicators that those PSTs would never become good teachers; quite on the contrary, experiencing in person difficulties and negative affects with proof can help a teacher to better understand students’ troubles, without taking understanding for granted, provided these negative aspects are suitably addressed and overcome. Discussing with the class such negative relationships is useful also to those PSTs who, on the contrary, always had a good relationship with proof, since this opens for them a different perspective and can hence help them to be in a better position to understand the difficulties of their future students.

An example of positive “permanence story” is Betta’s narration:

Being very rational, I always loved proofs (even the most boring ones!), because they allow me to verify theses that I would be unwilling to accept otherwise.

An example of negative “permanence story” is given by Biagio:

My relationship with proof was always a turmoil. In mathematics, even if I knew that this was wrong, I have always seen proof as something “boring”. Something where someone smarter than me had been able to find the way to reach his goal.

Analysing the stories, a number of causal links emerge. In the evolution stories it is interesting to see which factors are perceived as causes of the evolution. In some cases, it was fostered by the intervention of a teacher (see Anna’s quotation above), in other cases, the evolution took place as a consequence of personal growth. Alessio, e.g., explains:

As regards the first part of my life, that ended with the start of my university studies, my idea of proof was linked to argumentative and discursive forms, certainly of logical-deductive nature, sometimes axiomatic; in few cases also to inductive reasoning. In a second phase of my life I started to understand the “sense” of proof from a mathematical point of view: on one side, I started to catch its inner and deep sense when studying it at the university; on the other side, I used proof to clarify, justify, explain, memorize and give meaning to the topics that I had to teach to my students.

In a third phase, I started to reflect on the philosophical and mathematical meaning of proof, looking back at the questions that emerged spontaneously during the preceding phases. Important suggestions came from studying Mathematical Logic.

Also in the permanence stories some factors are seen at the basis of permanence, which is described as linked to one's personal characteristics, as well as to the very idea of proof. An example is Betta's excerpt reported above, where interest for proof is seen as an expression of personal rationality.

The crucial role of the teacher is also underlined; Caterina, for example, writes: "*My relationship with proof is essentially linked to the teachers who taught me it*". Explicitly stating facts like this is very important in teacher training, because, even though PSTs are very much aware of the importance of good teaching, it is anyway useful for them to remind the positive and negative influences they had from teachers in their school days.

The relationship with proof is often described in a strong "affective" way, independently of the author's attitude towards it: "*I always loved proofs*" (very good relationship; Betta), "*I loved those proofs that were taught me by teachers who loved mathematics*" (partially good, conditioned relationship; Caterina), "*My relationship to proof was always a turmoil*" (difficult relationship; Biagio). The stories brought to the fore the issue of affective involvement during the proving process or during the study of a proof: "*Usually I don't remember the final proof, what I like is proving*" (Clara); "*When I worked on proofs by myself at home and I managed to understand and to reach the end, I felt very satisfied*" (Brigitta). Such stories provided an opportunity to raise the point of the importance of affective factors in the teaching and learning of mathematics, and in particular in the teaching and learning of proof.

Beliefs about the nature of proof also emerged. Most of the PSTs focus on proof as a mathematical object: "*Proof is an important and necessary tool in mathematics, otherwise you risk to do philosophy*" (Claudio); "*In a proof, each step must be a natural consequence of the hypothesis or of the preceding steps, or an obvious application of theorems that were proved before*" (Cecilia). Others focus on the roles of proof in the teaching and learning of mathematics: "*I always used proof to better understand the theorems*" (Bianca); "*in order to frame the theorems, rather than memorizing them, I understood that it was easier to prove them, by means of the knowledge already at disposal*" (Agnese). If we refer to the functions of proof pointed out in the literature (de Villiers, 1990; Hanna, 1990), we may note that for some PST proof **convinces**, ("*Proofs allow me to verify a thesis, that otherwise, if not proved, I would not accept*"; Betta), while, for others, proof, besides convincing, **illuminates and explains** ("*I liked to see an ordered path of the reasons why from some hypotheses it is possible to reach some theses, besides the formal justification of what I found evident*"; Anita).

Personal narrations sometimes include reflections that tell the story of one's understanding. This is the case of Biagio who points out the gap existing between proof in mathematics and proof in the classroom:

In mathematics, you do each step coherently... to its system of rules. If somebody proved a theorem and the scientific community accepted this proof, the theorem is correct. But, for the student, the proof loses its main meaning and its role is "only" to help understand the theorem itself. But if the student does not understand the proof, what is the benefit of proving?.

Proof as a process or proof as a product is another dimension that emerges from narrations. Most PSTs conceive proof essentially as a product that the teacher presents in its final form and the students must learn and reproduce; other PSTs view proof as a process, as a special case of problem solving. The importance of taking into account both the process and the product aspects is well expressed by Biagio:

I often had difficulties with proofs, when I met sentences like "if we note that this term may be written in this way...", or "now, it is sufficient to note that...". Well, if the student "does not note", he will never finish the proof, and proving will be reduced to memorizing and reproducing that "awful step" created by the first who proved the theorem, rather than to justifying and really understanding the theorem.

Another important theme that emerged from some individual stories is the special nature of proof in mathematics, if compared with proof in the other sciences:

In the last years of secondary school [...] I started to distinguish empirical sciences from mathematics: the differences rely in the different kinds of proof that are accepted: empirical-experimental proof in sciences, abstract and formalized in mathematics (Alessandro).

Proof is different in experimental physics. [...] In physics, proving means planning some experiments to investigate the existence or non-existence of a phenomenon, and explaining why it "works" that way (Biagio).

Online discussion

Nine of the 16 pre-service teachers attending the course took part in the online discussion. Our PSTs had little previous experience with forums and distance-learning, and the participation in the online discussion was not part of the usual didactical contract in the teacher training school. We anyway proposed an online discussion for two reasons. Firstly for lack of time, as the course was not long enough to discuss in depth all relevant topics during class hours. Second, but not least, we wanted to grant the participants a looser time, allowing the possibility to reflect on similarities and differences before writing. The fact that part of the group never intervened in the forum was due mainly to difficulty to find time to log in, as several of the PSTs were commuters or did not have Internet connection at home.

Totally, 28 comments to the individual stories were posted. Not all stories were mentioned and discussed, but only those that were making some strong point with which the others felt the need to agree or disagree.

In general, we can single out different themes emerging from the posts:

- the feeling of pleasure in relation to creating and/or understanding proof;
- the differences and similarities between proving a statement and other school activities, such as translating from Latin;
- previous experiences as students (and their possible influence on the future profession of teacher);
- the complex work of a teacher.

Some themes are clearly linked to beliefs on proof, others are more general but anyway consistent with the aim of fostering reflectivity on teaching and learning as a crucial feature for professional development.

Particularly interesting appears the stream of discussion started with a comment on Clara's story (*Usually I don't remember the proof, what I like is proving*), which deepened the theme of proof-related pleasure besides raising other interesting points, such as the product/process nature of proof:

Daria: I agree with those who wrote that s/he never remembers proof, because the pleasure is building it. Actually, for me more than a pleasure the proof was a necessity, because I never remember things, and proving helped me to learn and memorize formulas and statements. Anyway, I think that reaching the end of a proof gives a real satisfaction!

Anita: I'm curious about this issue of satisfaction at the end of proof. At the university, whenever I reached the end of a long and complex proof, I felt satisfied because I had understood more deeply why the theorem held. But, to be honest, I felt even more satisfied in secondary school when I was able, after squeezing my brain, to prove by myself some geometric fact. But maybe, I had a great teacher...

Brigitta: When I think of proof, only geometric proofs come to my mind. And do you know why? Because geometric proofs were the most challenging and demanding for me. [...] I did not have the opportunity to see a lot of proofs. And I was never asked to prove by myself. Therefore, over the years I conceived proof as "something that must be done"... but I never felt a strong motivation to reach the end. I'm a little lazy in this. I prefer to get proofs already done, and just understand them. [...] I remember that when the teacher showed the proof I was very attentive, so as not to lose any step, and after, at home, I felt satisfied, when I was able to reach the end (but the proof was already done!).

We may add that the theme of proof enjoyment raised again during classroom discussion. We suppose that our PSTs felt allowed to talk of satisfaction, enjoyment and other "affective" aspects because these had already been mentioned online, hence their role had been recognized as relevant to mathematics teaching and was worth detailed attention.

DISCUSSION AND CONCLUSION

The outcomes presented in the previous section suggest that the experience was successful as concerns reaching its educational aim, in that it actually managed to make PSTs' beliefs about proofs emerge, and contributed, together with other activities on proof that were developed afterwards, to build a good level of awareness and understanding.

The PSTs produced a large variety of stories. The whole set of them included all aspects that the concise literature review presented above spots as characterizing proof, such as the difference between creating and reproducing proof and the different roles it can play in mathematics understanding. No narration, however, mentioned all aspects. Hence, sharing and discussing individual narrations was very important, because it allowed our PSTs to remark, on practical accounts rather than on a scholarly essay, how complex and articulated is the concept of proof, and how useful (even though demanding) may be its use to support mathematics learning.

The reported narrations show that a few PSTs had a negative relationship with proof, and hence were like to give scarce importance to this topic in their future teaching. This, together with the lack of a complete vision of the many aspects of proof, stresses the need to address proof in mathematics teacher training, and to do it starting from detecting trainees' beliefs, so as to allow the overcoming of negative dispositions and incomplete understanding. Eliciting beliefs at the beginning of the course resulted very useful to the teacher educator as well, in that it allowed her to know her students better and hence to adjust the course development as necessary.

The experience appeared successful also from a methodological point of view. The narrative approach resulted a gentle and effective way to make beliefs emerge, while the use of the written form, for both narrations and comments, stimulated reflection and self-study.

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