

# A non-local formulation for the level-set modeling approach

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When it comes to numerically model hyper-surfaces, the level-set approach is certainly the most efficient. In this approach, hyper-surfaces are defined implicitly using a scalar field. In the framework of structural mechanics, this approach encountered a rising success past decade as they were used for the modeling of cracks, and phase change interfaces.

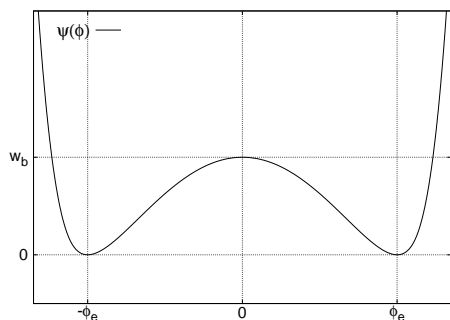
However, the level-set technique is not easy to handle: it implies three different equations –propagation, velocity extension and signed distance reinitialization– at different moments of the computation. Moreover, the propagation equation turns out to be difficult to solve with a Galerkin method without appropriate stabilization terms.

In this work, we propose a non-local level-set formulation that enables to get rid of the aforementioned problems: only one equation is now necessary to handle correctly the level-set problem, and, second, the added non-local term acts as a stabilization term which unlock the implementation in a Galerkin framework.

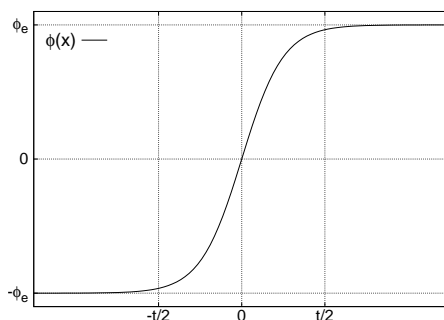
The non-local approach is based on the coupling between the derivation of a stationary field and the construction of a non-local term using the phase field approach for order parameter fields. The generic partial differential equation issued from this approach is:

$$\frac{\partial \phi}{\partial t} + F \|\nabla \phi\| + M \left( \psi'(\phi) - \frac{\kappa}{\nabla \phi \cdot \nabla \phi} \underline{H} \phi \cdot \nabla \phi \right) = 0, \quad (1)$$

where  $\underline{H}$  represents the Hessian differential operator,  $\phi$  is the level-set field,  $F$  the normal velocity of the level-set field,  $M$  and  $\kappa$  are parameters, and  $\psi$  is a prototypical double-well function as depicted on figure 1(a).



(a) Prototypical double-well function  $\psi$ .



(b) Spatial evolution of the one dimension equilibrium solution of the non-local level-set formulation.

Figure 1: Double-well function and characteristic solution for the non-local formulation.

Using classical phase field results, it can be shown that this equation models the propagation of an hyper-surface at the local velocity  $F$ . The term inside parenthesis in equation 1 enforces

a non-null value of the field gradient norm around the iso-0, and, elsewhere, a constant field value close to the double-well function local minima values  $-\phi_e$  and  $\phi_e$ . Those aspects are depicted on the 1-D equilibrium solution plotted in figure 1(b).

Introducing the following definition of interface thickness  $t$ :

$$t = \frac{\phi_e}{\|\nabla\phi\|_{|\phi=0}}, \quad (2)$$

it can be shown that the parameter  $\kappa$  and  $w_b$  controls the interface thickness through the relationship:

$$\frac{\kappa}{w_b} = \frac{1}{2} \left( \frac{t}{\phi_e} \right)^2. \quad (3)$$

Thus the thickness of the interface can be set by properly choosing the ratio  $\kappa/w_b$ . In the numerical application, the thickness is taken to be of the order of the element size of the mesh.

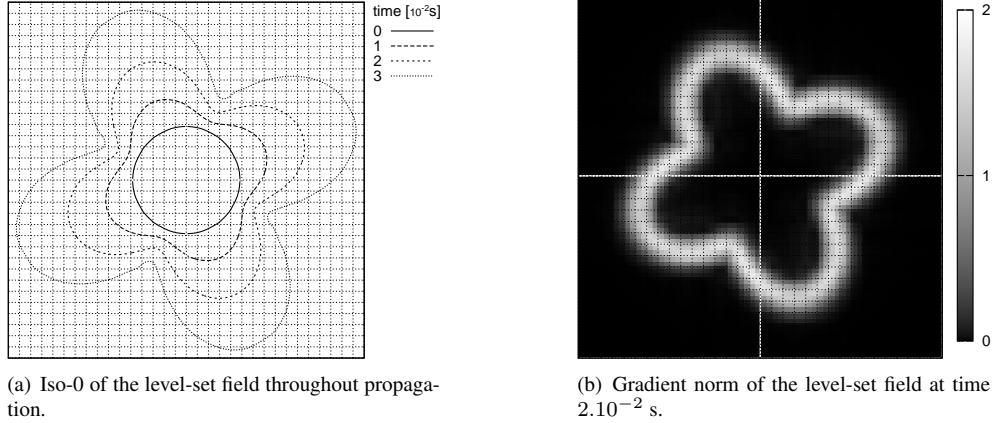


Figure 2: Numerical result for the non-local formulation with the velocity field  $\hat{F}$ ; the mesh is  $\mathcal{C}^1$ -continuous with 32 by 32 regular elements.

The non-local derived formulation has then been implemented in an iso-geometric framework. The time integration is done using a generalized- $\alpha$  scheme, whereas the time stepping is automated through a two stage solving process. Figure 2 displays the evolution of the iso-0 contours at different times for a velocity field given by:

$$\hat{F} = ((r - 3.)^2 + 1) \cdot (2. + \sin(4.\theta)), \quad (4)$$

where  $(r, \theta)$  are the cylindrical coordinates of the current point. This problem belongs to the classical benchmarking tests for the level-set approach.

From figure 2(a), it can be seen that the propagation is very smooth even though the mesh is relatively coarse. Moreover, as can be observed on figure 2(b), the gradient norm around the interface is kept constant. This formulation appears then to be ideal for the numerical modeling of hyper-surfaces.