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# On the ideal gas law

Jacques ARNAUD <sup>\*</sup>, Laurent CHUSSEAU <sup>†</sup>, Fabrice PHILIPPE <sup>‡</sup>

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## Abstract

When a cylinder terminated by a piston, containing  $N$  corpuscles, and with height  $h$  is raised at temperature  $\theta$ , the force that the corpuscles exerts on the piston is, to within constant factors:  $F = N \theta / h$ . This law, called the “ideal gas law”, is valid at any temperature (except at very low temperatures when quantum effects are significant) and for any collection of corpuscles, in a space of arbitrary dimensions. It is usually derived under the assumption that the temperature is proportional to the corpuscle’s kinetic energy. We show that this law rests only upon the democritian concept of corpuscles moving in vacuum, postulating that it is *independent* of the law of motion. This view-point puts aside restrictive assumptions that are sources of confusion. The present paper should accordingly facilitate the understanding of the physical meaning of the ideal gas law. The mathematics is elementary.

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# 1 Introduction

The purpose of this paper is to show that the ideal gas law may be obtained on the sole basis of the democritian model according to which nature consists of corpuscles moving in a vacuum, and a principle of simplicity: namely, the fundamental laws we are talking about are *independent* of the laws of motion. Besides, we are showing that ideal gas internal energies depend only on temperature, from which follows the Carnot result: The maximum efficiency of thermal engines is:  $\eta_C = 1 - \frac{\theta_l}{\theta_h}$ , where  $\theta_l$  is the cold bath temperature and  $\theta_h$  the hot bath temperature. The concept of temperature will be made precise.

**Concepts relating to heat since the antiquity:** Let us recall that Democritus, who lived about 300 years B.C., described nature as a collection of un-seccables corpuscles moving in vacuum. These corpuscles differ from one-another in form, position and weight. In the case of a gas, interaction between corpuscles may be neglected, but they collide with the container's walls. Platon [1] ascribed heat to corpuscular motion: "Heat and fire are generated by impact and friction, but that's motion".

Much later, Francis Bacon (1561-1626) wrote: "The very nature of heat is motion, and nothing else". This view-point is more explicit in Daniel Bernoulli writing (1738): "Gas atoms are moving randomly, and pressure is nothing else but the impact of the atoms on their container walls". Lastly, Carnot wrote more precisely *circa* 1830: "Heat is nothing else but motive power, or rather another form of motion. When motive power is destroyed, heat is generated precisely in proportion of the motive power destroyed. Likewise, when heat is destroyed, motive power is generated".

**Experimental results relating to air:** The first accurate experiments relating to gases are tied up to the invention of the thermometer by Galileo and the barometer by his assistant Torricelli; then, to Pascal experiments on atmospheric pressure.

Let us recall the crucial experiments performed in the seventeenth century concerning the properties of air. Air, consisting mostly of di-azote, may be viewed as an ideal gas. When a tight box contains some amount of air, the volume-pressure product is a constant at room temperature. This law has been enunciated for the first time by Boyle in 1660: "Pressure and expansion are in reciprocal proportions". Boyle employed a J-shaped glass tube, with the sealed small side full of air, and the other full of mercury. The left-side height was a measure of the volume, and the right-side height was a measure of the pressure. Subsequent experiments have shown that this law is applicable at any constant temperature, for example at various liquid boiling temperatures, within some experimental range. We shall call "generalized Boyle law" the expression:  $V(P, \theta) = f(\theta)/P$ , where  $P$  represents pressure,  $V$  volume, and  $f(\cdot)$  some unknown function.  $\theta$  is any temperature measure.

Let us now agree that a temperature  $\theta$  is, by convention, the pressure relating to some amount  $N$  of gas contained within some volume. This may be a one-cubic meter container filled up with air at standard pressure and temperature. As the temperature gets higher, the volume staying constant, the pressure increases. This pressure defines  $\theta$ . That definition of the temperature is applicable to any substance. However, it is only in the case of rarefied helium that it can be identified very accurately with the thermodynamic temperature (that is, the temperature that should be used in dealing with thermal engines). Indeed, rarefied helium may be described by independent corpuscles. The theory presented in this article shows that, on the basis of that model,  $\theta$  is indeed the thermodynamic temperature.

Gay-Lussac has shown in 1802 that, at atmospheric pressure, the volume increment of various gases from freezing to boiling water temperatures is 37.5% [2]. A helium thermometer (see

above for a definition of temperature) enabled experimentalists to establish the proportionality of volume and temperature, again at constant pressure. This measurement was made at various pressures, for example at various altitudes. The generalized Gay-Lussac law may then be enunciated as follows: The two-variable function  $V(P, \theta) = \theta g(P)$ , where  $g(P)$  is some unknown function of pressure. Comparison of the generalized Boyle and Gay-Lussac laws shows that:  $PV = \theta h(N)$ , where  $h(N)$  defines the amount of gas considered.

In 1803, Dalton, on the basis of his studies of chemical compounds and gaseous mixtures suggested that matter consists of atoms of different masses that combine in simple ratios. He discovered the partial pressure law according to which the total pressure exerted by a gas mixture is equal to the sum of the pressure that each one of the gases would exert if it occupied the full volume alone. Finally, in 1811, Avogadro concluded that equal volumes of gases at the same temperature and pressure contain the same number of molecules (or corpuscles). This entails that  $PV/\theta$  is proportional to  $N$ , which is now interpreted as the number of corpuscles. One calls ‘‘Avogadro number’’,  $N_A$ , the number of corpuscles contained in 0.0224 cubic meters of gas in standard conditions. In 1865, Loschmidt established from a measurement of the air viscosity that  $N_A$  is on the order of  $10^{23}$ . Many other methods have been employed since then for that purpose.

The ideal gas law may therefore be written:

$$PV = N\theta \tag{1}$$

where  $V$  denotes the volume,  $P$  the pressure,  $N$  the number of corpuscles, and  $\theta$  the temperature.

**A simple proof:** Let us present a simple proof leading to that law. Let us consider a cylinder of unit area, with a tight piston that can move freely along the axis. This cylinder of height  $h$  contains  $N$  corpuscles and is raised at temperature  $\theta$ .  $\theta$  is defined as above as the force  $F$  that the gas exerts on the piston when  $h = 1$  and  $N$  has some value  $N_o$ . That cylinder may, for example, be filled up by air at standard temperature and pressure. Because the corpuscles are independent,  $N$  corpuscles exert  $N$  times as much pressure as a single corpuscle,  $\theta$  and  $h$  remaining unchanged. We can therefore set in general:  $F = N f(\theta, h)$ , where  $f(., .)$  is the two-variables function we are seeking for. According to the above definition of  $\theta$  ( $F = \theta$  if  $h = 1$  and  $N = N_o$ ), we have:  $F = \theta = N_o f(\theta, 1)$ .

For simplicity, let us select  $N_o = 1$  (a single corpuscle), and consider a cylinder of height 1. According to what precedes the force is:  $F = \theta = f(\theta, 1)$  (setting  $N_o = 1$  in the previous expression). If we now superpose  $h$  cylinders of that kind, the total height becomes  $h$  and the number of corpuscles becomes  $h$  also. The force  $F$  exerted on the upper piston gets transmitted unchanged to each of the cylinders, if we neglect the gas and cylinder weight. This amounts to saying that each cylinder remains in the same conditions as before. But the system presently considered has height  $h$ , contains  $h$  corpuscles, and the force is:  $F = \theta$ . Substituting these values of  $h$ ,  $N$  and  $F$  in the general expression:  $F = N f(\theta, h)$ , we get:  $\theta = h f(\theta, h)$ . Thus  $f(\theta, h) = \theta/h$ , and the general formula becomes:  $F = N\theta/h$ , which is the ideal gas law.

We have implicitly assumed above that one can remove the intermediate pistons without modifying the system equilibrium; this is plausible if the corpuscles do not interact. A proof is given in Appendix.

**Our model:** We are considering a unit-area cylinder, with vertical axis in uniform gravity, resting on the ground ( $z = 0$ ) at some temperature. A tight piston can move in the vertical  $z$  direction. The cylinder height is denoted by  $h$ , and it contains a single corpuscle.

This configuration enables us to concretely define the corpuscle energy. Indeed, the maximum height reached by a corpuscle of weight  $P$  and with energy  $E$  is:  $z_m \equiv \frac{E}{P}$ . The corpuscle

is bouncing elastically on the ground. The average force exerted on the ground, equal to the corpuscle weight, is the product of the impact  $i$  and the number of impacts per unit time. Accordingly, the impact is the product of the corpuscle weight  $P$  and the time period of the motion denoted by  $\tau(z_m)$ , avec  $\tau(0) = 0$ .

Next, we introduce a rigid plane at  $z = h$ , and we consider corpuscle bounces on the  $z = 0$  and  $z = h$  planes, alternately. The time period becomes:  $\tau(z_m) - \tau(z_m - h)$ . We shall call  $F$  the force exerted on the  $z = h$  rigid plane, viewed as a piston, averaged over a time period.

As a consequence of the slight shivering of the cylinder lower end (thermal motion), the corpuscle energy varies in the course of time. We weight the force  $F$  just defined in such a way that the average force  $\langle F \rangle$  be *independent* of the corpuscle equation of motion. In the limit where  $h \ll \theta$  we obtain the ideal gas law in the form:  $\langle F \rangle = \theta/h$ , where  $\theta$  is a quantity having the dimension of an energy. Defining as above the temperature as the average force when  $h = 1$ , we prove that  $\theta$  is the thermodynamic temperature. To conform with international standards, one may define a temperature  $T$ , proportional to  $\theta$ , such that  $T = 273.16$  at the water triple-point temperature.

**Usual kinetic theories:** The ideal gas law has been partly explained by the gas kinetic theory by Waterston [3] in 1843. That theory postulates that the temperature is proportional to the corpuscle's kinetic energy, evaluated from Newton's laws. A recent reference [4] lists the assumptions on which the gas kinetic theory rests. Some of them express the democritian hypothesis and are indeed essential. The usefulness of the others, listed below, however, may be questioned:

1. Gases consist in corpuscles having non-zero mass. (Comment: zero-mass corpuscles are allowed. However, one should avoid a confusion between such corpuscles and light, sometimes described as a collection of zero-weight corpuscles called "photons" [5]).
2. The corpuscles are quickly moving. (Comment: If the corpuscles are slowly moving, it suffices to perform averages over large time durations).
3. They are perfectly spherical and elastic. (Comment: This assumption is unnecessary. The ideal gas law applies equally well to non-spherical molecules such as di-azote molecules and to spherical molecules such as helium atoms.
4. The average kinetic energy depends only on the system temperature.
5. Relativistic effects are negligible.
6. Motion laws are time reversible.
7. The number of molecules is so large that a statistical treatment is appropriate.

Comment: None of the above assumptions, 1 to 7, is needed. It suffices that the (perhaps unique) corpuscle be in thermal contact with the ground.

It thus appears that the ideal gas law is perfectly well comprehensible on the sole basis of the democritian philosophy.

Alternative treatments based on Quantum Theory involve the concept of micro-states, Boltzmann equation, and other assumptions that are not readily understood by students.

## 2 Motion of corpuscles in uniform gravity and the concept of impact

A corpuscle motion is shown in Fig. 1. The function  $z = z(t)$  is intentionally shown asymmetrical because the particular form of this function does not play any role in the establishment of the ideal gas law. The maximum value of  $z$  is denoted  $z_m$  ( $z_m = E$  in the figure). The time period is denoted  $\tau(z_m)$ . Because gravity is uniform, various motions differ from one another only by vertical translations.

We consider firstly a corpuscle of weight  $P$  moving along the vertical axis and bouncing without damping on a balance. The average weight measured by the balance is equal to  $P$ , irrespectively of the corpuscle energy. The corpuscle impact  $i$  is the product of the weight and the temporal motion period as said before.

## 3 Force

Let us now introduce a plane in  $z = h$  which can move vertically and which we call a “piston”, shown in Fig. 1 as a dotted line. The corpuscle is bouncing alternately on the ground at  $z = 0$  and on the piston at  $z = h$ .

The force  $F$  exerted by the corpuscle on the piston is obviously equal to zero if  $h \geq z_m$  since then the corpuscle does not reach the piston. If  $h \leq z_m$  this force is the ratio of  $P \tau(z_m - h)$  and the temporal period of the motion. Looking at the zig-zag shown on Fig. 1, one sees that the latter is equal to  $\tau(z_m) - \tau(z_m - h)$ . It follows that the force  $F$  exerted by the corpuscle on the piston is [6]:

$$\begin{aligned} F &= 0 & z_m &\leq h \\ F &= P \frac{\tau(z_m - h)}{\tau(z_m) - \tau(z_m - h)} & z_m &\geq h. \end{aligned} \quad (2)$$

However, a given temperature  $\theta$  corresponds, not to a fixed energy, but to an energy *distribution*. We will weight the expression of the force given above in such a way that the unknown function  $\tau(z_m)$  cancels out.

## 4 Average force

Let us select as a weighting factor:

$$\begin{aligned} \exp(-P z_m/\theta) \tau(z_m) & & z_m &\leq h \\ \exp(-P z_m/\theta) (\tau(z_m) - \tau(z_m - h)) & & z_m &\geq h, \end{aligned} \quad (3)$$

where  $\theta$  has the dimension of an energy [7].

The average force becomes, using (2) and (3):

$$\begin{aligned} \langle F \rangle &= \frac{P \int_h^\infty dz_m \exp(-P z_m/\theta) \tau(z_m - h)}{\int_0^h dz_m \exp(-P z_m/\theta) \tau(z_m) + \int_h^\infty dz_m \exp(-P z_m/\theta) (\tau(z_m) - \tau(z_m - h))} \\ &= \frac{P}{\exp(P h/\theta) - 1} \approx \frac{\theta}{h}, \quad h \ll \theta. \end{aligned} \quad (4)$$

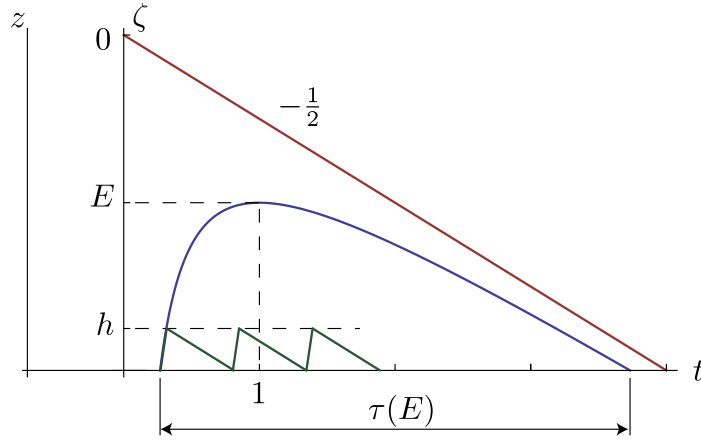


Figure 1: The curve represents the motion  $z = z(t)$  of a corpuscle submitted to uniform gravity, where  $z$  is the vertical axis and the time  $t$  the horizontal axis. We have assumed that the corpuscle weight  $P = 1$ . Its energy is accordingly  $E = z_m$ , where  $z_m$  denotes the maximum height reached above the ground level:  $z = 0$ . The  $\zeta$  coordinate and the slope  $-1/2$  line are aids to the construction of the motion curve for the case of the relativistic law of motion (this curve is intentionally a-symmetrical).  $\tau(E)$  represents the motion period. A reflecting plane is shown at the height  $z = h$  as a dotted line, with the corpuscle bouncing alternately between the two planes. From a simple inspection of the figure, one sees that the period is in that case:  $\tau(E) - \tau(E - h)$ .

In the integrals going from  $h$  to  $\infty$  we have replaced  $\exp(-P z_m/\theta)$  by:  $\exp(-P h/\theta) \exp(-P (z_m - h)/\theta)$  and introduced the variable  $z_m - h$ , so that all the integrals are going from zero to infinity, and cancel out.

The weighting factor that we have introduced is the only one that enables us to obtain a result (last line of (4)) which is independent of the particular form of  $\tau(z_m)$ , and therefore of the law of motion. For a purely mathematical reason, the term:  $\tau(z_m) - \tau(z_m - h)$  must be there to cancel a similar term in the denominator of the expression of  $F$ . On the other hand, the only function of  $u \equiv P z_m/\theta$  that enables us to cancel out the integrals is:  $\exp(-u)$ . Physical explanations are given in [8]: the weighting factor may be written as  $\exp(-E(f)/\theta)$ , where  $f$  denotes the motion action, equal to the  $z(t)$  motion area. In quantum mechanics this action is an integer (times the Planck constant).

In the case of a three-dimensional space, one may ascribe speeds  $v_x, v_y$  to the corpuscle at the top of its trajectory where  $v_z = 0$ . This modifies the function  $\tau(z_m)$ , but is un-consequential as far as the ideal gas law is concerned since that law does not depend on  $\tau(z_m)$ . An arbitrary  $v_x, v_y$  distribution would likewise be un-consequential.

The corpuscles being independent, and the ideal gas law being independent of their weight, it is clear that the law is applicable to an arbitrary collection of corpuscles. In contradistinction, in the case where gravity is significant and  $h$  is not small, the expression (4) *depends* on the corpuscle weight  $P$ . The ideal gas law should then be modified in the case of corpuscles of different weights.

We may now *define* the temperature as the force exerted by the gas on the piston for  $h = 1$ . This shows that the energy  $\theta$  introduced above corresponds to the temperature as we defined it earlier.

## 5 Barometric law

We are considering a corpuscle bouncing off the ground ( $z=0$ ) and reaching the altitude  $z_m = E/P$ , where  $E$  is its energy and  $P$  its weight. The motion period is  $\tau(z_m)$ . The fraction of the time during which the corpuscle is above the altitude  $z_o$  for some value of  $z_m$  is  $\tau(z_m - z_o)/\tau(z_m)$ . According to (3) the weighting factor to employ is  $\exp(-Pz_m/\theta)\tau(z_m)$  ( $h$  is presently infinite). It follows that the probability that the corpuscle altitude exceed  $z_o$  at some temperature  $\theta$  is, after integration of  $\tau(z_m - z_o)/\tau(z_m)$  over  $z_m$  from  $z_o$  to infinity:

$$p(z_o) = \exp(-Pz_o/\theta). \quad (5)$$

The density (defined as the number of corpuscles located between  $z_o$  and  $z_o + dz_o$  divided by  $dz_o$ ) is therefore:  $\rho(z_o) = -dp(z_o)/dz_o = (P/\theta) \exp(-Pz_o/\theta)$ . For  $N$  corpuscles this quantity must be multiplied by  $N$ . This is the barometric law.

The potential energy of a corpuscle of weight  $P$  located at the altitude  $z_o$  is, by definition, equal to  $Pz_o$ . The gas potential energy, according to the above expression of the density, is therefore  $\mathcal{P} = P \int_0^\infty dz_o z_o \rho(z_o) = P \int_0^\infty dz_o p(z_o) = \theta$ .

**Case of a plane at  $z = h$ :** Let us now suppose that there is a plane at  $z = h$  as in §4. The density  $\rho(z_o)$  is given by the previous expression, but it should now be normalized so that its integral from  $z_o = 0$  to  $z_o = h$  be unity. We obtain:  $\rho_h(z_o) = (P/\theta) \exp(-Pz_o/\theta)/(1 - \exp(-Ph/\theta))$ . The potential energy is therefore, setting  $a \equiv Ph/\theta$ ,

$$\mathcal{P}_h = P \int_0^h dz_o z_o \rho_h(z_o) = \theta \frac{\int_0^a dx x \exp(-x)}{1 - \exp(-a)} = \theta \left( 1 - \frac{a}{\exp(a) - 1} \right) \quad (6)$$

Accordingly, setting  $P = 1$  for brevity, the potential energy is:

$$\mathcal{P}_h = \left( \theta - \frac{h}{\exp(h/\theta) - 1} \right). \quad (7)$$

Let us observe that  $\mathcal{P}_h$  tends to 0 when  $h$  tends to 0, and to  $\theta$  monotonically if  $h \rightarrow \infty$  as said above.

## 6 Internal energy

The internal energy  $U$  (for a single corpuscle) is simply the average value of  $E$ , considering only the corpuscle motion along the vertical  $z$  axis. Setting  $P = 1$  in (3) for simplicity ( $E = z_m$ ), we obtain:

$$\begin{aligned} U = \langle z_m \rangle &= \frac{\int_0^h dz_m z_m \exp(-z_m/\theta)\tau(z_m) + \int_h^\infty dz_m z_m \exp(-z_m/\theta)(\tau(z_m) - \tau(z_m - h))}{\int_0^h dz_m \exp(-z_m/\theta)\tau(z_m) + \int_h^\infty dz_m \exp(-z_m/\theta)(\tau(z_m) - \tau(z_m - h))} \\ &= \frac{\int_0^\infty dz_m z_m \exp(-z_m/\theta)\tau(z_m)}{\int_0^\infty dz_m \exp(-z_m/\theta)\tau(z_m)} - \frac{h}{\exp(h/\theta) - 1} \equiv \mathcal{K} + \mathcal{P}_h \end{aligned} \quad (8)$$

where the kinetic energy:

$$\mathcal{K} = \frac{\int_0^\infty dz_m z_m \exp(-z_m/\theta)\tau(z_m)}{\int_0^\infty dz_m \exp(-z_m/\theta)\tau(z_m)} - \theta \quad (9)$$



depend only on temperature, and the potential energy  $\mathcal{P}_h$  is given in (7).

We have therefore obtained, without making any assumptions concerning the corpuscle motion, the following result: At some given temperature  $\theta$ , the internal energy of a gas is bounded from above. We will see that in a simple model of thermal engine the work performed may be infinite as a result, the gas internal energy may be negligible in comparison to the work performed. This enables us to obtain the Carnot expression for the maximum efficiency without knowing the law of corpuscle motion.

**Thermodynamic temperature** We may now prove in a very straightforward manner that the maximum efficiency of a thermal cycle is the Carnot efficiency:  $\eta_C = 1 - \theta_l/\theta_h$ , where  $\theta_l$  is the cold-bath temperature and  $\theta_h$  is the hot bath temperature. It suffices indeed to vary  $h$  from  $h_1$  to  $h_2$  when the gas is in contact with the cold bath, to transfer it to the hot bath without any change in volume (with therefore no work being performed or received), then changing  $h$  from  $h_2$  back to  $h_1$ , followed by a return to the cold bath without any volume change. We will see that the Carnot efficiency obtains in the limit where  $h_1/h_2 \rightarrow \infty$  [8].

Indeed, according to the ideal gas law previously established,  $F(h) = \theta/h$  at temperature  $\theta$ . If  $h$  varies from  $h_1$  à  $h_2$ , the work delivered is:  $W = \theta \ln(h_2/h_1)$ , after integration. In particular, the work delivered when the gas is in contact with the cold bath is:  $W_l = \theta_l \ln(h_2/h_1) < 0$  and the work delivered when the gas is in contact with the hot bath is:  $W_h = \theta_h \ln(h_1/h_2) > 0$ . In the limit considered the heat  $Q_h$  delivered by the hot bath is  $W_h$ , because the internal energy change is bounded from above, as we have shown, while the delivered work tends to infinity. We have therefore:  $\eta = W/Q_h = (W_h + W_l)/Q_h \approx 1 + W_l/W_h = 1 - \theta_l/\theta_h \equiv \eta_C$ . We may therefore conclude that:  $\theta$  is the so-called “thermodynamic temperature”.

This temperature of course is defined only to within a multiplicative factor. This factor is determined by agreeing that the water triple-point temperature is 273.16 kelvins. The rescaled temperature is denoted  $T$ . One generally considers an amount of gas called a “mole” occupying a volume of 0.0224 cubic meters at standard pressure and temperature (approximately one atmosphere or 100 000 pascals, and 300 kelvins). We then write: *pressure*  $\times$  *volume* =  $RT$ , with the ideal gas constant:  $R \approx 8.314$  joules per kelvin.

To obtain explicitly the internal-energy function  $U(\theta)$  one must specify the period  $\tau(z_m)$ . In the case of the relativistic dynamics, setting  $m = 1$ ,  $c = 1$ , we have [8]:  $\tau(z_m) = 2\sqrt{z_m^2 + 2z_m}$ . The function  $U(\theta)$  may then be expressed in terms of Bessel’s functions. Within the non-relativistic approximation:  $U = T/2$  and within the hyper-relativistic approximation:  $U = T - 1$ .

## 7 Conclusion

Let us briefly recall the concepts introduced in the present paper. One can think that Democritus, after having introduced the corpuscular concept, observed the elastic bounces of a unit weight on a balance, and defined the “impact” as the motion period. Not knowing the nature of the motion (parabolic?, hyperbolic?) nor the period, he may have thought of introducing a weighting factor such that the average force  $\langle F \rangle$  *does not depend* on the law of motion. This, as we have seen, may be done. In this weighing factor, for dimensional reasons, one must introduce a quantity denoted  $\theta$  having the dimension of energy. Considering a very simple thermal engine operating between two baths at different temperatures (subscripts  $h$  and  $l$ ), one finds on the basis of the principles just stated that the maximum efficiency is:  $1 - \theta_l/\theta_h$ . This allows us to identify  $\theta$  to the thermodynamic temperature. As side results, one obtains the decrease of the air density as a function of altitude (for uniform gravity and temperature), as well as the gaussian molecular speed distribution.

Guillaume d’Occam (or Ockham: 1285-1347) set as a matter of principle that one should not employ more concepts than those that are strictly necessary to explain the observed phenomenas [9]. Accordingly, it seems important to elucidate the assumptions on which rest, in particular, the ideal gas law, which plays an essential role in theoretical physics and many applications. Our thesis is that this law may be obtain on the sole basis of the Democritus model of corpuscles and vacuum. It is indeed unnecessary to specify the laws of motion. One can also show that the ideal gas internal energy depends only on temperature. To evaluate this energy it is, however, necessary to know the law of motion. From a pedagogical standpoint, and in application of the Occam principle, it appears important to avoid introducing the students to principles which, without being erroneous, are unnecessary.

## Appendix: Barometric equation with a plate

We consider the case where the corpuscle is bouncing, not only on the ground  $z = 0$  but also on a plate at  $z = h$ , and we are seeking for the density:  $\rho(z_o)$ , where  $0 < z_o < h$ . In the main text, we supposed that this density is the same as if the plate were absent. Even though this assumption is highly plausible, it is useful to verify it to prove the formalism consistency. As in the main text we denote by  $z_m$  the maximum height that the corpuscle would attain if the plate were not there, as a consequence of its energy. We set  $P = 1, \theta = 1$  for brevity.

To evaluate the probability  $p$  that the corpuscle be *below*  $z_o$ , one must distinguish three cases:

$$\begin{aligned} 0 < z_m < z_o : & \quad p = 1 \\ z_o < z_m < h : & \quad p = \frac{\tau(z_m) - \tau(z_m - z_o)}{\tau(z_m)} \\ z_m > h : & \quad p = \frac{\tau(z_m) - \tau(z_m - z_o)}{\tau(z_m) - \tau(z_m - h)} \end{aligned} \quad (10)$$

On the other hand, the weighting factors relating to  $z_m$  given in (3) are:

$$\begin{aligned} z_m < h : & \quad \exp(-z_m)\tau(z_m) \\ z_m > h : & \quad \exp(-z_m)(\tau(z_m) - \tau(z_m - h)) \end{aligned} \quad (11)$$

The probability that the corpuscle be below  $z_o$  is therefore:

$$\int_0^{z_o} dz_m \exp(-z_m)\tau(z_m) + \int_{z_o}^h dz_m \exp(-z_m)(\tau(z_m) - \tau(z_m - z_o)) \quad (12)$$

$$+ \int_h^\infty dz_m \exp(-z_m)(\tau(z_m) - \tau(z_m - z_o)) = (1 - \exp(-z_o)) \int_0^\infty dz_m \exp(-z_m)\tau(z_m). \quad (13)$$

After normalization, we see that the probability that the corpuscle be below  $z_o$  is:  $1 - \exp(-z_o)$ , which is the expected result if the density is the same as in the absence of the plate at  $z = h$ , namely:  $\exp(-z_o)$ . QED.

## References

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- [6] For small  $h$ -values this expression becomes:  $Fh = P\tau(z_m)/\tau'(z_m)$ , where  $\tau'(z_m) \equiv d\tau(z_m)/dz_m$ . The usual method employed to obtain the force  $F$  consists in showing in the first place that:  $Fh = pv$ . Indeed, the impact of a corpuscle on the piston is equal to the product of  $2p$  and the number of impacts per unit time  $v/2h$ ,  $p$  being the momentum and  $v$  the corpuscle speed. Setting  $P = 1$  and  $\xi \equiv z_m - z$ , let us set the Hamiltonian:  $H(p, \xi) = E(p) - \xi$ , where  $E(p)$  is any function of  $p$ , supposed even. The speed  $v = dE/dp$ . According to the Hamilton equations, we have:  $p = t$  and:  $d\xi/dt = dE(p)/dp$ . Thus:  $E(t) = \xi(t)$ , omitting constants. It follows that  $pv = t d\xi/dt$  is equal to  $\tau(\xi)/\tau'(\xi) = t(\xi)/t'(\xi)$ , where  $t'(\xi) \equiv dt/d\xi$ . Accordingly, a classical treatment leads to the main text result.
- In particular, in special relativity, we have to within constant factors:  $(1 + E)^2 - p^2 = 1$ , and we obtain:  $Fh = (E^2 + 2E)/(E + 1) = \tau/\tau'$ , where  $\tau = 2\sqrt{E^2 + 2E}$  and  $\tau' = d\tau/dE$ . Within the galileo approximation  $E(p) = p^2/2$ , and for zero-mass corpuscles,  $E = |p|$ .
- [7] When  $z_m \gg h$ , and within the non-relativistic equation ( $\tau = \sqrt{E}$  with  $E \equiv z_m$ , constants being omitted), the energy distribution is:  $f_E(E)dE = \frac{\exp(-E)}{\sqrt{E}}dE$ . The speed distribution for  $v_z > 0$ , obtained from:  $f_E(E)dE = f_{v_z}(v_z)dv_z$  and  $E = v_z^2$ , is therefore a gaussian:  $f_{v_z}(v_z) = \exp(-v_z^2)$ , which is the Maxwell’s result. In a three-dimensional space, the  $v_x, v_y, v_z$  distributions are independent. Hence the speed distribution:  $f_v(v) = v^2 \exp(-v^2)$  since  $v \equiv \sqrt{v_x^2 + v_y^2 + v_z^2}$ .
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- [9] Some authors consider that the Occam philosophy predates the advent of modern science by insisting on facts and the kind of reasoning employed rather than on speculations about essences.