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Guess my X and other techno-pedagogical patterns

Toward a language of patterns for teaching and learning mathematics

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Abstract

Most people see learning mathematics as a demanding, even threatening, endeavour. Consequently, creating technology-enhanced environments and activities for learning mathematics is a challenging domain. It requires a synergism of several dimensions of design knowledge: usability, software design, pedagogical design and subject matter. This paper presents a set of patterns derived from a study on designing collaborative learning activities in mathematics for children aged 10-14, and a set of tools to support them.

Introduction

This paper considers the question of designing educational activities for learning mathematics and the technologically-enhanced environments to support them. It is grounded in a techno-pedagogical approach with prioritises the function of educational technologies over their structure. This function is determined as much by their mode of use as it is by their static features; hence I see a discussion of tools as meaningful only with respect to the activities in which they are employed. My perspective is in line with the “instrumental genesis” approach to the design and analysis of educational technology (Ruthven 2008; Trouche, 2005; Folcher, 2003; Rabardel and Bourmaud, 2003; Trouche, 2003). As Rabardel and Bourmaud phrase it, “design continues in usage” (Rabardel and Bourmaud, 2003, p 666). In view of this position, the patterns presented in this paper do not

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make a distinction between tool and activity. These patterns can be implemented by developing new software components, by configuring and orchestrating existing components or even – in some cases – with analogue technologies such as paper and pencil.

Target

This paper addresses designers of educational activities and the technology to support such activities. It takes an inclusive approach, following Herbert Simon's position that – "Everyone designs who devises courses of action aimed at changing existing situations into desired ones" (Simon, 1969, p 129). The focus of this paper is on designing opportunities for learning mathematics. Thus, the designers in mind are educational practitioners who are considering ways of using technology to enhance their teaching, as well as technology designers who wish to support innovative educational practices,. A third potential audience are researchers experimenting with new practices or technologies. Likewise, the notion of technological design extends beyond the production of new tools from scratch, and includes the appropriation and adaptation of existing tools to enable particular practices.

Therefore,

Educators could use these patterns in designing educational activities, using off-the self tools.

Educators and learners could use these patterns as a language for participatory learning design, where the learners are included as equal partners in the educational process.

Educators and developers could use these patterns as a language for discussing the design and development of new educational technologies.

Developers could use these patterns as an entry point into contemporary learning theories, by embedding these in the familiar form of a pattern language.

Historical background

The patterns below are derived from The *WebLabs* Project (www.weblabs.eu.com), which has been described in detail elsewhere (Mor et al, 2006; Simpson et al, 2006). The project aimed at exploring

new ways of constructing and expressing mathematical and scientific knowledge in communities of young learners. The WebLabs project involved several hundred students, aged ten to fourteen, across sixteen schools and clubs in six European countries. Our approach brought together two traditions: *constructionist learning* as described by Papert & Harel (1991) and collaborative *knowledge-building* in the spirit of Scardamalia & Bereiter (1994). The former was largely supported by the programming language ToonTalk (Kahn, 1996; 2004; Morgado and Kahn, in press) (www.toontalk.com), whereas for the latter we have designed and built a web-based collaboration system called *WebReports* (Mor, Tholander & Holmberg, 2006). The central design intention of our approach is that students should simultaneously *build* and *share* models of their emerging mathematical knowledge.

ToonTalk is an animated language and a programming environment designed to be accessible by children of a wide range of ages, without compromising computational and expressive power. Following a video game metaphor, the programmer is represented by an avatar that acts in a virtual world. Through this avatar the programmer can operate on objects in this world, or can train a robot to do so. Training a robot is the ToonTalk equivalent of programming. The programmer leads the robot through a sequence of actions, and the robot will then repeat these actions whenever presented with the right conditions. ToonTalk programmes are animated: the robot displays its actions as it executes them.

The *WebReports* system was set up to serve both as a personal memory aid and as a communication tool. *web report* is a document that is composed and displayed online, through which a learner can share experiences, questions and ideas derived from her activities. The uniqueness of our system is that it allows the author to share her ideas not just as text, but also graphics and animated ToonTalk models. This last point is crucial: rather than simply discussing what each other may be thinking, students can share what they have built, and rebuild each others' attempts to model any given task or object.

A main concern was the careful design of a set of activities, aiming to foster learning of specific mathematical topics, such as sequences, infinity and randomness. The choice and design of technologies was subordinate to this cause.

The Patterns

This paper assumes a pedagogical approach which combines construction, communication and collaboration. The patterns presented here are focused on this perspective. They were derived from a three-year European educational research project. These patterns are viewed as the first steps towards a language. Figure 1 offers a “fantasy map” of this language – a possible draft of the form it may eventually take. This map is by no means comprehensive. To cover the whole field of technology-enhanced mathematical education would be a project of immense scale. The patterns actually discussed in this paper are highlighted in this map. The lines in the map show the links between patterns to other patterns they extend or use.

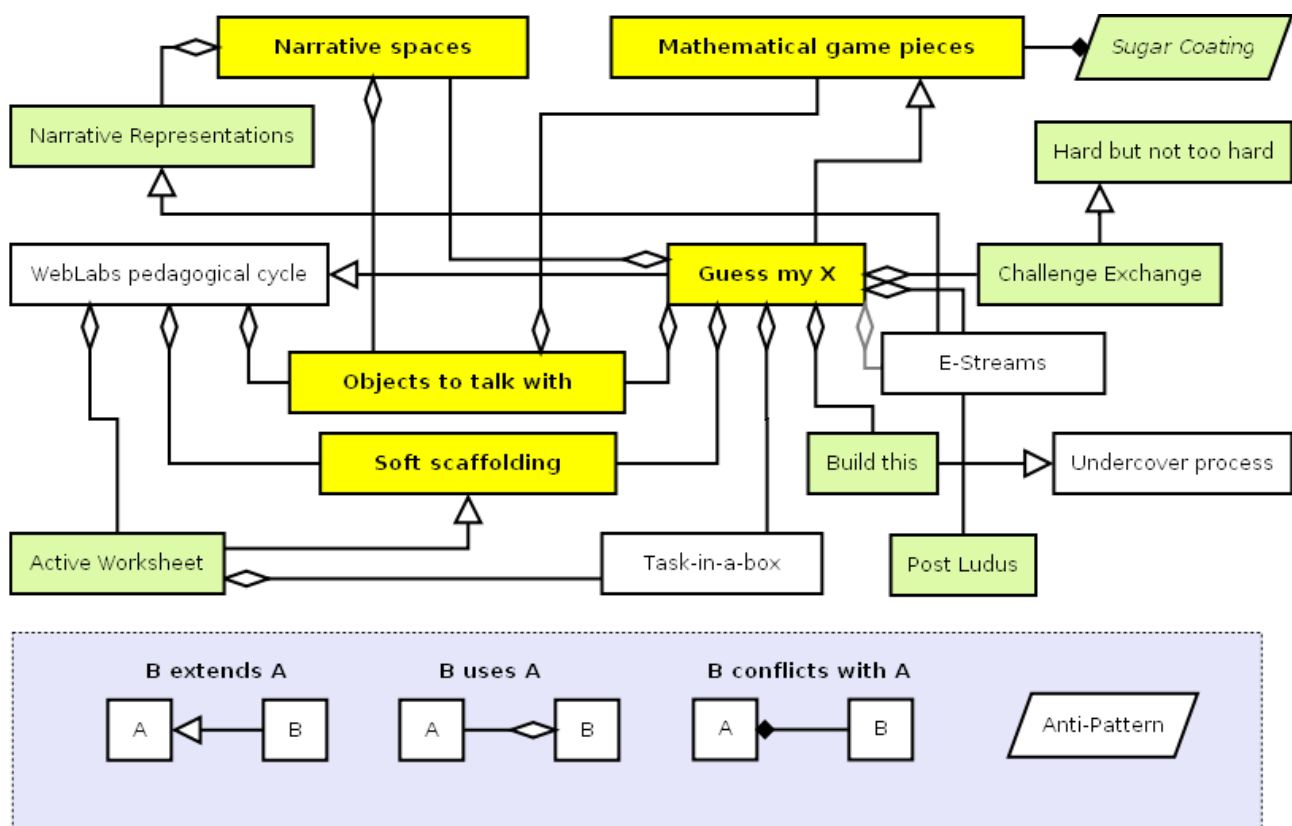


Figure 1: A “fantasy map” of the emerging pattern language. Dark highlighted patterns are presented in this paper. Light highlighted ones are mentioned. Italics denote anti-patterns.

The most elaborate of these is GUESS MY X (GmX), which was elicited from the guess my robot activity (Mor et al, 2006) and other activities cast in its mould by my colleagues Michelle Cerulli and Gordon Simpson. Other patterns were derived from this one as higher-order abstractions or as

components.

The five patterns described in detail are followed by “thumbnail” descriptions of other patterns mentioned in the text.

The structure of the design patterns aims to balance precision and simplicity. Visualising these patterns is often challenging, since they relate to intangible aspects of learning and teaching processes. I have tried to identify imagery which invokes a useful metaphor.

The problem descriptions utilize an innovative form of visual force-maps. These maps are based on Alexander’s force diagrams (1964), extended with iconic illustrations. While a formal methodological discussion of these force maps is called for, it is out of the scope of this paper. It is hoped that these maps will prove of intuitive value and assist readers in understanding the problems under inspection.

1. Mathematical Game Pieces

Mathematical content is often injected artificially into games or other activities, as SUGAR-COATING. This has a dual effect of ruining the game and alienating the mathematics. By contrast, for many mathematicians, mathematics **is** the game.



The problem

How do you design (or choose) a game to convey mathematical ideas in an effective and motivating manner? How do you judge if a proposed game is an adequate tool for teaching particular mathematical concepts?

Forces

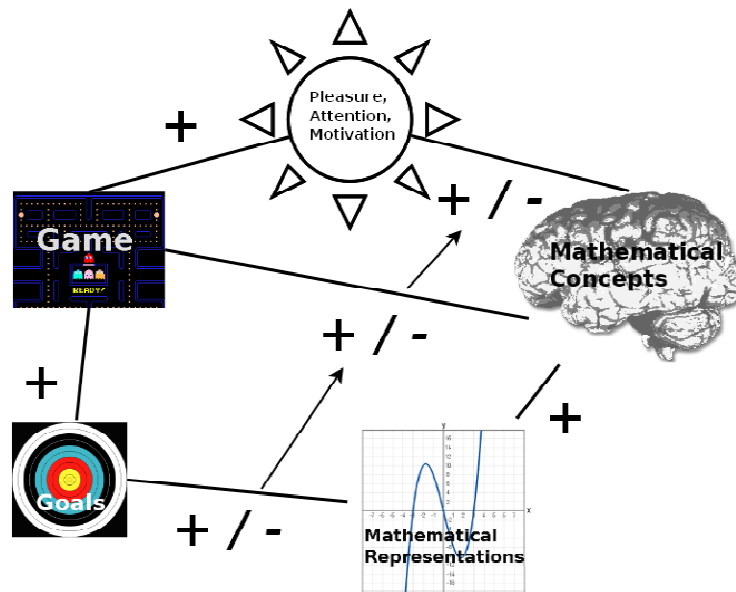


Figure 2: Force map for Mathematical Game Pieces pattern

- A game used in education has to provide a good game experience, or else it is “just another boring task”.
- Learners need to engage with the mathematical content that the game aims to promote.
- The chosen representations need to be consistent both with the game metaphors and with the epistemic nature of the content domain.

Context

An educator wishing to use games as part of her teaching, either evaluating existing “educational” games, appropriating “entertainment” games, or designing and developing her own games.

A developer wishing to develop games for the educational market.

Solution

- Identify an element of the mathematical content you wish to address in this game.

- Find a visual, animated or tangible representation of this element which is consistent with the game metaphors.
- Design your game so that these objects have clear PURPOSE AND UTILITY as game elements in the gameplay structure.

The objects representing the mathematical content should have a meaningful intrinsic role in the game. Manipulating these objects can be part of the game rules or goals, or understanding them could be a necessary condition for success.

If the game includes or is followed by communication between participants, then the mathematical game pieces should become OBJECTS TO TALK WITH.

Examples

In Chancemaker (Pratt, 1988) users manipulate the odds of various chance devices, such as dice and roulette wheels. The game pieces are representations of probability (Figure 3).

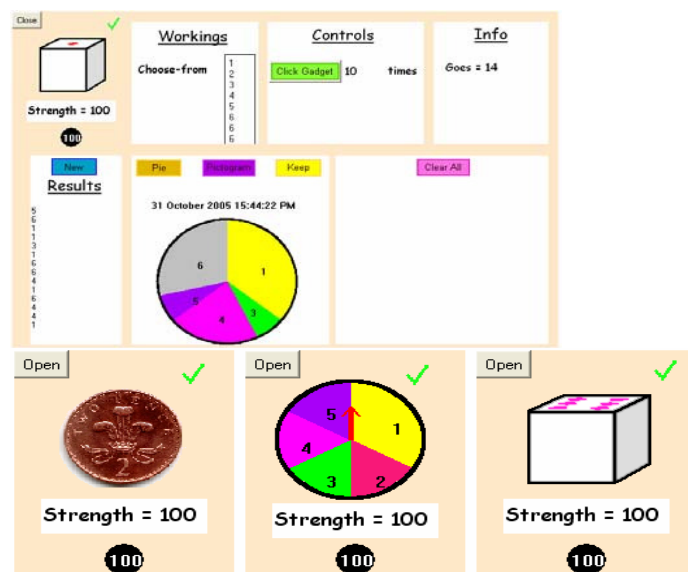


Figure 3: Chancemaker gadgets are probability game pieces

In Programming Building Blocks (<http://www.thinklets.nl>) the object of the game is to reconstruct a 3D shape from its 2D projections. The mathematical content is the focus of the game, and the objects used in the game are straightforward representations of that content (Figure 4).

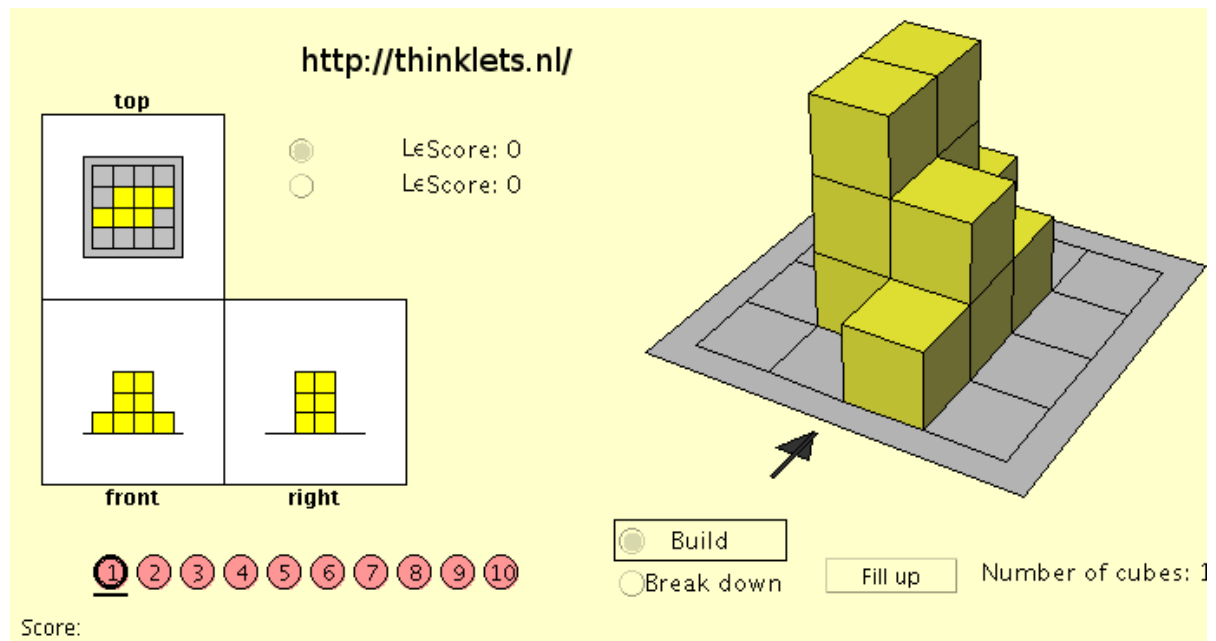


Figure 4: Building blocks constructions as 3D geometry game pieces

Related Patterns

Used by: OBJECTS TO TALK WITH.

Contradicts: *SUGAR-COATING* (anti-pattern)

Elaborated by: GUESS MY X

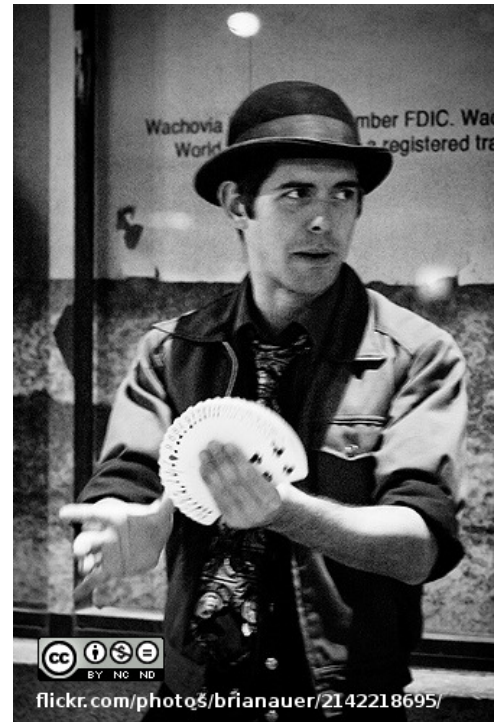
Notes

This is a very high level pattern which needs to be elaborated per specific game and content classes. For example, in a quest type game it might spawn different sub-patterns than in puzzle type games.

Likewise, factual and procedural knowledge might lead to different strategies than meta-cognitive skills. Nevertheless, it is useful as a guideline for evaluating design proposals. Its absence marks a game as a weak tool for learning.

2. Guess My X

Use a CHALLENGE EXCHANGE game of BUILD THIS puzzles to combine construction and conversation, promoting an understanding of process-object relationships and lead to meta-cognitive skills such as equivalence classes, proof and argumentation.



The problem

A teacher wants to design a game for learning concepts, methods and meta-cognitive skills in a particular mathematical domain. This game should use a combination of available technologies.

Many complex concepts require an understanding of the relationship between the structure of an object and the process which created it. Novices may master one or the other but find it challenging to associate the two. Constructing objects helps build intuitions, and discussing them espouses abstracting from intuitions and establishing socio-mathematical norms (Yackel & Cobb, 1995).

Learning mathematics is fundamentally learning to be a mathematician. It requires the learner to internalize a range of mathematical skills as regular habits: computation, analysis, conjecturing and hypothesis testing, argumentation and proof. For this to happen, the learner needs to be deeply engaged in meaningful mathematical inquiry, problem solving and discussion. Games provide a natural setting for the kind of “flow” needed, but how do we ensure that the focus of this flow is

mathematical activity and discourse?

Forces

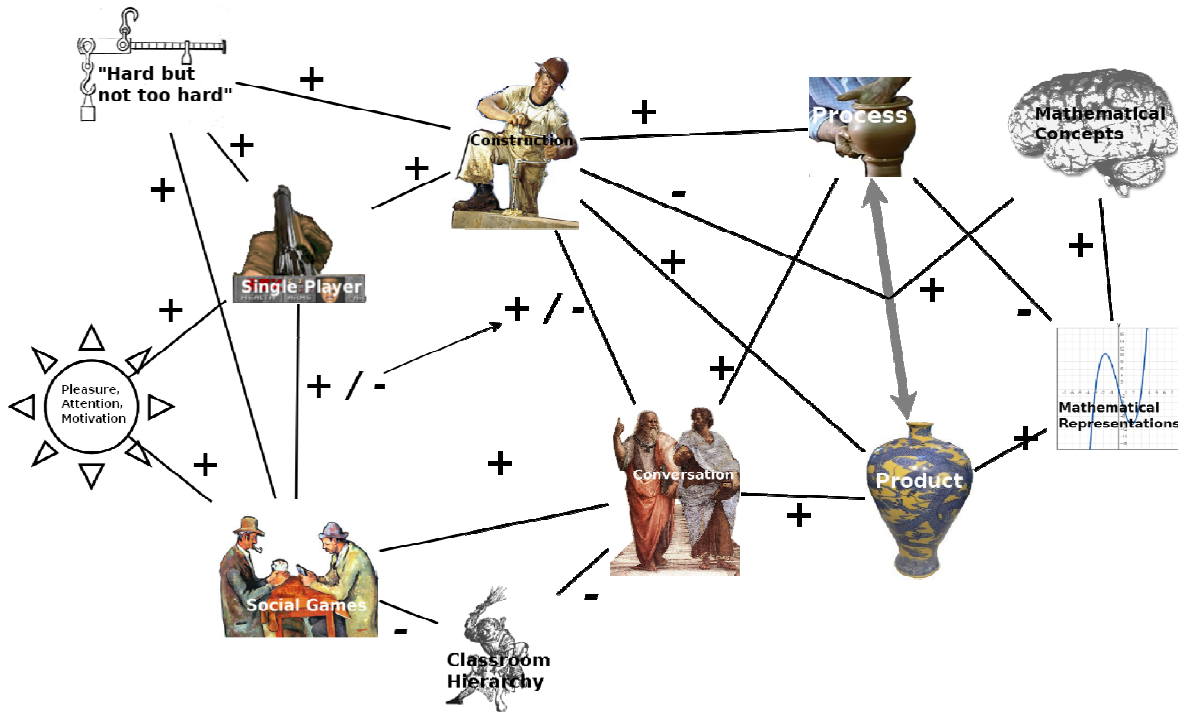


Figure 5: Force map for the Guess my X design pattern

- ◆ Many mathematical domains require learners to understand the relationship between mathematical objects and the process which generated them. This is a challenge for many learners.
- ◆ The teacher needs a non-invasive monitoring mechanism to assess students' performance.
- ◆ The communicational approach (Sfard, 2006) sees learning mathematics as acquiring a set of language rules and meta-rules. In order to achieve this, learners need to engage in meaningful and sustained discussion of mathematical topics.
- ◆ The classroom hierarchies and the perception of a teacher as more knowledgeable causes learners to be cautious and restrained in their mathematical discourse.

Context

Primarily, a classroom supported by a technological environment which provides a shared and protected web space (e.g. wiki or forum), common tools (programming environment, spreadsheets, etc.) and sufficient access time for all students.

The game relies on sustained interaction over a period of several sessions. It can be used as a short introduction to a topic, but the greater meta-cognitive benefits may be lost if not enough time is allowed for conversations to evolve.

Also works for several groups collaborating over a web-based medium.

Solution

Guess my X is a pattern of game structure, which can be adapted to a wide range of mathematical topics. It extends CHALLENGE EXCHANGE to encourage discussion and collaborative learning, and to break down classroom hierarchies. It uses BUILD THIS to engender reflection and discussion about the relationships between mathematical objects and the processes that produce them. The core of the pattern is described in Figure 6.

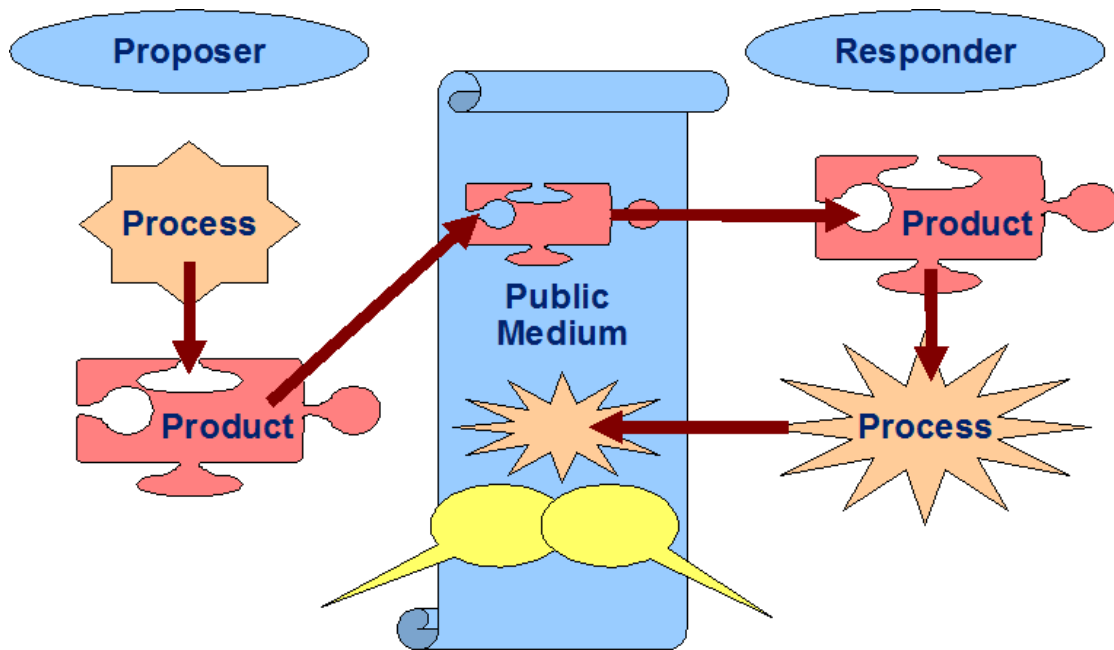


Figure 6: Schematic diagram of GmX

GmX involves players in two roles, *proposers* and *responders*, and a facilitator. An implementation of the game would specify a domain of mathematics and rules for constructing processes in that domain. A proposer sets a challenge, in the form of a mathematical object which she constructed. The explicit rules of the game define the nature of the process by which this object can be created, but not its details. A proposer would construct such a process, and capture its product. The proposer then saves the process model in a private space and publishes the product as a challenge. Responders then need to “reverse engineer” the process from the product. If they succeed, they publish their version as a response to the challenge. The proposer then needs to confirm the responder’s solution of provide evidence for the contrary.

It is important to keep the mathematical content explicit from the start. The game is not a SUGAR-COATING to disguise the mathematics: it is a game with MATHEMATICAL GAME-PIECES. The rules of the game are intentionally left vague, in the sense that the evaluation function used to determine the responders' success is not fully specified. This requires students to negotiate what constitutes a correct answer, and in doing so collaboratively refine the underlying mathematical concepts. These negotiations can lead to discussions of issues such as proof, equivalence and formal descriptions. The quality and extent of these discussions depends on the scaffolding and provocations provided by the teacher, but a necessary condition for them to emerge is that the medium of the game provide

a NARRATIVE SPACE, Where the MATHEMATICAL GAME-PIECES of the game can become OBJECTS TO TALK WITH.

Set-up phase

Before the game begins, the teacher needs to verify that the players have a minimal competence in analysing and constructing the mathematical objects to be used.

1. Teacher introduces the rules of the game and the game environment.
2. Teacher simulates one or two game rounds during a whole class discussion.
3. Students may need to initialize their game space on the chosen collaborative medium.

If the game uses separate media for construction and communication, consider using a TASK IN A BOX to streamline the transition between them.

Game session

The game sessions for the proposer and the responder are different, although the same player can play both parts in parallel.

1. Proposer initiates the game, by constructing an object according to the game rules and publishing it. She then waits for responses.
2. Responder chooses an attractive challenge, and attempts to resolve it. If she believes she has succeeded, she responds to the challenge by posting the object she constructed and the method she used.
3. Proposer reviews the response, and confirms or rejects it. If the response is rejected, an argument needs to be provided.

Play session

Each play session involves a single iteration of the game. Students tend to prolong their interaction in the game, by providing secondary challenges, etc. Since the iterations are asynchronous, there

may be a time gap of several days between turns.

The communication medium chosen for the game should afford NARRATIVE SPACES for the proposer and the responder. Although the rules of the game are limited to the exchange of mathematical objects, the ability to augment these with personal narratives is crucial for personal reflection as well as for collaborative knowledge building.

Set-down phase

The POST LUDUS discussion should highlight the issue of the evaluation function and its resolution.

Examples

In the Guess my Robot game (Mor et al, 2006) students exchanged challenges in the domain of number sequences. Proposers would program a ToonTalk robot to produce a sequence, keep the robot to themselves, and publish the first few terms of the sequence (Figure 10). Responders would then solve the challenge by recreating a robot to produce the same sequence and posting it as a comment on the challenge page (Figure 11). Often the response robot was different from the original, which led learners to discussions about issues such as proof and equivalence. The same structure was then used in the Guess my graph game in the domain of function graphs (Simpson, Hoyles and Noss, 2006) and the Guess my garden game in the domain of randomness and probability (Cerulli, Chiocciariello and Lemut, 2007).

Guess my robot



Created by [B Santos](#) - Created: 03-03-04 - Modified: 17-03-04

Bárbara's Guess My Robot Page

My Sequence:



Hints:

Solution:

After someone posts a Robot that generates your sequence, mention him / her and post your original robot here.

Figure 10: Example guess my robot challenge



Figure 11: Example guess my robot response

Related Patterns

Elaborates: MATHEMATICAL GAME-PIECES;

Uses: CHALLENGE EXCHANGE; UNDERCOVER PROCESS; BUILD THIS; TASK IN A BOX; OBJECTS TO TALK WITH; NARRATIVE SPACES;

Leads to: POST LUDUS;

Notes

Guess my X assumes a degree of social and technical sophistication which suggests it would be suitable for young teens and above. It can, however, be adapted for younger children.

The game requires flexibility in time to allow learning dynamics to emerge. It can be interleaved with other activities.

It is suitable for concrete, well-bounded content domains, such as computation, modelling or analysis. It uses these as a stratum for developing meta-cognitive skills of problem solving, analysis, argumentation and general mathematical discourse.

The fact that the game dynamics are driven by participants makes the educators' role subtle and critical. The educator needs to facilitate fruitful interactions, and monitor these to divert them to

high standards of mathematical discourse.

The game can be played by individuals, pairs or teams. The number and spread of participants can also vary. However, it is crucial to allow enough time for a culture to emerge. This can be achieved by interleaving the game with other activities, e.g. playing it for the last 10 minutes of each session over several weeks.

Both proposers and responders tend to converge to challenges which are HARD BUT NOT TOO HARD. When the environment encourages social cohesion, players seem to reciprocate 'good' challenges.

3. Soft scaffolding

Technology should be designed to scaffold learners' progress, but an interface that is too rigid impedes individual expression, exploration and innovation.



www.flickr.com/photos/docman/6107473

The problem

Scaffolding is a powerful tool for accelerating learning. It is a fundamental principle in many interactive learning environments, such as OISE's Knowledge Forum, and is a guiding principle in Learner-centred approaches (c.f. Quintana et al, 2004). However, scaffolds can become straitjackets when they are too imperative.

How do you provide direction and support while maintaining the learners' freedom, autonomy and sense of self, as well as the teachers' flexibility to adapt?

Forces

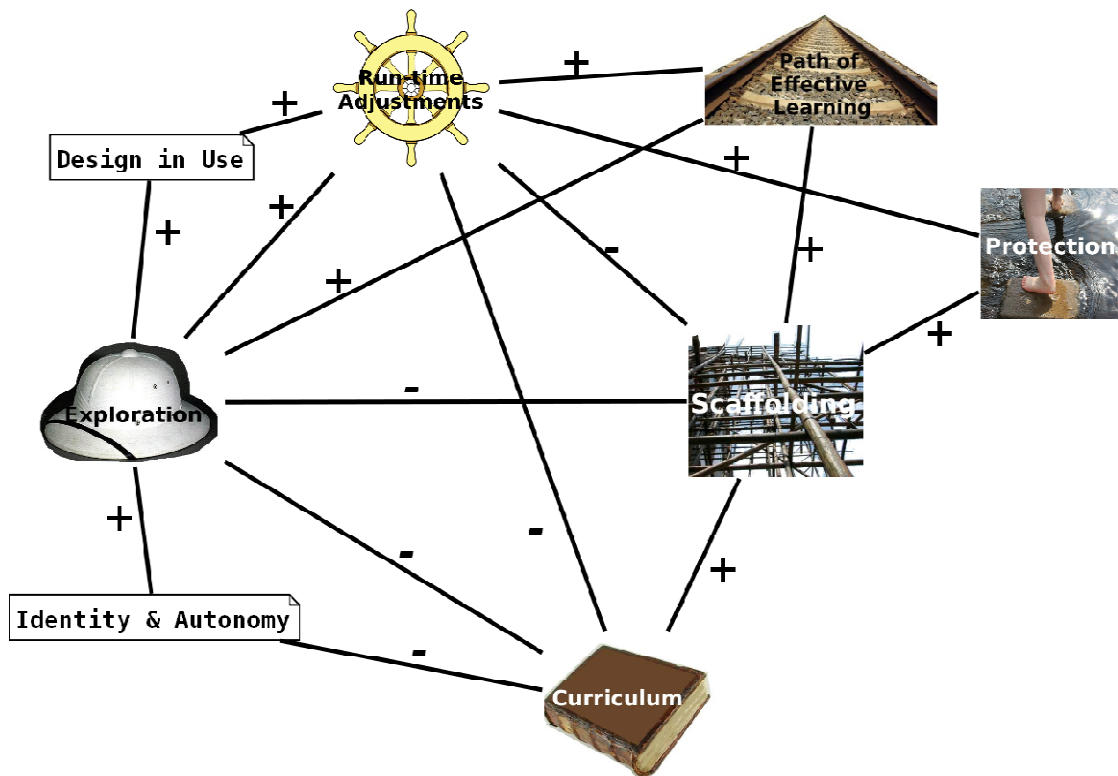


Figure 7: Force map for the Soft Scaffolding pattern

- The role of the educator, and by extension educational tools, is to direct the learner towards a productive path or enquiry.
- If the educational tool adamantly leads the learner through a set sequence, it risks failure on several accounts:
 - ◆ There is no leeway for mistakes, innovations, explorations or personal trajectories of learning.
 - ◆ Learners feel deprived of personal voice, and their motivation may falter.
 - ◆ It is hard to bypass design flaws discovered in the field or adjust to changing circumstances.

Context

Scaffolding is a term commonly used in educational design to describe structure that directs the learner's experience along an effective path of learning. This pattern originates from interactive web-based interfaces, where users can express themselves in writing. However, it should apply to almost any interactive learning interface.

Solution

Provide scaffolding which can easily be overridden by the learner or by the instructor. Let the scaffolding be a guideline, a recommendation which is easier to follow than not, but leave the choice in the hands of the learner.

- When providing a multiple-selection interface, always include an open choice, which the user can specify (select 'other' and fill in text box).
- When the user is about to stray off the desired path of activity, warn her, ask for confirmation, but do not block her.
- When providing templates for user contributions, include headings and tips but allow the user to override them with her own structure.

Examples

The ACTIVE WORKSHEETS used in the WebReports system (Mor, unpublished) provided participants a structure to work within, but allowed them to take control and change this structure as their confidence grew (Figure 8).

Explore

Can you think of a way to use your robot to produce these sequences?

| Sequence | Explain |
|---|--|
| | If yes, explain how you would do it. If you think it is impossible, explain why. Add any ToonTalk object that helps support your argument. |
| 2, 3, 4, 5... | |
| -1, -2, -3, -4... | |
| -7, -6, -5, -4.. | |
| 2, 4, 6, 8... | |
| 5, -1, -7... | |
| Write down a sequence of your own, which can be generated by your robot. | |
| Write down a sequence of your own, which cannot be generated by your robot. | |

Explain

How would you explain to a friend what kind of sequences your robot can generate, and how it can be used to generate those sequences?

Describe one sequence that cannot be generated by it, and explain why.

How would you convince a friend of your claims?

Figure 8: Example of Active worksheet. Learners were given a template in which to report on their exploration, but could edit it freely and replace the structure with their own.

The ToonTalk tool packaging convention (Mor et al, 2006), which was the basis for TASK IN A BOX, prompted learners to package their own productions in a particular way by providing them with useful examples. It did not block them from developing their own packaging style, but the ToonTalk-weblabs interface did give precedence to conventionally packaged constructions (Figure 9).

Training Add number

Train a robot to repeatedly add 1 to produce the numbers 0, 1, 2, 3.... Call the robot *Add number*.



(Click the box to get started, post your robot here when you're done)

Figure 9: Task-in-a-box demonstrating the ToonTalk packaging convention. Learners received their tasks in the recommended style for submitting their answers, but could edit and modify it to their preference.

Related Patterns

Used by: GUESS MY X; WEBLABS PEDAGOGICAL CYCLE; NARRATIVE SPACES

Elaborated by: ACTIVE WORKSHEET

Notes

The forces of this pattern are present in face-to-face learning situations. Experienced educators resolve them by providing ADAPTIVE SUPPORT; varying the learners freedom in response to their confidence. This could be implemented by intelligent tutoring systems, but simple learning environments lack this flexibility, and tend to compensate by being over-directive.

4. Narrative spaces

Constructing narrative is a fundamental mechanism for making sense of events and observations. To leverage it, we must give learners opportunities to express themselves in narrative form.



The problem

How can the epistemic power of narrative be harnessed by educators and learners in the construction of mathematical meaning?

Forces

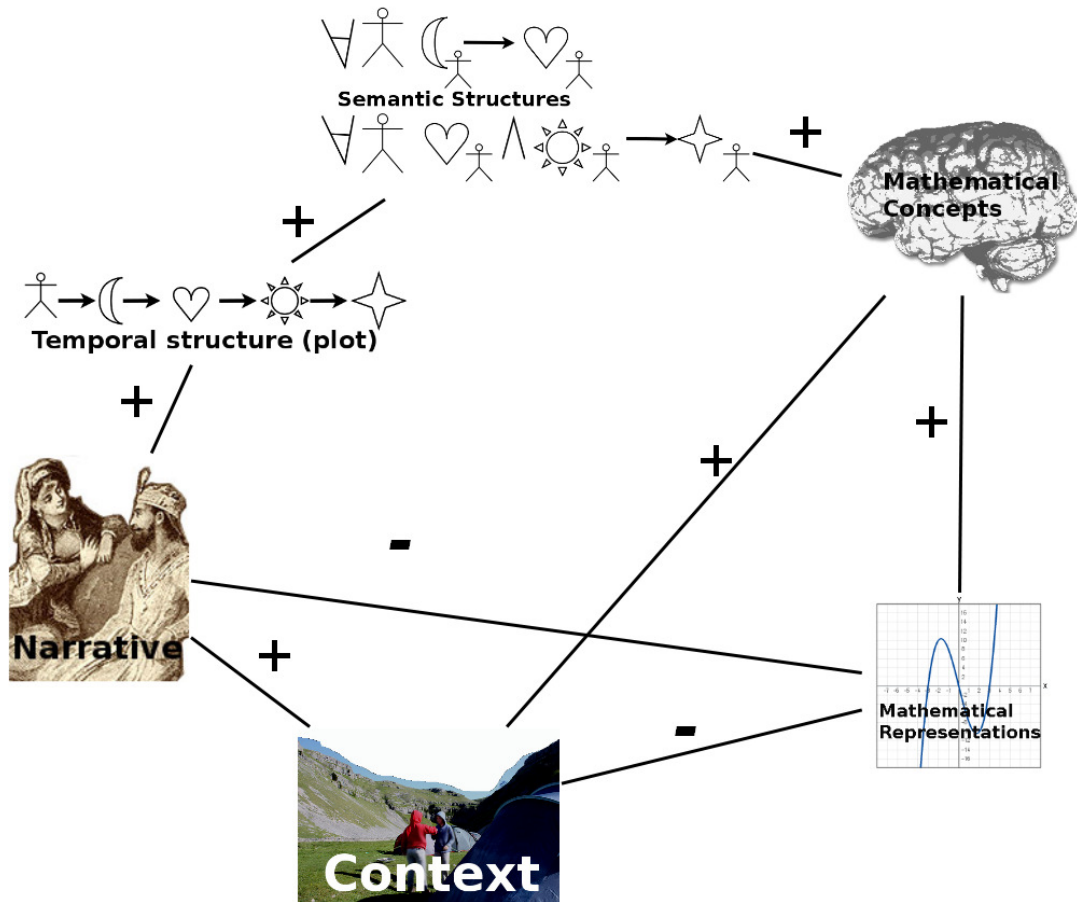


Figure 10: force map for Narrative Spaces

- Narrative is a powerful cognitive and epistemological construct (Bruner 1986; 1990; 1991).
- Mathematics appears to be antithetical to narrative form, which is always personal, contextual and time-bound.

Context

Digital environments for collaborative learning of mathematics.

Solution

Provide learners with a narrative space: a medium, integrated with the activity design, which allows learners to express and explore ideas in a narrative form:


- Allow for free-form text, e.g. by supporting SOFT SCAFFOLDING.
- Choose NARRATIVE REPRESENTATIONS when possible.

Mark narrative elements in the medium:

- Clearly mark the speaker / author, to support a sense of voice.
- Date contributions to support temporal sequentiality ('plot').
- Use SEMI-AUTOMATED META-DATA to provide context.

Examples

The webreports system allowed learners to comment on any page using a free-form WYSWYG editor (Figure 6). This allowed them to express their mathematical ideas in a personal narrative, as well as the path by which they arrived at these ideas. Using this feature, learners expressed and developed arguments which they could not yet formalize, and shared their learning process.

 **I try explain**

Posted by: Rita at 23-02-04

To create my sequence I thought thus: My first term is 2 and each one of the other terms is gotten of the previous one adding 2 and multiplying 4 to the result.

I created a box with 4 holes. In the first hole I put the first term (2), in thesecond hole I put the number that I wanted to add (2), in the third hole I put the number that I wanted to multiply (x4) and in the fourth hole I put a bird. I gave the box to the robot and I went in to the robot thought.

In the robot thought, I copied with magic wand the first 2 and gave to the bird, I copied with magic wand the second 2 and put in the first hole, I copied with magic wand x4 and put in the first hole. I Clicked in Esc and I left the robot thought.

I cleaned the first number of the robot thought box and i tested my robot...
And my sequence born...

Figure 11: Example comment from the Guess my Robot game (described below). Rita expresses mathematical ideas far beyond the curriculum for her age, and shares her learning process with her peers.

One participant in the guess my robot game (described below) used ToonTalk robots in an unexpected way: he trained the robot to “act out” the way in which he solved a challenge (Figure 7).

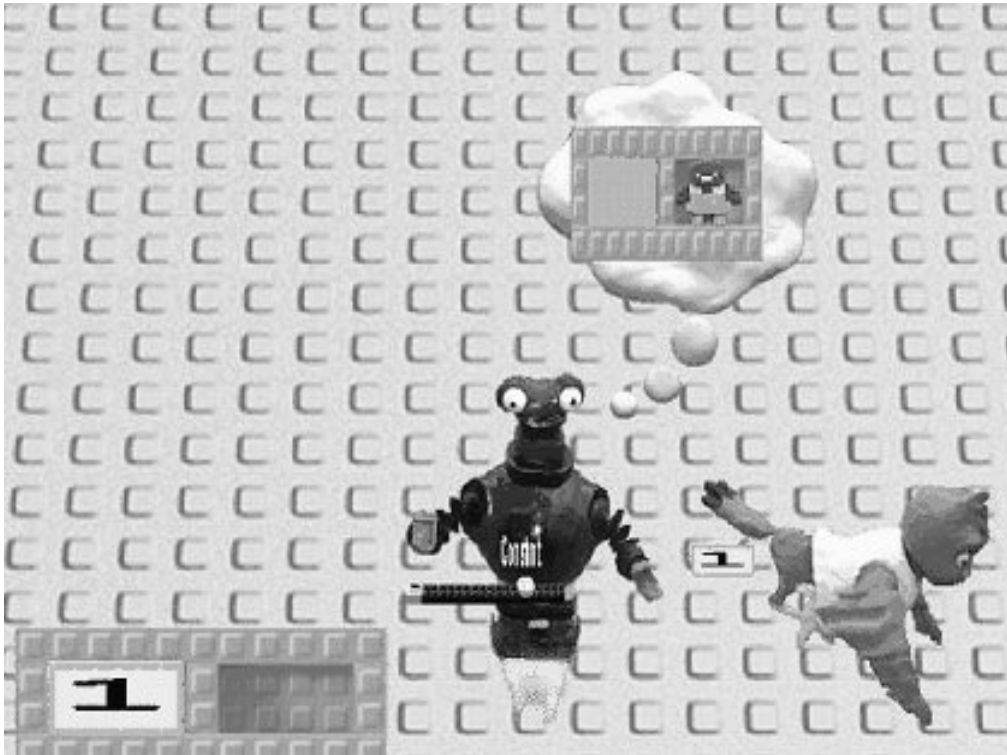


Figure 12: A robot acting out the way in which its programmer solved a mathematical challenge

Related Patterns

Uses: NARRATIVE REPRESENTATIONS; OBJECTS TO TALK WITH; SEMI-AUTOMATED META-DATA

Used by: GUESS MY X

5. Objects to talk with

Natural discourse makes extensive use of artefacts: we gesture towards objects that mediate the activity to which the discussion refers. This dimension of human interaction is often lost in computerized interfaces. When providing tools for learners to discuss their experience, allow them to easily include the objects of discussion in the discussion.



The problem

Several approaches to mathematics education highlight the importance of conversation and collaboration. The communicational approach (Sfard, 2008) equates thinking with communication, and sees learning mathematics as acquiring certain rules of discourse. Yackel and Cobb (1995) talk of the establishment of socio-mathematical norms through classroom discourse. Hurme and Järvelä (2005) argue that networked discussions can mediate students' learning, allowing students to co-regulate their thinking, use subject and metacognitive knowledge, make metacognitive judgments, perform monitoring during networked discussions and stimulates them into making their thinking visible.

Most computer-mediated discussion tools are strongly text-oriented, prompting users to express their thoughts lucidly in words or symbols. Yet two important elements of natural conversation are lost: the embodied dimension, i.e. gestures, and the ability to directly reference the objects of discussion.

Forces

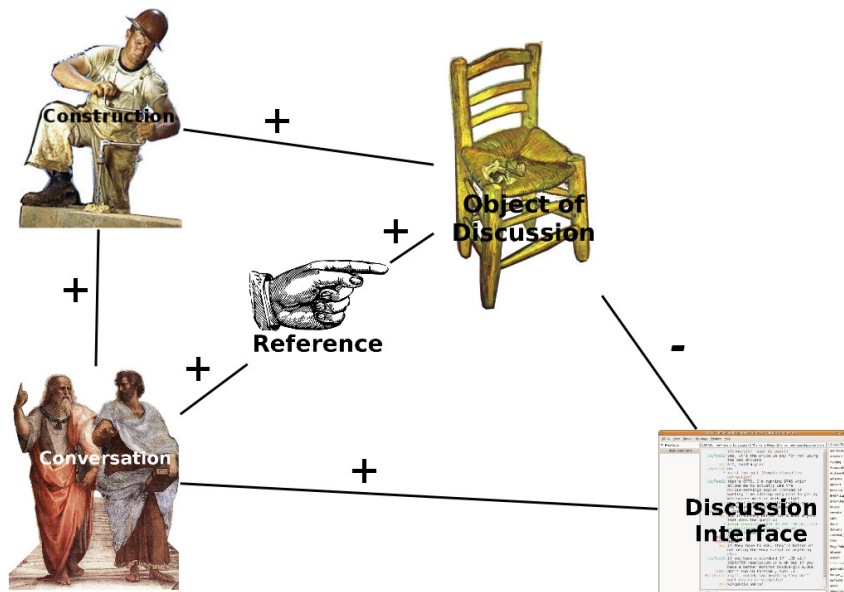


Figure 13: force map for the Objects to Talk With pattern

- Conversation is a powerful driver of learning, it:
 - Prompts learners to articulate their intuitions and in the process formulate and substantiate them.
 - Establishes mathematical norms of discourse.
 - Enables learners to share knowledge and questions.
- Conversation is even more powerful when building on personal experience or constructing or exploring mathematical objects.
- However, text based conversation media may obstruct learners, by forcing them to describe verbally the objects of enquiry which they would naturally gesture at.

Context

This pattern refers to interfaces which allow learners to converse about a common activity.

Solution

Learning activities often involve the use or construction of artefacts. When providing tools for learners to discuss their experience, allow them to easily include these artefacts in the discussion. If the activity is mediated by or aims to produce digital artefacts, then the discussion medium should allow embedding of these artefacts. The medium should support a visual (graphical, symbolic, animated or simulated) 1:1 representation of these objects.

When providing a NARRATIVE SPACE, allow the user to seamlessly embed the objects of discussion in the flow of narrative, so that learners can refer to these objects in a naturalistic manner.

In POST LUDUS discussions, the game's MATHEMATICAL GAME PIECES should become the OBJECTS TO TALK WITH. If the game is supported by a NARRATIVE SPACE, this emerges from the game flow.

Examples

This pattern identifies one of the WebReports system's primary design objectives. When developing the final version of the system, significant effort went into providing streamlined integration, which would allow students to select objects in ToonTalk and with a few clicks embed them in a webreport. The embedded objects were represented by their graphical image. When clicked, this image would invoke the original ToonTalk object in the viewers' ToonTalk environment. Likewise, when the activity involved graphs, learners could embed these in their report (Figure 14).

This is the real graph that was produced by the cumulate total of the halving-a-number robot. It looks like the top of my graph but I made the fatal mistake of thinking it started at zero. I also said it wouldn't go over 100, which was very wrong.

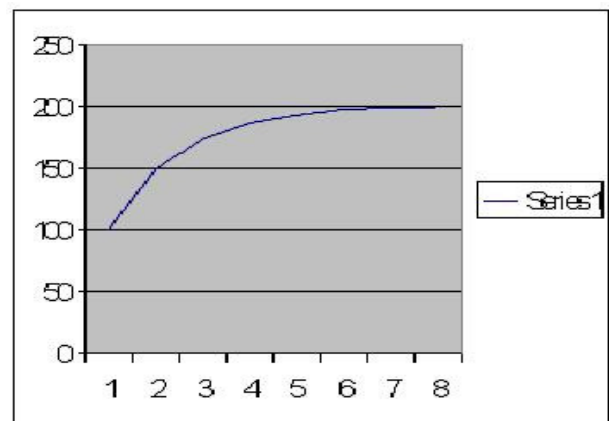


Figure 14: Webreport discussing a graphing activity, with the graph embedded as an object to talk with.

Related Patterns

Used by: GUESS MY X; TASK IN A BOX; ACTIVE WORKSHEET; SOFT SCAFFOLDING; NARRATIVE SPACES; POST LUDUS

Uses: MATHEMATICAL GAME-PIECES

Notes

The wide range of patterns which use this one indicate that it is indeed a fundamental component, applicable to most systems aiming to support discussion and collaborative learning.

Thumbnails

| | |
|--|--|
| NARRATIVE REPRESENTATIONS | Prefer forms of representation which have narrative qualities, or afford narrative expression. These are not only textual representation, but also visual or animated forms which include elements of context, plot, and voice. |
| POST LUDUS | Follow a game (or any other exploratory activity) with a discussion in which participants are prompted to articulate their learning experience and acquired knowledge. Such a discussion brings intuitions to the surface, strengthening structured knowledge and exposing discrepancies. |
| HARD BUT NOT TOO HARD | A challenge has to be set at a level which is slightly above the participants current level. A challenge too easy will be perceived as boring, while a challenge too hard will result in frustration – both leading to disengagement. |
| CHALLENGE EXCHANGE | A self-regulating mechanism for implementing HARD BUT NOT TOO HARD: allow learners to set challenges for one another. |
| BUILD THIS | A type of challenge / game, where learners are shown an object and asked to reconstruct it. |
| TASK IN A BOX | When using environment A to provide tasks in environment B, package these tasks in a compact form that can be embedded in A and unpacked in B. Each task package should include the task description, any tools required to perform it, and the mechanism for reporting back the results. |
| UNDERCOVER PROCESS | A challenge / game / task where learners need to reverse-engineer a process by observing its effects. |
| <i>SUGAR-COATING</i> (anti-pattern) | Mathematical (or any other subject) matter is injected into a game in a disconnected way, so that the game is the sugar-coating used to help the learner swallow the bitter content. As a result the game loses its appeal, and the learner receives the message that the educational content is inherently un-enjoyable. |

Conclusions

This paper presented five patterns out of an emerging language of patterns for techno-pedagogical design for mathematics learning and teaching, and outlined another seven. These design patterns reflect a techno-pedagogical approach which sees the design of educational activities and tools as a holistic endeavour. Thus, while some patterns emphasise certain features of technology and others highlight structures of activity, they all relate to some extent to both. The pedagogical approach underlying these patterns combines construction, communication and collaboration.

Above all, these patterns demonstrate the immense complexity of designing for learning. This complexity calls for further efforts towards identifying methodical frameworks for describing, aggregating and mapping design knowledge. The prime example of this complexity is the GUESS MY X pattern. At first, guess my robot may seem a simple game with surprising effects. The detailed analysis embodied in the GUESS MY X pattern, along with its 'ancestor' patterns – such as CHALLENGE EXCHANGE and BUILD THIS, suggests that the games success is not a fluke, but rather a result of an intricate assemblage of multiple design elements relating to the tools, the activity and the ways in which they interact.

Apart from the patterns themselves, this paper introduces the construct of force-maps as an innovative form supporting design pattern problem descriptions.

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