

# Coupling of finite element and boundary integral methods: Motion of ellipsoidal capsules in simple shear flow

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**Résumé** — We introduce a novel numerical method to model the fluid-structure interaction between a microcapsule and an external flow. An explicit finite element method is used to model the large deformation of the capsule wall, which is treated as a bidimensional hyperelastic membrane. It is coupled with a boundary integral method to solve for the internal and external Stokes flows. In particular, we are interested in characterizing the motion and deformation of an initially-ellipsoidal capsule subjected to a simple shear flow.

**Mots clés** — fluid-structure interaction, finite element method, boundary integral method.

## 1 Introduction

Bioartificial capsules, which consist of an internal liquid protected by a thin hyperelastic wall, have many practical uses in pharmaceutical, cosmetic and bioengineering applications. They can also be seen as simplified models of red blood cells. When suspended in another flowing liquid, capsules undergo large deformations and strong fluid-structure viscous coupling due to the low-Reynolds-number flows of the internal and suspending liquids. The membrane mechanical behaviour has been extensively studied numerically for initially-spherical capsules in flow (*e.g.* [1, 2]) : it has been shown to be dominated by in-plane membrane tensions, but compressive tensions can occur locally, creating wrinkles. The first objective of this work is to introduce a novel coupling method that has been found to remain stable even in the presence of wrinkling. It is based on a finite element (FE) membrane model of the capsule wall, coupled with a boundary-integral (BI) formulation of the Stokes flow equations [3].

Capsules with a non-spherical initial shape have only recently been considered in more details [4, 5], even though they provide more realistic models of red blood cells. The extensive wrinkling that appear when such capsules deform had rendered most numerical simulations unstable until then. We presently study initially-ellipsoidal capsules placed in a simple shear flow such that their revolution axis is in the shear plane. We consider both oblate and prolate spheroids to characterize the influence of the capsule shape on the deformation.

## 2 Problem statement

A spheroidal microcapsule is suspended in a simple shear flow, the inner and outer fluids having the same viscosity. Two main parameters influence its deformation : the capillary number  $Ca$ , which compares the viscous stresses in the flow to the elastic stresses in the membrane, and the aspect ratio  $\alpha$ , ratio of the ellipsoid semi-axis lengths. The spheroid is oblate when  $\alpha < 1$  and prolate when  $\alpha > 1$ .

Following [6], we treat the membrane as a purely bidimensional sheet of hyperelastic material. In particular, we neglect strain variations across the thickness and therefore the bending stiffness of the material. The numerical procedure consists in following the position of the capsule membrane after the start of flow. At each time step, the position of the membrane material points is thus known. The deformation of the capsule may be computed, along with the elastic tension tensor  $\underline{\underline{\tau}}$ , having assumed a material law. The finite element method is used to solve the equilibrium of the membrane

$$\nabla_s \cdot \underline{\underline{\tau}} + \underline{\underline{q}} = \underline{\underline{0}}. \quad (1)$$

It provides the value of the load  $\underline{q}$  exerted by the fluids on the membrane,  $\nabla_s \cdot$  being the surface divergence operator in the deformed configuration.

The velocity of the points of the capsule can then be calculated through an integral equation over the deformed surface  $S$ , using the boundary integral method :

$$\forall \underline{x}_0 \in S, \quad \underline{v}(\underline{x}_0) = \underline{v}^\infty(\underline{x}_0) + \frac{1}{8\pi\mu} \int_S \underline{J}(\underline{x}_0, \underline{x}) \cdot \underline{q}(\underline{x}) dS, \quad (2)$$

where  $\underline{v}^\infty$  is the undisturbed flow velocity,  $\underline{J}$  is the Green's single layer kernel,  $\underline{r} = \underline{x}_0 - \underline{x}$  and  $r = \|\underline{r}\|$ . The velocity is finally integrated with an explicit second-order Runge-Kutta method to obtain the new position of the membrane points at the following time step.

### 3 Results

We show that spheroidal capsules exhibit two types of behaviour depending on the value of  $Ca$ . For low  $Ca$ , a ‘solid-like’ regime is found, called *tumbling*. The capsule rotates under the influence of the vorticity of the external flow. The internal flow is almost stationary with respect to the membrane. At high  $Ca$ , the membrane, however, rotates around the deformed shape of the capsule in a tank-treading motion. The capsule therefore has a ‘fluid-like’ behaviour. Its overall shape varies with time, the capsule deformation and orientation oscillating periodically. This behaviour is due to the rotation of the reference shape during the tank-treading motion and is called *swinging*.

During the tumbling-to-swinging transition, the capsule transits through an almost circular profile in the shear plane, for which a long axis can no longer be defined. Oblate and prolate capsules are found to have a similar qualitative behaviour, particularly at large capillary numbers, when the influence of the initial state fades out. However, the capillary number at which the transition occurs is significantly lower for oblate than for prolate spheroids.

### 4 Conclusion

Coupling FE and BI methods leads to accurate capsule deformation modelling. It allows to sustain some compressive tensions in the wall even though the mechanical wall model does not account for bending stiffness. This is a convenient feature when the bending effects remain localised and weak. Still, to model exactly the physical behaviour of the capsule wall, a proper shell model remains to be implemented.

The study of the deformation of initially-ellipsoidal capsules has shown that the capsule initial shape mainly influences the transition between the tumbling and swinging regimes. In the case of oblate spheroids, the tumbling regime is limited to a very small range of capillary numbers, as the value of the critical capillary number at transition is low.

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