

# A Lagrangian finite element method for non-Newtonian free-surface fluid flows and fluid-structure interaction problems

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**Résumé** — A Lagrangian formulation of the Navier-Stokes equations is applied to the simulation of free-surface fluid-structure interaction problems. Both Newtonian and non-Newtonian fluid behaviours are considered. Different examples are performed to show the potentialities of the approach.

**Mots clés** — Lagrangian fluid, free-surface, fluid-structure interaction, non-Newtonian fluids.

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## 1 Introduction

In many fields of engineering, technology and biology, the numerical simulation of fluid-structure interaction phenomena is increasingly important. A critical aspect in fluid-structure interaction simulation is the computational treatment of the free surfaces and of the interfaces between solid and fluid. A possibility to solve this kind of problems consists in the Arbitrary Lagrangian Eulerian method (ALE), in which the movement of the fluid particles is independent of that of the mesh nodes. Other possible solutions are based on the volume of fluid or level set algorithms. Meshfree and meshless methods are often used for their ability to recover the free-surfaces and the interfaces. To overcome the difficulties concerning the tracking of the interfaces, a possibility is to adopt a Lagrangian approach for both fluid and structure. In the present work a fluid-structure interaction algorithm is presented based on a staggered scheme in which the fluid is treated in a Lagrangian framework using a new implementation [2] of the so called Particle Finite Element Method (PFEM), originally proposed by Oñate, Idelsohn and coworkers [1], and the structure using a classical finite element method.

## 2 Description of the method

One of the main difficulty of a Lagrangian formulation for the solution of a fluid-dynamics problem is the necessity to frequently regenerate the mesh. In fact, in a Lagrangian framework the position of element nodes is updated as a consequence of the fluid flow, and so, if a fixed finite element mesh is used, very soon the element distortion becomes excessive. A remedy which alleviates the burden of a complete remeshing consists of regenerating the connectivity of the elements by means of efficient implementation of a Delaunay triangulation. Moreover, to correctly impose the boundary conditions and to define the integration domain, the so-called “alpha shape method” [1] has been used. This method allows to identify the external boundary using a criterion based on the mesh distortion. The same method is used to define the inclusion and the separation of the individual particles.

When the connectivity between elements is regenerated, data have to be transmitted from the old mesh to the new one. To avoid interpolation from mesh to mesh, only degrees of freedom of particles located at the vertices of triangles are used, so that only linear shape functions can be used for both velocity and pressure unknowns. However, it is well known that this type of discretization does not satisfy the LBB inf-sup compatibility condition and so a stabilization method is required. To this purpose a pressure-stabilizing Petrov-Galerkin (PSPG) stabilization is used.

The proposed Lagrangian Finite Element Method is particularly suited for the solution of fluid-structure interaction problems in the presence of free-surfaces. To this purpose a staggered scheme is developed in which the PFEM is used to solve the fluid part of the problem and a classical finite element method is used for the solid part. A critical issue of fluid-structure interaction schemes is the identification

of the contact interfaces between the solid and fluid domains. The evolution of the interaction surfaces is tracked using a novel algorithm which exploits the features of the Lagrangian approach based on the continuous re-triangulation introduced for the fluid solution. The proposed algorithm is based on the superposition of a set of fictitious fluid particles to the nodes of the solid domain, which can come in contact with the fluid domain. When the Delaunay triangulation is performed, the alpha-shape criterion selects those parts of the interface where the fluid particles can possibly come into contact with the structure. If the two discretized domains are not in contact, the fluid and the solid analyses are performed separately without any interaction. If instead parts of the two discretized domains are in contact, a coupled analysis is performed with a Dirichlet-Neumann iterative approach.

### 3 Example : simulation deformable landslide simulation

The proposed scheme has been applied to both Newtonian and non-Newtonian fluids. Comparisons with numerical benchmarks and with experimental results are performed to show its potentialities. Here a simulation of the wave generated by a deformable mass of granular material sliding in a water channel is shown. This example is particularly interesting because impulse waves in bays, lakes or reservoirs generated by landslides or rock falls may propagate with possibly catastrophic consequences for the surrounding environment. For the numerical simulation of the sliding mass a non-Newtonian regularized Bingham fluid behaviour is assumed, while a classical Newtonian behaviour is considered for the water. Figure 1 shows snapshots of the analysis at different time steps.

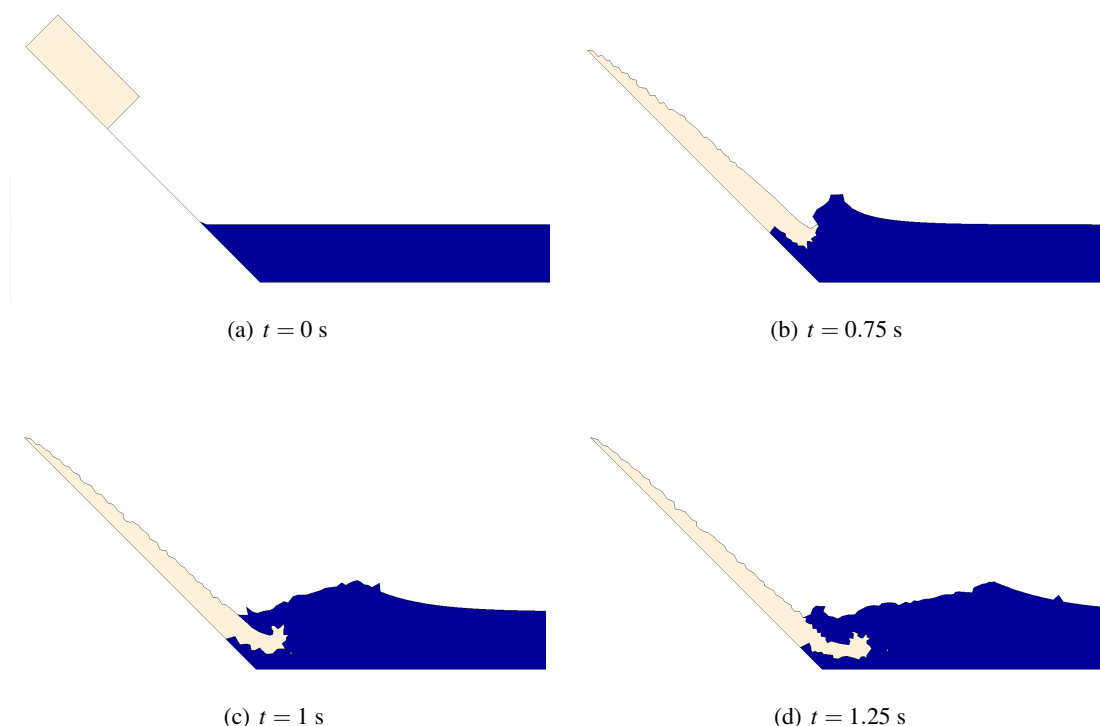


FIGURE 1 – Landslide simulation. Snapshots at different time steps.

### Références

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