Numerical convergence and stability of mixed formulation with X-FEM cut-off

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Abstract

The Finite Element Methods (F.E.M.) are widely used in engineering analysis, and has deep mathematical foundations. However, the treatment of incompressible or nearly incompressible problems, with only one unknown field variable, leads to numerical locking of this approximation. For the analysis of such problems, it is necessary to use a mixed formulation. However, mixed finite element methods are not stable in all cases, some of them show spurious pressure oscillations if displacement and pressure spaces are not properly chosen [1]. For a mixed formulation to be stable it must verify consistency, ellipticity and the so called inf-sup (or LBB) condition. The later is a severe condition which depends on the connection between the approximation spaces for the displacement and pressure [2]. The presence of a crack in a structure reveals two types of discontinuities: a strong discontinuity that requires an adapted mesh to the shape of the crack, hence the domain is meshed at each time step; and a weak discontinuity that requires refinement at the crack tip. These two operations lead to a huge computational cost. In order to overcome these difficulties and to make the finite element method more flexible, many alternative methods had been developed. Apart from mixing meshless methods and finite elements, another alternative consists in using the concept of "partition of unity" introduced by Babuška, to creates an independence between the mesh and the crack. Among the class of partition of unity finite element methods, the Generalised Finite Element Method (GFEM) and the eXtended Finite Element Method (X-FEM) are the most advanced. The X-FEM allows to model cracks, material inclusions and holes on nonconforming meshes. It was introduced by [3]. It consists in enriching the basis of the classical finite element method by a step function along the crack line and by some non-smooth functions representing the asymptotic displacement around the crack tip. In spite of the singular enrichment of the finite element basis, the obtained convergence error of X-FEM remains of order if linear finites elements are used. To improve this convergent rate and to obtain an optimal accuracy, [4] introduced a new enrichment strategy: the so called X-FEM cut-off. This enrichment strategy uses a cut-off function to locate the crack tip surface. In their work, Chahine et al. have shown that the X-FEM cut-off has an optimal convergence rate of order h and that the conditioning of the stiffness matrix does not deteriorate. In this work, we extend the theoretical and numerical results given by Chahine et al. [4] to an incompressible isotropic linear plane elasticity problem in fracture mechanics. In particular, this formulation must satisfy the famous inf-sup condition (or LBB). We present the problem of linear elasticity in cracked domains (formulation of the problem and general concepts). We then introduce the concept of enriching the space of finite elements. We write our mixed formulation with and without X-FEM. Then we formulate our problem as a linear system. Finally we discuss the stability of our mixed formulation. We prove the LBB condition with P_2/P_0 - mixed formulation. We also give a mathematical result of quasi-optimal error estimate. We numerically validate the numerical result found.

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