

RECENT DEVELOPMENTS IN LOCAL/GLOBAL REDUCTION TECHNIQUES FOR THE SIMULATION OF LOCAL FAILURE IN STRUCTURES.

P. KERFRIDEN¹, P. GOSSELET², S. BORDAS¹

¹ CARDIFF UNIVERSITY
QUEEN'S BUILDINGS, THE PARADE, CARDIFF CF24 3AA, WALES, UK
² ECOLE NORMALE SUPERIEURE DE CACHAN
61 AVENUE DU PRÉSIDENT WILSON, F-94230 CACHAN, FRANCE

1. INTRODUCTION

Predictive failure models usually describe structures at a very fine scale, and the finite element discretization of the underlying partial differential equations leads to large, potentially highly nonlinear, numerical problems. Various multiscale computational strategies have been developed to tackle this important issue, such as enrichment techniques, homogenization strategies, domain decomposition methods (a recent survey of condensation-based strategies is proposed in [1]), or model order reduction [2, 3, 4, 5].

Among the later family of methods, the Proper Orthogonal Decomposition-based (POD) (see for instance [3] for a survey), and more generally model order reduction strategies based on a projection in a Ritz basis, provide extremely valuable tools for an automatic derivation of a multiscale computational method (no *a priori* physical understanding of the different scales involved is required). They consist in using a set of potential solutions to the initial problem (snapshot) and extracting, by a spectral analysis a few basis vectors spanning a space of small dimension in which the solution to the, initially large, numerical problem is well approximated [2]. These techniques have been proved extremely efficient when speed is more important than accuracy, for instance if the goal is to provide an approximation of the response of a complex structure in quasi real-time, in the case of early stage design. This type of applications is our main focus.

In the context of structural problems involving plasticity or damage, a severe drawback limits the direct application of projection-based model order reduction: The initial snapshot might be too poor to represent accurately the solution of the damaged structure. This can happen if:

- The “a priori” mechanical understanding of the structure is weak, which results in a bad choice of the snapshot simulations.
- The number of parameters involved is too important. The snapshot needed to obtain a relevant reduced basis might be very large, and its reduction inefficient.
- Strong topological changes in the structure occur, which cannot be represented by a small number of Ritz basis vectors.

We propose a two-stage enhancement of classical projection-based model order reduction strategies. We first develop an efficient enrichment strategy of the projection basis [5]. The resulting adaptive reduction technique performs well for moderate nonlinearities, for instance during the initiation phase of failure mechanisms. The second stage of our work is to develop

a global/local approach for the reduction of structural problems involving strong topological changes (propagation phase of the damage mechanisms).

2. METHODOLOGY

The adaptive projection technique proposed in this work relies on the use of projected Krylov algorithm. In the past years, particular studies [6, 7] have shown that the convergence rate of Krylov solvers could be increased by using a reduce model as a preconditionner, through a projection technique. We propose the reverse operation. At a given iteration of the Newton-like nonlinear solution algorithm, the *a priori* reduced model is updated with a solution vector obtained by solving the non-reduced problem with a Krylov algorithm. The resulting technique is an "on-the-fly" enrichment procedure. As proposed in [7], the correction of the reduced model make use of both the reduce model and some spectral properties (selective reuse of Krylov subspaces) of the non-reduced operator to derive a good preconditioner.

Our first results show that the enrichment technique becomes prohibitively expensive as the nonlinearity of the problem increases. Yet, in the case of failure, the strong topological changes are usually localized. We can turn this property to our advantage, by performing the reduction of the undamaged zone only. Two local/global strategies are investigated. The first one is directly inspired by the work published in [8]. The zone of maximum increment of damage is extracted and analysed at the fine scale (i.e.: without reduction). The second method, called the substructured reduction method (SRM), reduces the local problem in Schur-based domain decomposition methods, allowing defining a very versatile reduction framework. Depending on the level of local nonlinearity, the local problems are, or ar not, reduced by projection on a Ritz basis.

3. RESULTS

We show that the coupling of local/global reduction technique and adaptive Ritz basis provide a very versatile framework for the reduction of highly nonlinear structural problems. The efficiency of the method is illustrated on particular examples involving damage mechanisms, from initiation to propagation.

REFERENCES

- [1] P. Gosselet and C. Rey. Non-overlapping domain decomposition methods in structural mechanics. *Archives of Computational Methods in Engineering*, 13:515–572, 2006.
- [2] L. Sirovich. Turbulence and the dynamics of coherent structures. part I: coherent structures. *Quarterly of Applied Mathematics*, 45:561–571, 1987.
- [3] Y. C. Liang, H. P. Lee, S. P. Lim, W. Z. Lin, K. H. Lee, and C. G. Wu. Proper orthogonal decomposition and its applications - part I: theory. *Journal of Sound and Vibration*, 252(3):527–544, 2002.
- [4] D. Ryckelynck. A priori hyperreduction method: an adaptive approach. *Journal of Computational Physics*, 202(1):346–366, 2005.
- [5] P. Kerfriden, P. Gosselet, S. Adhikari, and S. Bordas. Bridging proper orthogonal decomposition methods and augmented newton-krylov algorithms: an adaptive model order reduction for highly nonlinear mechanical problems. *Computer Methods in Applied Mechanics and Engineering*, doi:10.1016/j.cma.2010.10.009, 2010.
- [6] Z. Dostál. Conjugate gradient method with preconditioning by projector. *International Journal of Computer Mathematics*, 23(3):315–323, 1988.
- [7] F. Risler and C. Rey. Iterative accelerating algorithms with krylov subspaces for the solution to large-scale nonlinear problems. *Numerical algorithms*, 23:1–30, 2000.
- [8] O. Allix, P. Kerfriden, and P. Gosselet. A relocalization technique for the multiscale computation of delamination in composite structures. *Computer Modeling in Engineering and Sciences*, 55(3):271–291, 2010.