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## Recent developments in local/global reduction techniques for the simulation of local failure in structures

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**Résumé** — This paper proposes a novel technique to reduce the computational burden associated with the simulation of structural failure by concentrating the computational effort where it is most needed, i.e. in the localisation zones. To do so, a local/global technique is devised where the global (slave) problem (far from the zones undergoing severe damage and cracking) is solved for in a reduced space computed by the classical Proper Orthogonal Decomposition, while the local (master) degrees of freedom (associated with the part of the structure where most of the damage is taking place) are fully resolved.

**Mots clés** — nonlinear fracture mechanics, model order reduction, multiscale solution strategy.

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### 1 Introduction

Simulating damage initiation and subsequent global structural failure is one of most active topics in computational mechanics. Mathematical models and numerical methods have been extensively developed over the years to assess limit states such as failure due to permanent deformations, cracks or decohesion. Yet, these models, be they damage based or relying on discrete cracks are computationally expensive, as they introduce a description of the structural and material properties at a fine scale ("micro" or "mesoscale"). Therefore, today's engineers are not able to use these state-of-the-art models for routine design. For the advances in the treatment of material failure to become useful in practice, it is thus important to devise techniques which are able to significantly reduce the computational effort required without sacrificing accuracy.

Historically, reducing the computational time associated with solving nonlinear problems in solid mechanics has mainly been addressed by developing homogenisation techniques. However, the application of homogenisation-based methods to failure is not straightforward, as the classical assumptions required to derive a proper separation of scales are not satisfied. We propose here to follow an alternative route which relies on recent advances in model order reduction strategies for highly nonlinear structural problems [16, 12, 19, 8]. We believe that this class of techniques can provide an interesting alternative (or complementary) approach to homogenisation-based solution procedures.

Model order reduction have in common an approximation of the unknown fields a space spanned by a limited number basis of global vectors (as opposed to locally supported shape functions in finite elements for example). Three families of reduction methods can be distinguished. The oldest propose to use intrinsic spectral properties of the system in order to define a valid approximation basis. These approaches have been extensively studied for the reduction of dynamical systems (modal synthesis, Craig-Bampton method). They are in general limited to linear or "smoothly nonlinear" (see for instance [9]), dynamical problems. When nonlinear or quasi-static evolution approach, a second family of reduced order modelling techniques is preferred, which uses the spectral properties of a pre-computed image of the system (set of representative responses, called snapshots [18]). The Proper Orthogonal Decomposition (POD) [14, 6] can be used to extract a relevant basis from the vectors, which in turn defines a coarse space where a solution can be cheaply searched (see for instance [1, 2] for the efficient reduction of parametric nonlinear problems). The last family of solvers propose to build, "on the fly", a global basis for the approximation of the solution [3, 16, 13, 10].

To the authors' knowledge, the application of snapshot-based reduced order modelling methods to

failure simulations has virtually never been addressed. The main reason for this is that the solution fields in the zones of the structure where damage or failure initiate and propagate are highly parameter-dependent, and cannot be known in advance. Hence, precomputed reduced basis are unable to represent the structural behaviour beyond the onset of strain localisation, damage and fracture. In the particular case of linear elastic fracture mechanics, the authors of [4] have proposed a mesh morphing technique that allows to keep an autosimilar description of the singularity during propagation. The use of a snapshot POD becomes relevant and allows for interesting computational savings. Corrective POD algorithms (therefore bridging the first and second family of solvers described in the previous paragraph) have also been proposed to alleviate this in the case of plasticity or damage models [16, 8]. These techniques are based on global corrections of the initial Ritz basis by Krylov algorithms. Though efficient for global nonlinearities, it was shown in [8] that in the case of damage assessment, the number of corrections increases when nonlinearities localise, i.e. : when failure happens. The purpose of this paper is to circumvent this phenomenon.

The first stage of the proposed method is to only reduce the set of balance equations which exhibits a “smooth” nonlinearity. In the context of the simulation of damage, one part of the degrees of freedom, corresponding to highly damaged zones, will be considered as master degrees of freedom, which will be fully solved for, while the remaining will be approximated as a linear combination of Ritz vectors, which are precomputed. In this respect, the proposed method has strong links with Craig-Bampton methods, or model order reduction by substructure such as the one proposed in [15, 11, 10]. After a brief introduction to reduced order modelling in section 2, this technique is described in section 3, and illustrated in section 4. The second stage of our developments, detailed in section 5, consists in performing efficient global correction to the reduced basis in order to transmit the long-range effect of the local failure to the far region, which are treated by model order reduction. We show that the combination of the locally enriched description and global adaptive reduced order modelling permit to treat parametrised nonlinear fracture problems efficiently .

## 2 Model order reduction by projection.

### 2.1 Generic discrete nonlinear problem of evolution

We consider the generic problem of quasi-static evolution of irreversible damage processes in a deformable solid. The rate-effects are neglected and the solution field (displacement) is looked for under the assumption of small perturbation. We introduce a classical finite element space discretization for the spatial derivatives. A time discretization scheme is performed to intergrate the resulting semi-discrete problem over the time interval  $[0, T]$ . The procedure consists in finding a set of consecutive solutions at times  $(t_n)_{n \in \llbracket 0, n_t \rrbracket}$ . At any time  $t_{n+1}$  of the analysis , one needs to solve a nonlinear vectorial equation of the form :

$$\underline{\mathbf{F}}_{\text{Int}} \left( \underline{\Delta \mathbf{U}}, \left( \underline{\mathbf{U}}_{|t_m} \right)_{m \in \llbracket 0, n \rrbracket} \right) + \underline{\mathbf{F}}_{\text{Ext}} = \mathbf{0} \quad (1)$$

where the vector of increment in the nodal displacement unknowns  $\underline{\Delta \mathbf{U}} \in \mathbb{R}^{n_u}$  ( $n_u$  is the number of nodal unknowns introduced in the finite element discretization) is defined by  $\underline{\Delta \mathbf{U}} = \underline{\mathbf{U}}_{|t_{n+1}} - \underline{\mathbf{U}}_{|t_n}$ ,  $\underline{\mathbf{F}}_{\text{Int}} \in \mathbb{R}^{n_u}$  and  $\underline{\mathbf{F}}_{\text{Ext}} \in \mathbb{R}^{n_u}$  are respectively the classical internal and external force vectors.

The set of successive solution vectors to problem (1) at times  $(t_n)_{n \in \llbracket 0, n_t \rrbracket}$ , obtained by a tangent Newton algorithm, will be considered as the reference solution in the presented piece of work. The algorithms proposed in the following should provide a solution that is close to the reference, at cheap computational cost. Hence, the relevance of the space and time discretizations will not be discussed.

### 2.2 Projection-based model order reduction of the discrete problem

The increments in the solution vector are searched for in a space of small dimension (several orders smaller than the number of finite element degrees of freedom). Let us call  $\underline{\mathbf{C}}$  the matrix whose  $n_c$  columns  $(\underline{\mathbf{C}}^k)_{k \in \llbracket 1, n_c \rrbracket} \in (\mathbb{R}^{n_u})^{n_c}$  (also called Ritz vectors) form a basis of this space. Applied to the reduction of problem (1), the increment in the solution field is approximated by :

$$\underline{\Delta \mathbf{U}} = \underline{\mathbf{C}} \underline{\alpha} \quad (2)$$

where we introduced the reduced state variables  $\underline{\alpha} \in \mathbb{R}^{n_c}$ . Problem (1) is overconstrained. It is classically solved by introducing a Galerkin orthogonality condition : the residual  $\underline{\mathbf{R}} = \underline{\mathbf{F}}_{\text{Int}}(\underline{\mathbf{U}}|_{t_n} + \underline{\mathbf{C}}\underline{\alpha}) + \underline{\mathbf{F}}_{\text{Ext}}$  of equation (1) is required to be orthogonal to any test vector  $\underline{\delta\mathbf{U}}^* = \underline{\mathbf{C}}\underline{\delta\alpha}^*$  belonging to the space spanned by the Ritz vector. The reduction of problem (1) takes the following form :

$$\underline{\mathbf{C}}^T \left( \underline{\mathbf{F}}_{\text{Ext}} + \underline{\mathbf{F}}_{\text{Int}}(\underline{\mathbf{U}}|_{t_n} + \underline{\mathbf{C}}\underline{\alpha}) \right) = \underline{\mathbf{0}} \quad (3)$$

This nonlinear problem can be solved by a Newton solution strategy.

The reduced basis used in the paper is computed prior to the actual simulation by the classical Snapshot Proper Orthogonal Decomposition approach [18]. Broadly, full-scale solutions are computed for a set of particular realisations of the considered parametric problem. A small orthogonal projection basis can be obtained by sorting the resulting solution vectors using a spectral analysis.

### 3 Local/global model order reduction strategy

Projection-based model order reduction techniques introduce a global approximation of the displacement. As such, even adaptive versions [17, 8] of these methods are not well suited to the analysis of localised nonlinearities. We propose in the following to use reduction techniques by projection for the “weakly nonlinear” equations of reference problem (1), namely equations for which reduced order modeling is relevant, while the remaining equations will be solved directly.

#### 3.1 Displacement approximation

The principle of the proposed local/global strategy is to split the unknown solution vector into two parts, only one of them being approximated as a linear combination of global vectors. We shall use superscript  $(r)$  for the approximated “slave” part of the solution vector, while superscript  $(f)$  (which stands for “fully resolved”) will be used to denote its complementary “master” part. More precisely, let  $\underline{\Delta\mathbf{U}}$  be the unknown increment vector at time  $t_{n+1}$ , the numbering being reorganised as follows :

$$\underline{\Delta\mathbf{U}} = \begin{pmatrix} \underline{\widetilde{\Delta\mathbf{U}}}^{(r)} \\ \underline{\widetilde{\Delta\mathbf{U}}}^{(f)} \end{pmatrix} = \begin{pmatrix} \underline{\mathbf{E}}^{(r)} \\ \underline{\mathbf{E}}^{(f)} \end{pmatrix} \underline{\Delta\mathbf{U}} \quad (4)$$

In the above notations,  $\underline{\widetilde{\Delta\mathbf{U}}}^{(r)} \in \mathcal{R}^{n_r}$  and  $\underline{\widetilde{\Delta\mathbf{U}}}^{(f)} \in \mathcal{R}^{n_f}$ , with  $n_r + n_f = n_u$  and  $n_f$  much smaller than  $n_r$ .  $\underline{\mathbf{E}}^{(r)} \in \{0, 1\}^{n_r} \times \{0, 1\}^{n_u}$  and  $\underline{\mathbf{E}}^{(f)} \in \{0, 1\}^{n_f} \times \{0, 1\}^{n_u}$  are two boolean extractors (rectangular matrices with one 1 per line, the other coefficients being null).  $\underline{\widetilde{\Delta\mathbf{U}}}^{(r)}$  is approximated as a linear combination of the Ritz vectors  $(\underline{\mathbf{C}}^k)_{k \in [1, n_c]} \in (\mathbb{R}^{n_u})^{n_c}$  :

$$\underline{\widetilde{\Delta\mathbf{U}}}^{(r)} = \left( \underline{\mathbf{E}}^{(r)} \underline{\mathbf{C}} \right) \underline{\alpha} \quad (5)$$

where  $\underline{\alpha} \in \mathbb{R}^{n_c}$  are the reduced degrees of freedom. In the initial numbering, the unknown increment vector can be expressed under the form :

$$\underline{\Delta\mathbf{U}} = \underline{\Delta\mathbf{U}}^{(r)} + \underline{\Delta\mathbf{U}}^{(f)} \quad \text{where} \quad \begin{cases} \underline{\Delta\mathbf{U}}^{(r)} = \underline{\mathbf{E}}^{(r)T} \underline{\widetilde{\Delta\mathbf{U}}}^{(r)} \\ \underline{\Delta\mathbf{U}}^{(f)} = \underline{\mathbf{E}}^{(f)T} \underline{\widetilde{\Delta\mathbf{U}}}^{(f)} \end{cases} \quad (6)$$

Introducing projector  $\underline{\mathbf{P}}^{(r)} = \underline{\mathbf{E}}^{(r)T} \underline{\mathbf{E}}^{(r)}$ , the approximation of the displacement increment finally reads :

$$\underline{\Delta\mathbf{U}} = \underline{\mathbf{P}}^{(r)} \underline{\mathbf{C}} \underline{\alpha} + \underline{\mathbf{E}}^{(f)T} \underline{\widetilde{\Delta\mathbf{U}}}^{(f)} \quad (7)$$

Let us introduce the new state vector  $\underline{\mathbf{X}} = \left( \underline{\alpha}^T \quad \underline{\widetilde{\Delta\mathbf{U}}}^{(f)T} \right)^T$  at time  $t_{n+1}$ . We now have :

$$\underline{\Delta\mathbf{U}} = \underline{\mathbf{A}} \underline{\mathbf{X}} \quad \text{where} \quad \underline{\mathbf{A}} = \left( \underline{\mathbf{P}}^{(r)} \underline{\mathbf{C}} \quad \underline{\mathbf{E}}^{(f)T} \right) \quad (8)$$

### 3.2 Locally reduced set of balance equations

Using the the Galerkin procedure, the balance equation are required to be orthogonal to any test vector  $\underline{\delta\mathbf{U}}^* = \underline{\mathbf{A}} \left( \underline{\alpha}^{*T} \quad \widetilde{\underline{\delta\mathbf{U}}}^{(f)*T} \right)^T$ , where  $\underline{\delta\alpha}^* \in \mathbb{R}^{n_c}$  and  $\widetilde{\underline{\delta\mathbf{U}}}^{(f)*} \in \mathbb{R}^{n_f}$  are test state variables. The orthogonality requirement applied to the full set of nonlinear equations (1) yields the following system :

$$\underline{\mathbf{R}}_R(\underline{\mathbf{X}}) = \underline{\mathbf{A}}^T \left( \underline{\mathbf{F}}_{\text{Int}} \left( \underline{\Delta\mathbf{U}}(\underline{\mathbf{X}}) + \underline{\mathbf{U}}_{|t_n} \right) + \underline{\mathbf{F}}_{\text{Ext}} \right) = \underline{\mathbf{0}} \quad (9)$$

### 3.3 Newton solution scheme

At each time step of the time discretization scheme, the reduced problem (9) is solved by a tangent Newton algorithm. The  $(i+1)^{\text{th}}$  Newton iteration consists in finding  $\underline{\delta\mathbf{X}}^{i+1}$  satisfying the following set of  $n_c + n_f$  linearized equation :

$$\left. \frac{\partial \underline{\mathbf{R}}_R(\underline{\mathbf{X}})}{\partial \underline{\mathbf{X}}} \right|_{\underline{\mathbf{X}}=\underline{\mathbf{X}}^i} \underline{\delta\mathbf{X}}^{i+1} = -\underline{\mathbf{R}}_R^i \quad (10)$$

where  $\underline{\delta\mathbf{X}}^{i+1} = \underline{\mathbf{X}}^{i+1} - \underline{\mathbf{X}}^i$  and  $\underline{\mathbf{R}}_R^i = \underline{\mathbf{R}}_R(\underline{\mathbf{X}}^i)$  is the residual of problem (9) computed at iteration  $i$ .

The expression of the tangent stiffness  $\underline{\mathbf{K}}_{\text{T,R}}^i = \left. \frac{\partial \underline{\mathbf{R}}_R(\underline{\mathbf{X}})}{\partial \underline{\mathbf{X}}} \right|_{\underline{\mathbf{X}}^i}$  is obtained by differentiation of equation (9) with respect to the set of reduced state variables. The derivation will not be detailed here. Using the notation  $\underline{\mathbf{K}}_{\text{T}}^i = \left. \frac{\partial \underline{\mathbf{F}}_{\text{Int}}}{\partial \underline{\Delta\mathbf{U}}} \right|_{\underline{\Delta\mathbf{U}}(\underline{\mathbf{X}}^i)}$  for the consistent tangent stiffness of the full system of equations (1), one obtains obtains the following linearized reduced system of equations :

$$\begin{aligned} & \left( \begin{array}{cc} \underline{\mathbf{C}}^T \underline{\mathbf{P}}^{(r)} \underline{\mathbf{K}}_{\text{T}}^i \underline{\mathbf{P}}^{(r)} \underline{\mathbf{C}} & \underline{\mathbf{C}}^T \underline{\mathbf{P}}^{(r)} \underline{\mathbf{K}}_{\text{T}}^i \underline{\mathbf{E}}^{(f)T} \\ \underline{\mathbf{E}}^{(f)} \underline{\mathbf{K}}_{\text{T}}^i \underline{\mathbf{P}}^{(r)} \underline{\mathbf{C}} & \underline{\mathbf{E}}^{(f)} \underline{\mathbf{K}}_{\text{T}}^i \underline{\mathbf{E}}^{(f)T} \end{array} \right) \underline{\delta\mathbf{X}}^{i+1} \\ & = - \left( \begin{array}{c} \underline{\mathbf{C}}^T \underline{\mathbf{P}}^{(r)} \\ \underline{\mathbf{E}}^{(f)} \end{array} \right) \left( \underline{\mathbf{F}}_{\text{Int}} \left( \underline{\Delta\mathbf{U}}(\underline{\mathbf{X}}^i) + \underline{\mathbf{U}}_{|t_n} \right) + \underline{\mathbf{F}}_{\text{Ext}} \right) \end{aligned} \quad (11)$$

Notice that problem (11) is considerably reduced compared to a direct linearisation of (1). Indeed,  $\underline{\mathbf{K}}_{\text{T,R}}^i \in \mathbb{R}^{n_c} \times \mathbb{R}^{n_f}$  where  $n_c \ll n_u$  and, applied to the analysis of localised nonlinearities, we can reasonably expect that  $n_f$  is at least one order of magnitude smaller than  $n_u$ , which means that only a few of the balance equations are strongly nonlinear. This assumption will be validated in section 4.

This framework is very general. It does not rely on domain decomposition but on a splitting of the balance equations. Yet, this technique will be particularized in section 4, and we will show that using the ideas inspired from domain decomposition methods and local/global methods [5, 7] provides efficient splitting in the case of damage mechanics.

## 4 Application to the adaptive reduction of parametrised damage problems

### 4.1 Damageable lattice problem

We consider a lattice structure made of straight bars under traction or compression. Each bar behaves as an elastic damageable unidirectional media. Our purpose is to check the relevancy of the proposed local/global reduction technique for obtaining a setof load/deflection curves (pre and post failure phases) of the structure under parametrised loading conditions (figure (1, left) shows one of the realisations).

### 4.2 Particularisation of the local/global approach

The splitting between reduced degrees of freedom  $\widetilde{\underline{\Delta\mathbf{U}}}^{(r)}$  and the complementary ones  $\widetilde{\underline{\Delta\mathbf{U}}}^{(f)}$  is made by considering that the part of the structure undergoing strong damage variations will not be correctly solved for when projected on a pre-computed Ritz basis.

The following procedure is adopted. At the end of a time step, the element undergoing the maximum damage increment is spotted, and a sphere of radius  $\rho_s$ , centred at the isobarycenter of the element is defined. Every degree of freedom belonging to a node located inside this sphere are set as a fully

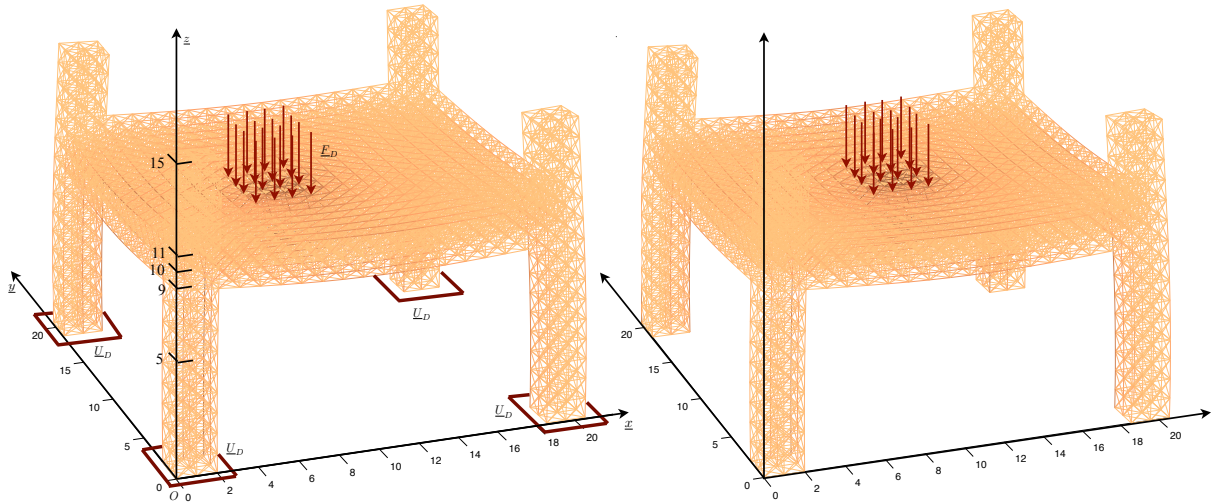


FIGURE 1 – Solution obtained at an intermediate time step (maximum value of prescribed forces before unstable propagation) of the full-scale simulation for a particular realisation of the parametrised problem. Darker bars undergo a higher damage state. The load is applied on a square surface which can be located anywhere in the  $(x, y)$  plane. On the right-hand side, an other realisation used as a snapshot to compute a Ritz basis by the POD.

resolved degrees of freedom. This procedure is repeated on the remaining elements, until a criterion on the maximum damage increment in the reduced part of the domain is satisfied. To illustrate this procedure, two different reduced domain are represented in figure (2).

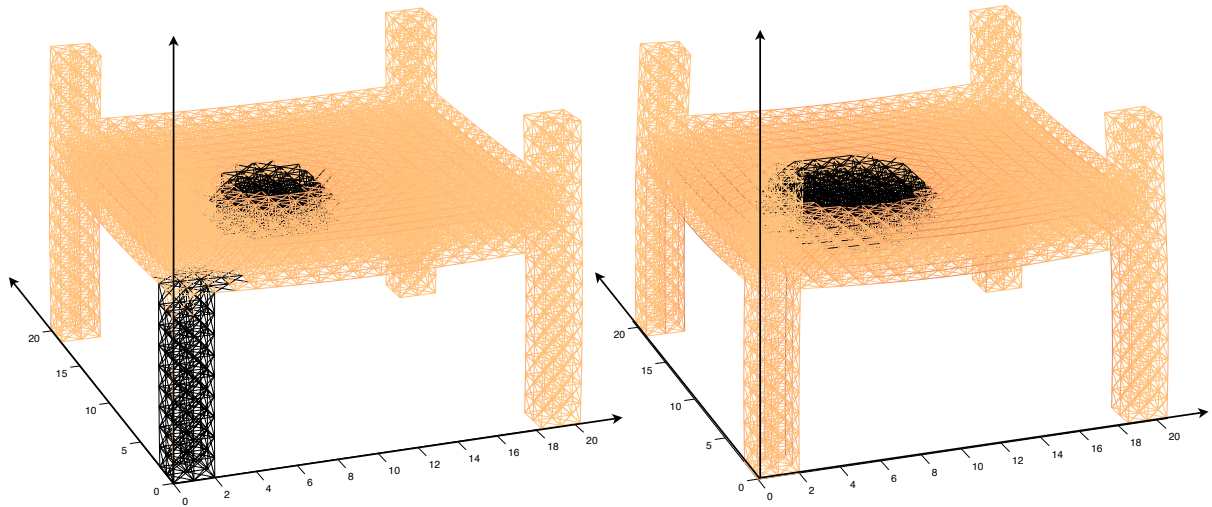


FIGURE 2 – Splitting obtained at two different stages of the simulation performed by the local/global reduction method (left : initiation, right : unstable propagation). Dark bars are connected to at least one fully resolved node. As time increases, the degrees of freedom for which no reduction is performed localises in the region of failure.

At the end of a time step, the Ritz basis is enriched in order to take into account the information provided by the local iterative solver. The procedure used here is simply to add the successive solutions to the initial snapshot, and sort them by a singular value computation.

### 4.3 Results

We search here the solution of the realisation of the parametric problem shown in figure (1, left). We use as a snapshot the displacement vectors obtained by solving a nearby realisation by the full-scale solver. The difference between the snapshot solution and the solution that is looked for is illustrated

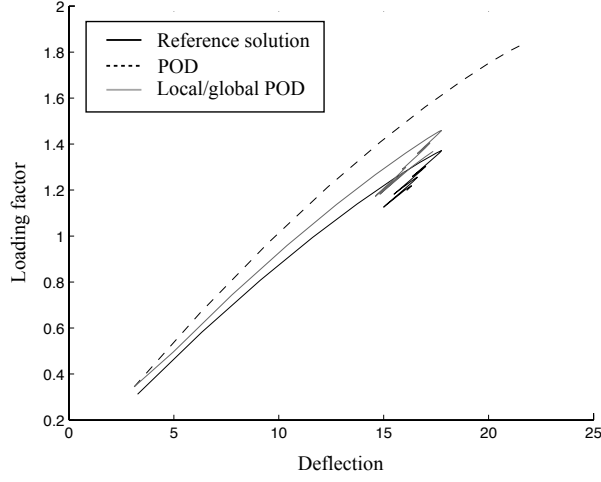


FIGURE 3 – Load/deflection curves obtained one using successively direct numerical simulation, simple proper orthogonal decomposition and local/global reduction technique. The snapshot used here is the solution of the direct numerical simulation to the nearby problem.

in figure (1). It should be noted that far away from the part of the structure where Neumann boundary conditions are applied, the solutions are qualitatively similar. Yet, they are very different within this process zone, which qualitatively justifies the local/global splitting proposed in section (4.2).

Figure (3) compares the load/deflection curves obtained in the reference case, and when using successively the basic POD strategy and the local/global reduction technique. Obviously, the POD strategy loses its relevancy very quickly, as the damage localises in the part of the structure where the damage localised during the snapshot simulation. The resulting model is too stiff, and the maximum strength of the structure is overestimated. When using the local/global approach, with the parameters described previously for the fine analysis of the damaged areas, a much more accurate load/deflection curve is obtained. Yet the peak load is still overestimated by 5%. This remaining inaccuracy is due to the fact that the information from the snapshot is not correct in the reduced zone. Therefore, global corrections of the Ritz basis are necessary.

## 5 Adaptive local/global reduction

### 5.1 "On-the-fly" global corrections of the Ritz basis based on Krylov iteration

The adaptive Ritz basis used in this paper was proposed in [8]. We recall here the basics of this algorithm. The updates are done during the Newton algorithm used to solve (3) at current time step  $t_{n+1}$ . At iteration  $i$  of the Newton solver, if the solution to the reduced problem (3) is sufficiently converged, one can evaluate the residual of the full problem (1). If the norm of this residual is larger than a certain tolerance (i.e. : an error criterion), the following linear system is considered to compute a correction to the current Ritz basis  $\underline{\underline{\mathbf{C}}}^i$  :

$$\underline{\underline{\mathbf{K}}}_T^i \delta \underline{\underline{\mathbf{U}}} = \underline{\underline{\mathbf{R}}}^i \quad (12)$$

$\underline{\underline{\mathbf{K}}}_T^i$  is an approximation of the tangent  $\underline{\underline{\mathbf{K}}}_T^i = \left. \frac{\partial \mathbf{F}_{\text{Im}}(\underline{\underline{\mathbf{U}}})}{\partial \underline{\underline{\mathbf{U}}}} \right|_{\underline{\underline{\mathbf{U}}}=\underline{\underline{\mathbf{U}}}^i}$  of the full problem, while  $\underline{\underline{\mathbf{R}}}^i$  is the residual of the initial system of equations (1) for  $\underline{\underline{\mathbf{U}}} = \underline{\underline{\mathbf{U}}}|_{t_n} + \underline{\underline{\mathbf{C}}}^i \underline{\underline{\alpha}}^i$ . An approximate solution to this problem is searched for in two supplementary spaces :

$$\delta \underline{\underline{\mathbf{U}}} = \delta \underline{\underline{\mathbf{U}}}_C + \delta \underline{\underline{\mathbf{U}}}_K \quad \text{where} \quad \begin{cases} \delta \underline{\underline{\mathbf{U}}}_C \in \text{Im}(\underline{\underline{\mathbf{C}}}^i) \\ \delta \underline{\underline{\mathbf{U}}}_K \in \text{Im}(\underline{\underline{\mathbf{C}}}^i)^{\perp_{\underline{\underline{\mathbf{K}}}_T^i}} = \text{Ker}(\underline{\underline{\mathbf{C}}}^{iT} \underline{\underline{\mathbf{K}}}_T^i) \end{cases} \quad (13)$$

$\perp_{\underline{\underline{\mathbf{K}}}_T^i}$  denotes the  $\underline{\underline{\mathbf{K}}}_T^i$ -orthogonality.  $\delta \underline{\underline{\mathbf{U}}}_C$  is the exact solution of the linearisation of reduced problem (3).  $\delta \underline{\underline{\mathbf{U}}}_K$  is a correction searched by a Krylov solver projected in space  $\text{Ker}(\underline{\underline{\mathbf{C}}}^{iT} \underline{\underline{\mathbf{K}}}_T^i)$  and set to a loose tolerance ( $10^{-1}$  is a typical value for the stopping criterion). The resulting solution, which is  $\underline{\underline{\mathbf{K}}}_T^i$ -orthogonal

to the current reduced space by construction, is used to update the Ritz basis :

$$\underline{\underline{\mathbf{c}}}^{i+1} = \left( \underline{\underline{\mathbf{c}}}^i \quad \frac{\delta \bar{\mathbf{U}}_K}{\|\delta \bar{\mathbf{U}}_K\|} \right) \quad (14)$$

Newton iteration  $i + 1$  is solved using the corrected reduced basis. Successive corrections may be performed until the tolerance on the norm of the residual of the full problem is reached.

## 5.2 Results

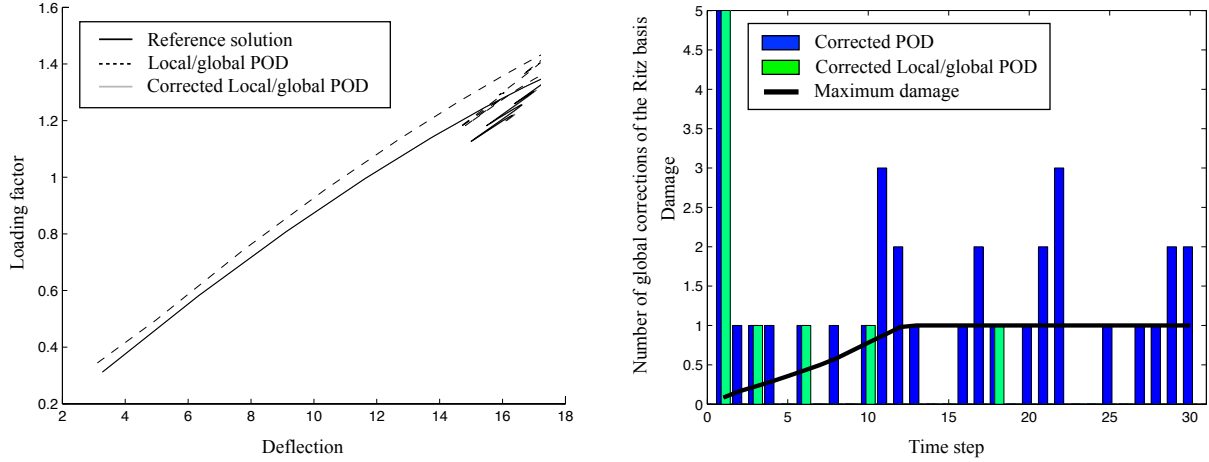


FIGURE 4 – Load/deflection curves obtained one using successively direct numerical simulation, local/global reduction technique and local/global reduction technique with "on-the-fly" global corrections (left). Number of global corrections performed when using the adaptive model order reduction on the one hand, and when using adaptive local/global model order reduction on the other hand (right).

The test case described in section the previous section is solved using successively the local/global technique on its own, the global correction algorithm on its own, and the local/global reduction technique coupled with global corrections. The target norm of the relative global residual norm is  $10^{-1}$ . The improvement in terms of error when using the global correction algorithm in the local/global approach, as opposed to the global/local method on its own is obvious in the load-displacement curve in figure (4).

More importantly, the number of relatively expensive global corrections required to achieve the target in terms of global residual norm is very low when using the local/global approach, as opposed to the one observed when using only global corrections. This effect is shown in figure (4). One can also notice that using only the global correction methods lead to an increase in the computational costs in the damage propagation phase of the analysis, which has been reported in [8]. With the local/global reduction approach, this effect disappears. The same trend has been observed in [7], where a local/global algorithm was developed in a domain decomposition framework to avoid unnecessary computations far away from the process zones. This suggests that the result obtained here is extensible, and that the number of global corrections required to achieve a given level of accuracy does not depend on the local nonlinearity (i.e. : independent on the damage state).

## 6 Conclusions and discussion

The work described in this paper focuses on decreasing the computational effort required in solving large scale damage and fracture problems, where small scale phenomena must be taken into account to accurately represent the global structural behaviour. Though most of the investigations being done to tackle this issue consider the extension of homogenisation frameworks, we followed an alternative route, choosing projection-based model order reduction as a starting point. The proposed local/global reduction technique was demonstrated on a damaging 3D frame structure, but we expect that the algorithm will carry forward to the more general case of the initiation and propagation of cracks in structures.



Two important conclusions of the work we presented in this paper are that (i) the proposed local/global model order reduction approach can significantly improve the relevancy of using a global Ritz basis to approximate the displacement field in the case of localised failure and (ii) the number of global corrections of the reduced model required to obtain a given level of accuracy is drastically reduced when excluding from the reduction the balance equations which exhibit the highest level of non-linearity.

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