



HAL
open science

A Proportional Integral Feedback for Open Channels Control through LMI Design

Valérie dos Santos, Mickael Rodrigues

► **To cite this version:**

Valérie dos Santos, Mickael Rodrigues. A Proportional Integral Feedback for Open Channels Control through LMI Design. 18th World IFAC Congress, Milano, Italia, Aug 2011, Italy. hal-00592399

HAL Id: hal-00592399

<https://hal.science/hal-00592399>

Submitted on 12 May 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A Proportional Integral Feedback for Open Channels Control through LMI Design

V. Dos Santos Martins M. Rodrigues

*LAGEP, Université de Lyon, Lyon, F-69003, France; Université
Lyon 1, CNRS, UMR 5007, LAGEP, Villeurbanne, F-69622, France;
e-mail: name@lagep.univ-lyon1.fr*

Abstract: This paper deals with the controller design for systems described by Partial Differential nonlinear Equation (PDE) of Saint-Venant. The proposed approach is based on Multi-Models concept which takes into account Linear Time Invariant models defined around a set of operating points. This method allows to describe the dynamic of this nonlinear system over a wide operating range. By the means of an Internal Model Boundary Control (IMBC), the design of a Proportional Integral (PI) feedback is performed through Linear Matrix Inequality (LMI). The method is applied to simulation and also compared to previous experimentations on a micro-channel, illustrating the new theoretical results developed in the paper.

1. INTRODUCTION

Regulation of open channels represents an economic and environment interest and many research are done in this area. Indeed, water losses in open channels, due to inefficient management and control, may be large. In order to deliver water, it is important to ensure that the water level and the flow rate in the open channel remain at certain values [21]. The difficulty of this control system is that only the gates positions are able to meet performance specifications so a specific design with boundary control laws satisfying the control objectives is required.

This problem has been previously considered in the literature using a wide variety of technics see [20, 29]. Some of them take into account the uncertainties and apply robust control approach (see [16, 17] e.g.). Studying directly the nonlinear dynamics is also possible as in [30, 15, 9]. Recent approaches consider the distributed feature of the system. Using the Riemann coordinates approach on the Saint-Venant equations, stability results are given in [13] for a system of two conservation laws, and for system of larger dimension. Lyapunov technics have been used in [5, 9].

In practice, process industries as chemical, water treatment processes are characterized by complex processes which often operate in multiple operating regimes. It is often difficult to obtain nonlinear models that accurately describe plants in all regimes. Also, considerable effort is required for development of nonlinear models. An attractive alternative to nonlinear technique is to use a multi-linear model strategy. Multi-linear models methods are based on the partitioning of the operating range of a system into separate regions and applying local linear models to each region [22]. The Multi-Models structure is well adapted for nonlinear systems because this structure allows to determine a set of linear models defined around some predefined operating points. Each local model (sub-model) is defined as a LTI dynamic system defined for a specific operating point. The Multi-Models philosophy is based on weighting functions which ensure the transi-

tion between the different locals models. These functions represent the degree of validity of each local models and it depends on the system inputs and outputs which vary with time. The multi-model approach has been often used in recent years for modelling and control of nonlinear systems [27, 24, 2] and for fault diagnosis [3, 11, 26]. In the Multi-Models concept, some authors speak about gain scheduling strategy which is well detailed in [14] or for interpolated controllers or switching controllers [23, 1].

The use of Multi-Models representation for stability study of systems described by nonlinear PDE is not present in the literature. More generally, common approaches are based on a finite dimensional approximation of the nonlinear PDE and adaptive control. The stability for such systems is still an open problem. In this paper, an analysis of the stability of the nonlinear PDE of Saint-Venant is proposed by the use of the Multi-Models and IMBC [28] structures. A previous work from the authors has been published with the use of an Integral control. The goal of this paper is to extend the theory with the use of a PI Control. The stability in Multi-Models framework is often performed by Linear Matrix Inequality (LMI) due to the effectiveness for calculating a gain solution for multiple models [18, 25].

The paper is organized as follows: firstly, the Saint-Venant equations are presented as well as the control problem. The Internal Model Boundary Control is explained and the physical constraints are given. Secondly, the linearized systems are developed around equilibrium sets which depend on the space variable. Their insertion into the LMI formalism are also described into this second part. The third part of the paper is dedicated to the design of the feedback gain by LMI which ensures the stability of the system: a Proportional Integral (PI) controller is implemented using a LMI approach and the local stability of each systems. The last section is dedicated to the simulations and comparison with previous experimentations. Comparisons between initial experimental results using a PI-controller (done some years ago) and simulations

with the new PI-controller using the LMI gain calculated through this paper, are realized.

2. PROBLEM STATEMENT ABOUT CHANNEL REGULATION

Let us consider the following class of water channels represented on the figure (1), i.e. a reach of an open channel delimited by underflow and/or overflow gates where:

- $Q(x, t)$ is the water flow rate,
- $Z(x, t)$ is the height of water channel,
- L is the length of the reach taken between the upstream $x = 0$ and the downstream $x = L$,
- $U_0(t), U_L(t)$ are the opening of the gates at upstream and downstream.

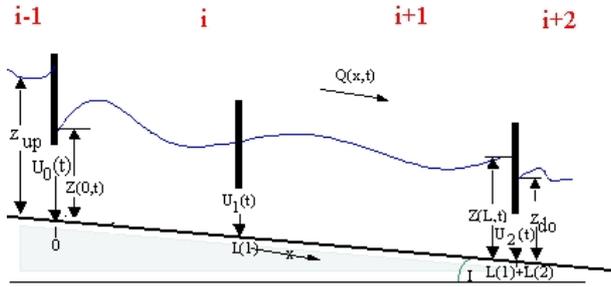


Fig. 1. Channel scheme: two underflow gates

The regulation problem concerns the stabilization of the water flow rate and/or the height of the water around an equilibrium for a reach denoted by $(z_e(x), q_e(x))$. A linear model with variable coefficients can be deduced from the nonlinear PDE, in order to describe the variation of the water level and flow on an open channel. Let recall these models of [6].

2.1 A model of a reach

We suppose that the channel has a sufficient length L such that we can consider that the lateral movement is uniform. Nonlinear PDE of de Saint-Venant which describe the channel are the following [12, 19]:

$$\partial_t Z = -\partial_x \frac{Q}{b}, \quad (1)$$

$$\partial_t Q = -\partial_x \left(\frac{Q^2}{bZ} + \frac{1}{2}gbZ^2 \right) + gbZ(I - J), \quad (2)$$

$$Z_0(x) = Z(x, 0), \quad Q_0(x) = Q(x, 0), \quad (3)$$

where I is the slope, b is the channel width, g is the gravity constant.

J is the friction slope from the formula of Manning-Strickler and R is the hydraulic radius. J and R are defined such that:

$$J = \frac{n^2 Q^2}{(bZ)^2 R^{4/3}}, \quad R = \frac{bZ}{b + 2Z}. \quad (4)$$

The different limits conditions bring us to consider 2 control cases (mono or multi variable control). Here, the control of two underflow gates is considered.

Multi-variable control :

The equation of the upstream condition of the reach ($x = x_{up}$) is given by $Q(x_{up}, t) = U_{up}(t)\Psi_1(Z(x_{up}, t))$, with $\Psi_1(Z) = K_1\sqrt{2g(z_{up} - Z)}$. The other control at downstream of the reach, i.e. in $x = x_{do}$ (Fig. 1) is given by: $Q(x_{do}, t) = U_{do}(t)\Psi_3(Z(x_{do}, t))$, where $\Psi_3(Z) = K_2\sqrt{2g(Z - z_{do})}$ and $U_{do}(t)$ is the downstream control of the reach, z_{do} is the water height on downstream of the gate (cf. figure 1).

Remark 1. Upstream and downstream depend of the considered reach, it is the same thing for abscissa and gates.

2.2 A regulation model

An equilibrium point of the system verifies the following equations:

$$\begin{aligned} \partial_x q_e &= 0 \\ \partial_x z_e &= gbz_e \frac{I + J_e + \frac{4}{3}J_e \frac{1}{1+2z_e/b}}{gbz_e - q_e^2/bz_e^2}, \end{aligned} \quad (5)$$

Remark 2. The fluvial case is considered and it follows that:

$$z_e > \sqrt[3]{q_e^2/(gb^2)}. \quad (6)$$

Let denote that q_e is constant but that z_e depends of variable space. The linearized model around an equilibrium point $(z_e(t), q_e(t))^t$ is, with

$\xi(x, t) = (z(x, t), q(x, t))^t, \forall t > 0, x \in \Omega =]0, L[$
the linearized state variables:

$$\partial_t \xi(x, t) = A_1(x)\partial_x \xi(x, t) + A_2(x)\xi(x, t) \quad (7)$$

$$\xi(x, 0) = \xi_0(x)$$

$$q(x_{up}) = u_{up,e}\partial_z \Psi_1(z_e(x_{up}))z(x_{up}) + u_{up}\Psi_1(z_e(x_{up})),$$

$$q(x_{do}) = u_{do,e}\partial_z \Psi_3(z_e(x_{do}))z(x_{do}) + u_{do}\Psi_3(z_e(x_{do}))$$

$\forall t > 0$, where $u_{up,e}, u_{do,e}$ are the openings gates for the upstream and downstream at the equilibrium and $u_{up}(t), u_{do}(t)$ are the variations of these openings gates to be controlled. The matrices $A_1(x), A_2(x)$ are given by:

$$A_1 = \begin{pmatrix} 0 & -a_1 \\ -a_2 & -a_3 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & 0 \\ a_4 & -a_5 \end{pmatrix}, \quad (8)$$

with $a_1(x) = 1/b, a_2(x) = gbz_e(x) - \frac{q_e^2}{bz_e^2(x)}, a_3(x) = \frac{2q_e}{bz_e(x)}, a_4(x) = \frac{2gbJ_e(x)z_e(x)}{q_e}, a_5(x) = gb(I + J_e(x) + \frac{4}{3}\frac{J_e(x)}{1+2z_e(x)/b})$.

The control problem is to find the variations of $u_{up}(t)$ at extremity $x = x_{up}$ and $u_{do}(t)$ at the extremity $x = x_{do}$ of the reach such that downstream water level, $z(x_{do}, t) = z(L, t)$ (measured variables), track a reference signal $r(t)$. The reference signal $r(t)$ is chosen for all cases or constant or non-persistent (a stable step answer of a non-oscillatory system).

In this paper, the control scheme based on the Internal Model Boundary Control (IMBC) [8, 10] is adopted as illustrated on figure (2). This control strategy integrates the process model in real time and allows to regulate the water height in all the points of the channel by taking into account the error between the model and the system.

- M_f is the linear filtering model of finite dimension.

- M_r is the pursuit model which allows to set a dynamic in regards of the fixed instruction $r(t)$.

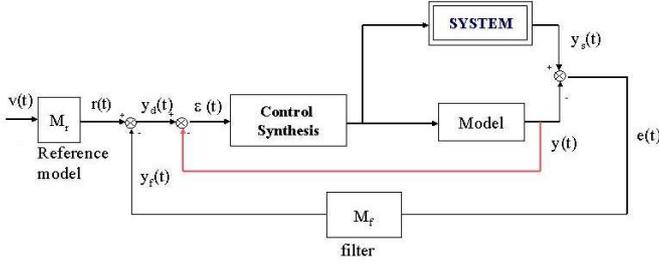


Fig. 2. IMBC structure: Internal Model Boundary Control

In order to control the water level over a wide operating range, we consider a set of models established around judicious operating points: each model is an approximation of the process in a small interval of the operating range and it should be activated to synthesize a control on this interval. The idea is to construct a set of predefined models in order to control the system over all the operating range.

2.3 A Multi-Models representation

The Multi-Models structure [26] allows to control the system over a wide operating range because it takes into account the different sub-models which can be activated under different operating regimes [22, 26]. The representation of Saint-Venant's PDE around N operating points by the Multi-Models approach is defined by the following equations:

$$\partial_t \xi(x, t) = \sum_{i=1}^N \mu_i(\zeta(t)) \mathcal{A}_i(x) \xi(x, t), \forall t > 0, x \in \Omega \quad (9)$$

$$\mathcal{A}_i(x) = A_{1,i}(x) \partial_x + A_{2,i}(x) \quad (10)$$

$$\xi_0(x) = \xi(x, 0)$$

$$F_b \xi(t) = B_b u(t) \text{ on } \Gamma = \partial\Omega, \forall t > 0 \quad (11)$$

- $\mathcal{A}_i(x)$ is the operator which corresponds to the i^{th} equilibrium state.
- $\zeta(t)$ is a function depending of some decision variables directly linked with the mesurables states variables and eventually to the input.
- $\mu_i(\zeta(t))$ is the weighting functions which determines the sub-model for the control law synthesis depending of the output height of the process z_L .

The equation (9) describes the system dynamic in open loop. In this representation, the state vector $\xi(x, t)$ is not explicitly linked with the boundary control. In order to design an output feedback and to study the stability in closed loop, an operator D of distribution of the boundary control is introduced, it is a bounded operator such that $Du \in \text{Ker}(A)$, [8]:

$$\xi(x, t) = \varphi(x, t) + Du(t). \quad (12)$$

This operator is naturally null in the domain of $A(x)$ as it is active only on the boundary of the domain. This change of variables allows to get a Kalman representation of the system:

$$\partial_t \varphi(x, t) = \sum_{i=1}^N \mu_i(\zeta(t)) [\mathcal{A}_i(x) \varphi(x, t) - D\dot{u}] \quad (13)$$

$$\varphi(x, 0) = \varphi_0(x) = \xi_0(x) - Du(0). \quad (14)$$

A Multi-Model approach can be developed and made possible the study of the stability by the second theory of Lyapunov.

In the following paragraph, we will focus on the synthesis of a control law by LMI technics. An output feedback is considered under a hypothesis of an integral control, so as to do a synthesis of a gain by *LMI* which ensures the stability of the system.

3. STABILITY STUDY BY LMI

In this part, the closed loop structure (Fig. 2) is studied under a proportional integral feedback. The pursuit model (M_r) and filtering model (M_f) are not considered.

3.1 Closed-loop structure for a Proportional-Integral feedback

For a control with an output feedback, K_i and K_p are defined as the gains, $u(t) = K_p \varepsilon(t) + K_i \int \varepsilon(t) dt$, it follows that [8]:

$$\varepsilon(t) = r(t) - y(t) \quad (15)$$

$$u(t) = K_p [r(t) - y(t)] + K_i \int [r(\tau) - y(\tau)] d\tau \quad (16)$$

with $y(x, t) = C\xi(x, t)$ and equation (12), one deduce:

$$y(x, t) = C\varphi(x, t) + CDu(t) \quad (17)$$

To clarify equations, the index i of the i -th equilibrium is omitted in the following (e.g. \mathcal{A} replace \mathcal{A}_i). By replacing $y(x, t)$ into the control equation:

$$\begin{aligned} u &= K_p [r - C\varphi - CDu] + K_i \int [r - C\varphi - CDu] d\tau \\ \Rightarrow \dot{u} &= K_p [\dot{r} - C\dot{\varphi} - CD\dot{u}] + K_i [r - C\varphi - CDu] \\ &= K_p [\dot{r} - C\mathcal{A}\varphi] + K_i [r - C\varphi - CDu] \end{aligned}$$

using the change of variables (12) and the fact that $\text{Im}(D) \subset \text{Ker}(A)$ (See [8]). Then \dot{u} is introduced into the equation (13) and, the closed-loop expression is then

$$\begin{aligned} \partial_t \varphi(x, t) &= \sum_{i=1}^N \mu_i(\zeta(t)) [DK_i (CDu(t) - r(t)) - DK_p \dot{r}(t) \\ &\quad + (\mathcal{A}_i(x) + DK_p C\mathcal{A}_i + DK_i C) \varphi(x, t)] \end{aligned} \quad (18)$$

Let define:

$$\widetilde{K}_{in} = DK_i \quad \widetilde{K}_{pr} = DK_p \quad (19)$$

The equation (18) can be written as

$$\begin{aligned} \partial_t \varphi &= \sum_{i=1}^N \mu_i(\zeta) [(\mathcal{A}_i + \widetilde{K}_{pr} C\mathcal{A}_i + \widetilde{K}_{in} C) \varphi \\ &\quad + \widetilde{K}_{in} (CDu - r) - \widetilde{K}_{pr} \dot{r}]. \end{aligned} \quad (20)$$

The conditions which ensure the stability are ensured by using a quadratic Lyapunov function [25, 4], in order to guarantee the convergence of the water height to the reference $r(t)$, over the widest operating range.

3.2 Stability study with a quadratic Lyapunov function

Let us consider:

$$V(\varphi(x, t), t) = \varphi^T(x, t)P\varphi(x, t). \quad (21)$$

The Multi-Models representation of the linearized PDE of Saint-Venant defined by equation (20) is asymptotically stable if there exists a matrix $P > 0$ such that¹:

$$\dot{V}(\varphi, t) = \dot{\varphi}^T P\varphi + \varphi^T P\dot{\varphi} < 0, \quad (22)$$

Then, it follows this inequality

$$\left[\sum_{i=1}^N \mu_i(\zeta) \left[\left(\mathcal{A}_i + \widetilde{K}_{pr} C\mathcal{A}_i + \widetilde{K}_{in} C \right) \varphi \right. \right. \quad (23)$$

$$\left. \left. + \widetilde{K}_{in} (CDu - r) - \widetilde{K}_{pr}\dot{r} \right] \right]^T P\varphi + \varphi^T P \left[\sum_{i=1}^N \mu_i(\zeta) \left[\left(\mathcal{A}_i + \widetilde{K}_{pr} C\mathcal{A}_i + \widetilde{K}_{in} C \right) \varphi \right. \right. \quad (24)$$

$$\left. \left. + \widetilde{K}_{in} (CDu - r) - \widetilde{K}_{pr}\dot{r} \right] \right] < 0 \quad (25)$$

The development of this inequality leads us to consider an inequality for each i such that:

$$\begin{aligned} & \varphi^T \left[\mathcal{A}_i + \widetilde{K}_{pr} C\mathcal{A}_i + \widetilde{K}_{in} C \right]^T P\varphi \\ & + \varphi^T P \left[\mathcal{A}_i + \widetilde{K}_{pr} C\mathcal{A}_i + \widetilde{K}_{in} C \right] \varphi \\ & + [\widetilde{K}_{in}(CDu - r)]^T P\varphi + \varphi^T P[\widetilde{K}_{in}(CDu - r)] \\ & - (\widetilde{K}_{pr}\dot{r})^T P\varphi - \varphi^T P(\widetilde{K}_{pr}\dot{r}) < 0 \end{aligned} \quad (26)$$

Remark 3. In a first approach, the particular case $K_{prop} = K_{int}$ is taken and so one get $\widetilde{K}_{pr} = \widetilde{K}_{in} = \widetilde{K}$. Then equation (26) become:

$$\begin{aligned} & \varphi^T \left[\mathcal{A}_i + \widetilde{K} C\mathcal{A}_i + \widetilde{K} C \right]^T P\varphi + [\widetilde{K}(CDu - r)]^T P\varphi \\ & + \varphi^T P \left[\mathcal{A}_i + \widetilde{K} C\mathcal{A}_i + \widetilde{K} C \right] \varphi + \varphi^T P[\widetilde{K}(CDu - r)] \\ & - (\widetilde{K}\dot{r})^T P\varphi - \varphi^T P(\widetilde{K}\dot{r}) < 0 \end{aligned} \quad (27)$$

In the inequality (27), which defines the stability condition of the system, the control parameter u appears in this inequality and it is a difficulty for the design of the gain \widetilde{K} . Let us consider the following equality deduced from (17):

$$CDu(t) - r(t) = C\xi(x, t) - r(t) - C\varphi(x, t) \quad (28)$$

Proposition 1. If there exists a matrix P positive definite, a matrix W and a scalar α such that the following statements hold true:

- $\varphi^T P\widetilde{K}(CDu(t) - r(t)) \leq \alpha\varphi^T P\widetilde{K}C\varphi$, (29)
- $\mathcal{A}_i^T P + P\mathcal{A}_i + WC_i + C_i^T W^T < 0$, with $\widetilde{K} = P^{-1}W$ and with $C_i = C\mathcal{A}_i + (1 + \alpha)C$
- r is constant by piecewise, then the system (13) with a proportional integral control input (16) is stable. ■

¹ We suppose that $\partial_t \psi = \dot{\psi}$ whatever the function ψ .

Proof 1. Only a sketch of the proof is given here. For the integral case, one can found it in [7].

Let consider the quadratic Lyapunov function

$$V(\varphi(x, t), t) = \varphi^T(x, t)P\varphi(x, t)$$

then one can wrote $\dot{V}(t) < 0$ such that (27) can be upper bounded. Indeed, the use of inequality (29) implies that (the same is done for the transposed expression)

$$\begin{aligned} & \varphi^T P \left[\mathcal{A}_i + \widetilde{K} C\mathcal{A}_i + \widetilde{K} C \right] \varphi + \varphi^T P[\widetilde{K}(CDu - r)] \\ & - \varphi^T P(\widetilde{K}\dot{r}) < \varphi^T P \left[\mathcal{A}_i + \widetilde{K} C\mathcal{A}_i + (1 + \alpha)\widetilde{K} C \right] \varphi \\ & - \varphi^T P(\widetilde{K}\dot{r}) < \varphi^T P \left[\mathcal{A}_i + \widetilde{K} C \right] \\ & = \varphi^T [P\mathcal{A}_i + WC_i] \varphi < 0 \end{aligned} \quad (30)$$

with $C_i = C\mathcal{A}_i + (1 + \alpha)C$, $W = P\widetilde{K}$ and using the condition a) firstly, the third c) and the second condition b) of the proposition.

The gain \widetilde{K} has been implemented into the model of simulations so as to verify the stability of the system. The results have been obtained for a single reach with two underflow gates. The aim is to compare the simulations and experimental curves obtained with this method and the ones obtained experimentally by Dos Santos Martins in this works [8, 10].

4. SIMULATIONS RESULTS

Firstly, let describe the benchmark used for the simulations and the experimentations, described in the second and third subsections respectively.

4.1 Configuration and data of the channel

For this study, the following set of parameters from the practical Valence's channel (Fig.3) is considered where the data are defined such that:

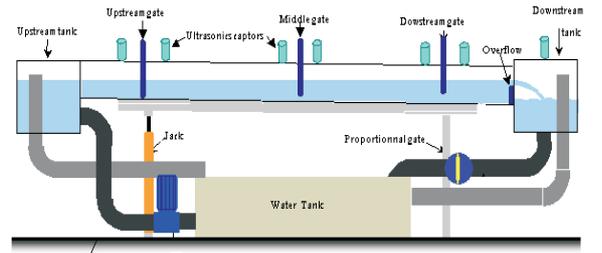


Fig. 3. Valence channel scheme

- $L = 64.5dm$ is the length of the channel,
- $b = 1dm$ is the width of the channel,
- $N = 20$ is the number of the discretized points,
- Z_L is the water height to regulate, such that $z_{min} < Z < 2 dm$; where z_{min} is the minimum critical fluvial water level, 2dm is the canal height.

In this single reach with two gates, the regulation of the water height Z_L at $x = L$, is done by controlling the openings $U_0(t)$ and $U_L(t)$ of the gates at upstream and

downstream respectively: it is a multi-variable control (cf Fig. (1)).

The equilibria profiles have been chosen such that the calculated control law from the local models can be efficient over all the operating range of the water height [8]. Let notice that it has been experimentally verified that a local model is valid around $\pm 20\%$ of an equilibrium profile. In order to assign references which are included between $0.6dm$ and $2dm$, the operating points at $x = 0$ are the following:

Table 1. Initial set points for the simulations

Simulations		
$z_{e1}(x = 0)$	$z_{e2}(x = 0)$	$z_{e3}(x = 0)$
$0.625dm$	$0.9375dm$	$1.40625dm$

In this application, the weighting function $\mu_i(\zeta(t))$ is equal to 1 if the output's height is included into the validity domain of the model and 0 in the other case for each operating state. The parameter $\zeta(t)$ exclusively depends on the output which is the only one variable of decision in this precise case.

4.2 Simulations

These results are obtained from an IMB Control and a Multi-Models approach with a LMI gain previously calculated. The figure (4) shows that the output converges to the reference even if this one strongly varies (variations > 100%). The reference tracks a slow dynamic and one can see that the convergence of the output is good.

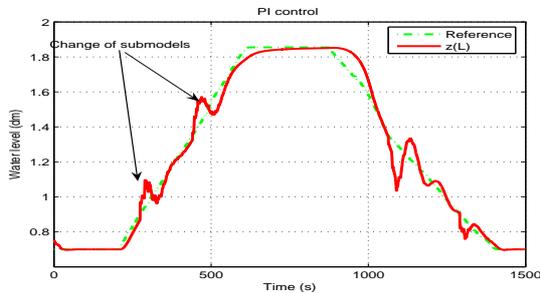


Fig. 4. Variations of the reference along the valued domain

The curves that describe the upstream and downstream gates openings of the reach are given by the figure (5). The convergence of the output to the reference is ensured even when the reference is decreasing or increasing.

The following simulations compare an integral and a PI controllers which gains have been calculated by the LMI approach, Fig (6)-(7). The PI performs better than the integral control as expected.

Next simulations are a comparison between simulations using the theoretical gain obtains through LMI approach (I and PI control) with the first tests realized some years ago by [10], using an experimental Multi-Models gain, without any theoretical study. The figure (8) represents the dynamic evolution of the simulated system and the experimental data obtained with the quite same references.

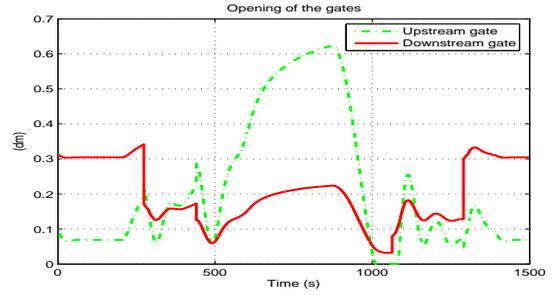


Fig. 5. Gates opening

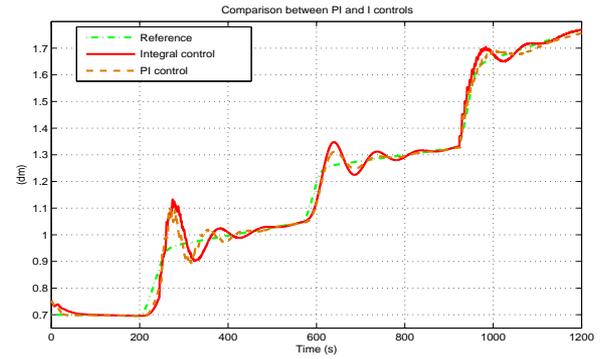


Fig. 6. Comparison of a PI and an integral which gains are simulated with the LMI approach

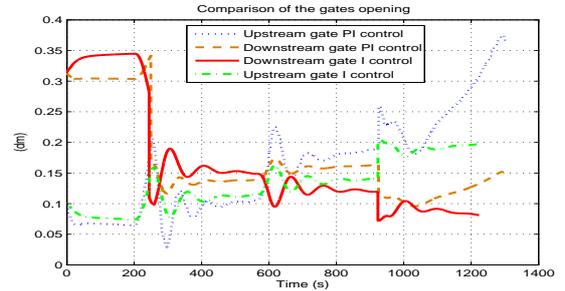


Fig. 7. Gates opening

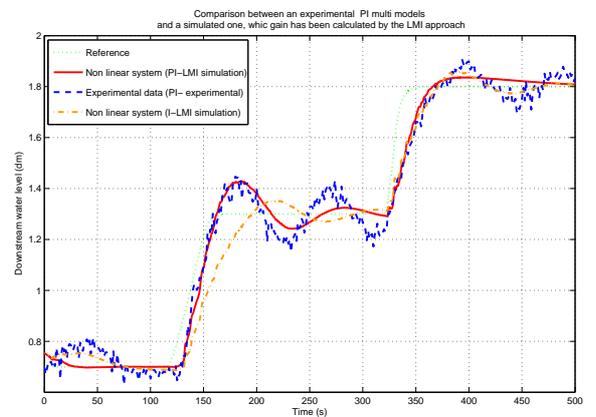


Fig. 8. Comparison of the downstream water level measured in the Valence channel with the first Multi-Models approach in 2004 versus the simulated one with the LMI approach

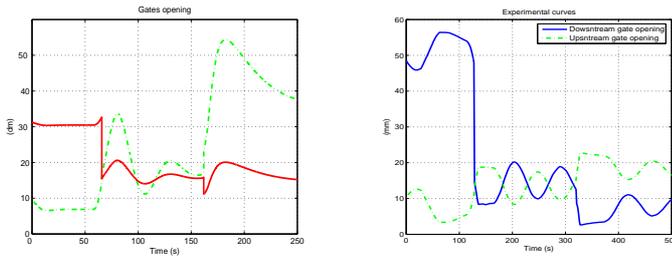


Fig. 9. Gates opening

The figure (9) compares the dynamic of the gate openings.

This study is based on the previous works of Rodrigues [26] and from works of Dos Santos [8, 10].

Experimentations have been realized into the Valence channel (Fig.3) with a Multi-Models approach and a gain calculated via the LMI approach. For these experimentations, the wide range of the accessible water level is attempt for an integral controller, but authors did not get time to implement their PI controller. For the integral case, relevant experimentations have been included into an article, which is actually under revision.

5. CONCLUSION

The first results trough the use of multi-models approach dedicated to irrigation channels control, under an IMBC structure, have been realized some years ago [8]. These good experimental results, but without stabilizable theoretical approach, were obtained. In this paper, the authors have formalized and extended their LMI approach of this regulation problem by the synthesis of a PI Controller. Simulations have shown the improvements realized towards the initial multi-models approach trough this new PI feedback controller designed by LMI.

REFERENCES

- [1] Aberkane, S., Ponsart, J.-C., Rodrigues, M., Sauter, D., 2008. Output feedback control of a class of stochastic hybrid systems. *Automatica* 44 (5), 1325–1332.
- [2] Athans, M., Fekri, S., Pascoal, A., 2005. Issues on robust adaptive feedback control. In: Proc. 16th IFAC World Congress, Prague, Czech Republic.
- [3] Bhagwat, A., Srinivasan, R., Krishnaswamy, P. R., 2003. Multi-linear model-based fault detection during process transitions. *Chemical Engineering Science* 58, 1649–1670.
- [4] Chadli, M., Maquin, D., Ragot, J., 2002. Output stabilisation in multiple model approach. In Proc. of the IEEE Conference on ControlApplication (CCA'02), Glasgow, Scotland, 1315–1320.
- [5] Coron, J. M., d'Andréa Novel, B., Bastin, G., 2007. A strict lyapunov function for boundary control of hyperbolic systems of conservation laws. *IEEE Transactions on Automatic Control* 52(1), 2–11.
- [6] de Saint-Venant, A. B., 1871. Théorie du mouvement non permanent des eaux avec applications aux crues des rivières et à l'introduction des marées dans leur lit. *Comptes rendus de l'Académie des Sciences de Paris* 73, 148–154, 237–240.
- [7] Diagne M., Dos Santos V., Rodrigues M., 2010. Une approche Multi-modèles des équations de Saint-Venant : une analyse de la stabilité par techniques LMI. In: Proc. of 6th IEEE Conférence Internationale Francophone d'Automatique (CIFA 2010), Nancy, France.
- [8] Dos Santos, V., 2004. Contrôle frontière par modèle interne de systèmes hyperboliques : Application à la régulation de canaux d'irrigation. Phd thesis, Université d'Orléans.
- [9] Dos Santos, V., Prieur, C., 2008. Boundary control of open channels with numerical and experimental validations. *IEEE Transactions on Control Systems Technology* 16, 1252–1264.
- [10] Dos Santos, V., Toure, Y., Mendes, E., Courtial, E., 2005. Multivariable boundary control approach by internal model, applied to irrigations canals regulation. In: Proc. 16th IFAC World Congress, Prague, Czech Republic.
- [11] Gatzke, E., Doyle, F., 2002. Use of multiple models and qualitative knowledge for on-line moving horizon disturbance estimation and fault diagnosis. *Journal of Process Control* 12, 339–352.
- [12] Georges, 2002. *Automatique pour la Gestion des Ressources en Eau*. Edts IC2, Systèmes automatisés, Hermès.
- [13] Greenberg, J.-M., Li, T., 1984. The effect of boundary damping for the quasilinear wave equations. *Journal of Differential Equations* 52, 66–75.
- [14] Leith, D. J., Leithead, W. E., 2000. Survey of gain-scheduling analysis and design. *International Journal of Control* 73 (11), 1001–1025.
- [15] Litrico, X., Fromion, V., Baume, J.-P., Arranja, C., Rijo, M., 2005. Experimental validation of a methodology to control irrigation canals based on saint-venant equations. *Control Engineering Practice* 13, 1425–1437.
- [16] Litrico, X., Georges, D., 1999. Robust continuous-time and discrete-time flow control of a dam-river system: (i) modelling. *J. of Applied Mathematical Modelling* 23(11), 809–827.
- [17] Litrico, X., Georges, D., 1999. Robust continuous-time and discrete-time flow control of a dam-river system: (ii) controller design. *J. of Applied Mathematical Modelling* 23(11), 829–846.
- [18] Lopez-Toribio, C., Patton, R., Daley, S., 1999. A multiple-model approach to fault-tolerant control using takagi-sugeno fuzzy modelling: real application to an induction motor drive system. In: European Control Conference, ECC 99, Karlsruhe.
- [19] Malaterre, P., 2003. Le contrôle automatique des canaux d'irrigation : Etat de l'art et perspectives. In: Proc. of Colloque Automatique et Agronomie, Montpellier, France.
- [20] Malaterre, P.-O., Rogers, D., Schuurmans, J., 1998. Classification of canal control algorithms. *J. of Irrigation and Drainage Engineering* 124(1), 3–10.
- [21] Mareels, I., Weyer, E., Ooi, S., Cantoni, M., Li, Y., Nair, G., 2005. Systems engineering for irrigation systems: Successes and challenges. *Annual Reviews in Control* 29(2), 191–204.
- [22] Murray-Smith, R., Johansen, T., 1997. *Multiple Model Approaches to Modelling and Control*. Taylor and Francis.
- [23] Narendra, K., Balakrishnan, J., Kernal, M., 1995. Adaptation and learning using multiple models, switching and tuning. *IEEE Contr. Syst. Mag.*, 37–51.
- [24] Porfirio, C. R., Neito, E. A., Odloak, D., 2003. Multi-model predictive control of an industrial c3/c4 splitter. *Control Engineering Practice* 11, 765–779.
- [25] Rodrigues, M., Theilliol, D., Aberkane, S., Sauter, D., 2007. Fault tolerant control design for polytopic lpv systems. *Int. Journal. Applied Math. Comput. Sciences* 17 (1), 27–37.
- [26] Rodrigues, M., Theilliol, D., Adam-Medina, M., Sauter, D., 2008. A fault detection and isolation scheme for industrial systems based on multiple operating models. *Control Engineering Practice* 16, 225–239.
- [27] Theilliol, D., Sauter, D., Ponsart, J., 2003. A multiple model based approach for Fault Tolerant Control in nonlinear systems. In: Proc. IFAC Symposium Safeprocess, Washington .D.C, USA, CD-Rom.
- [28] Toure, Y., Jossierand, L., 2004. Semigroup formalism and internal model control for a heat exchanger. *Revue electronique des sciences et technologies de l'automatique, e-STA*, vol. 1, no. 2.
- [29] Weyer, E., 2002. Decentralised pi controller of an open water channel. 15th IFAC world congress, Barcelona, Spain.
- [30] Zaccarian, L., Li, Y., Weyer, E., Cantoni, M., Teel, A. R., 2007. Anti-windup for marginally stable plants and its application to open water channel control systems. *Control Engineering Practice* 15(2), 261–272.