# CONNECTING THEORETICAL FRAMEWORKS: THE TELMA PERSPECTIVE

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*ABSTRACT: In this text, we report on a research project developed within the European research team TELMA (Technology Enhanced Learning in MAthematics) of the Kaleidoscope network of excellence created in 2004. We describe the conceptual and methodological tools we have progressively built for allowing productive research collaboration and overcoming the difficulties resulting from the diversity and heterogeneity of our respective theoretical backgrounds. We also show how these tools have contributed to give us a clearer idea of what is needed in terms of theoretical connection and integration in mathematics education, of what seems accessible today and how.*

## INTRODUCTION

Research in mathematics education does not obey a unified paradigm. On the contrary, it often appears as a field broken into a multiplicity of local communities that develop more or less independently, generating an overflow of conceptual and methodological tools poorly connected. In spite of the multiplicity of international conferences and groups, in spite of evident common trends, exchanges remain often superficial. Even if anyone understands the necessary sensitivity of the educational domain to social and cultural contexts, this situation conveys the negative image of an immature scientific field and does not encourage at considering the results obtained in it as convincing and valuable. Such a situation appears more and more problematic, increasing the attention paid to issues of comparison and connection between theoretical frames, as illustrated for instance by two recent issues of the Zentralblatt für Didaktik der Mathematik (ZDM 2005 Vol. 37(6), ZDM 2006 Vol. 38(1)), the chapter by Cobb in the second NCTM Handbook of Research on Teaching and Learning Mathematics (Cobb, 2007) or the existence of a working group especially devoted to these issues at the two last conferences of the European Association for Research in Mathematics Education (Bosch, 2006). Research concerning digital technologies does not escape this rule as evidenced for instance by the meta-study (Lagrange & al., 2003) but, due to the normal ambition of artefact designers to develop tools not restricted to one particular local community and able to

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migrate from one educational context to another one, researchers in that area are perhaps more sensitive to the problems raised by the current fragmentation of the field.

Within the European research team TELMA, we faced the difficulties generated by this situation when exploring possibilities for collaboration between the six different teams involved. In this paper, we report on the TELMA enterprise which began four years ago and led us to develop specific tools for overcoming these difficulties. We first briefly present the TELMA structure then focus on the conceptual and methodological tools that we have developed. After describing these, we try to show how these tools have contributed to give us a clearer idea of what is needed in terms of theoretical connection and integration in mathematics education, of what seems accessible today and how.

## TELMA: AIMS, CHARACTERISTICS AND FIRST STEPS

TELMA (Technology Enhanced Learning in Mathematics) is a sub-structure of the Kaleidoscope European Network of Excellence. It includes six European teams from four different countries (England, France, Greece and Italy), and its main aims is to promote networking and integration among such teams for favouring the development of collaborative research and development projects on the teaching and learning of mathematics with digital technologies. The TELMA teams have a long experience in that area but they live in different educational contexts, the digital technologies they have developed are diverse, ranging from half baked microworlds to diagnostic and remedial tools, and the theoretical frameworks they rely on are also quite diverse. A first attempt made for identifying these (ITD, 2004) showed the existence of at least eight main theoretical frameworks: theory of didactical situations, anthropological theory of didactics, activity theory, instrumental approach, theory of semiotic mediation, social semiotics, socio-constructivism and constructionism, not to mention the theoretical approaches referred to in the AIED community and mobilized in the design of digital artefacts (Grandbastien & Labat, 2006).

For facilitating research collaboration, TELMA teams decided first to structure their collaborative work regarding the design and use of digital technologies around two main issues: representations and contexts, and to produce a description of each team according to common categories: main research aims, theoretical frameworks of references, digital tools designed and used… in order to make visible similarities and differences. As mentioned above, the descriptions produced evidenced a striking diversity in terms of theoretical frameworks, language and concepts used, and the difficulty we had to understand up to what point and how these differences affected our respective research and perspectives on the issues at stake. The notion of didactical functionality (see below) was then introduced as a reading key, general enough and based on elements relevant for all the teams, to be used to describe and compare frameworks. It was also decided to ask each team to select some few publications it considered the most appropriate for promoting mutual understanding and to work on these. Soon enough we experienced the limitation of such an enterprise: the reading of selected papers gave us only a rather superficial view of the exact role played by theoretical frames in our respective research projects. Theoretical frames were of course evoked or even discussed but their links with the details of the actual research work were missing or remained fuzzy. The idea of developing a specific methodology: the cross-experimentation methodology, presented in the next part, emerged from the awareness of these limitations.

## TELMA CONSTRUCTS

The first construct introduced in TELMA was the notion of didactical functionality. It was seen as a reading key as mentioned above and a means to link theoretical reflection and practice, helping us approach theories in more operational terms, beyond the declarative level dominating in the set of selected papers.

### The notion of didactical functionality

The notion of *didactical functionality* (Cerulli et al, 2005) indeed individuates three different dimensions to be taken into account when considering a learning environment integrating one or several digital artefacts, for purpose of design or analysis of use:

- a set of features/characteristics of the considered digital artefact(s);
- one (or a few coordinated) educational goal(s);
- the modalities of use of the artefact(s) in the teaching and learning activity enacted to reach such goal(s).

These three dimensions are not independent of course: although characteristics and features of a digital tool can be identified through an *a priori* inspection, these features only become functionally meaningful when understood in relation to the educational goal for which the artefact is being used in a given context and to the modalities of its use. Nevertheless, identifying and distinguishing these dimensions helped us structure the reflection and analysis, and approach theoretical frameworks in operational terms. For progressing in the understanding of our similarities and differences, we needed then to complement this structure by appropriate descriptors or categories. This was the source of the notion of *key concern* we introduce below.

### The notion of key concern

In spite of its limitations, the analysis of selected papers carried out showed that the different teams shared evident common sensitivities (for instance common sensitivity to semiotic and instrumental issues, to the social and situated dimensions of learning processes), but they generally took these into consideration through different constructs and approaches. Retrospectively, the existence of such common sensitivities has nothing strange: even if we live in different educational cultures and have different trajectories, we are partly facing similar challenges and issues. Seeing theoretical frameworks and constructs as tools that we build for understanding and addressing challenges and issues, we thus conjectured that, for comparing and identifying possible productive connections between our respective theoretical frameworks and concepts, a good strategy could be to approach theories and concepts through the main sensitivities and needs they try to respond to. For tracing these common sensitivities and needs, we needed a common language not dependent on some particular theoretical approach. This was the source of the notion of key concern. A set of key concerns was thus attached to each dimension of the notion of didactical functionality, expressing the main sensitivities evidenced by the analysis carried out in the first phase of TELMA work (Artigue & al., 2005).

If we consider for instance, the first dimension of the notion of didactical functionality corresponding to the analysis of the tool for identifying potentially interesting characteristics, we distinguished between different dimensions, questioning the usability of the tool, how the mathematical knowledge of the domain is implemented in the tool and what kind of relationships with mathematical objects this implementation allows, the forms of social and didactic interactions offered by the tool, the distance with institutional and cultural objects. This resulted in a set of 8 different key concerns for this dimension.

The theoretical frame(s) that a team relies on contribute to creating a partial hierarchy between key concerns. We decided to use these hierarchies, once identified, for organizing the comparison and connection between theoretical frameworks that we wanted to achieve, considering that priority had to be given to the cases where the same key concern or set of key concerns was given a high position by two or more different teams. In such cases, we expected to be able to trace how similar or close needs were fulfilled by different theoretical constructions, better understand the functionality of these, and infer from that possible interesting connections.

We had thus a structure and the meta-language of concerns for approaching theoretical connection, but what made these tools productive was the cross-experimentation methodology we developed for supporting the analysis.

**The cross-experimentation methodology**

The cross-experimentation methodology was supposed to enable comparison among teams highlighting similarities and differences in their research approaches. In order to do this TELMA teams developed a set of simultaneous teaching experiments according to the principles described below.

First of all it was decided that *each team would develop a teaching experiment making use of an IT-based tool developed by another team*. This was expected to induce deeper exchanges between the teams, and to make more visible the influence of theoretical frames through comparison of the vision of didactical functionalities developed by the designers of the digital artefacts and by the teams using these in the cross-experimentation. These simultaneous experiments needed to be gathered together to allows comparisons. For this reason it was decided the *collaborative development of a common set of guidelines expressing questions to be addressed* by each designing and experimenting team in order to frame the process of cross-team communication. This document was meant to draw a framework of common questions providing a methodological tool for comparing the theoretical basis of the individual studies, their methodologies and outcomes. Furthermore, to increase the visibility of theoretical choices and discussions, and also to make the experimental situation more realistic, it was decided that in each team *PHD students and young researchers would be in charge of the experimentation*.

Finally the range of some variables was limited: in order to facilitate the comparison between the different experimental settings, it was agreed to address common mathematical knowledge domains (fractions and introduction to algebra), to carry out the experiments with students between the 5th to 8th grade, and to perform classroom experiments of about the same duration (one month).

These principles were put in practice through an on-line collaborative activity that brought the involved young researchers characterised by the 4 main phases: **1.** Production of a pre-classroom experiment version of the guidelines, containing plans for each experiment and answers to some questions (*a priori* questions); **2.** Implementation of the classroom experiments; **3.** Analysis of the experiments; **4.** Production of the final version of the guidelines containing answers to all of the addressed questions (including the *a posteriori* questions).

Each phase was interlaced with reflection tasks were the involved researchers were requested to review in-itinere the other teams' answers to the questions contained in the guidelines, and to comment on them and ask for clarifications. In this way a constant dialogue could be set up, enabling researchers to bring to light implicit assumptions and to compare the different teams' approaches (Cerulli & al, 2007). In a sense the guidelines may be considered both as a product and as a tool supporting TELMA collaborative work. A product in the sense that the final version contains questions and answers to questions as well as plans, descriptions of the experiments and results. A tool in the sense that the guidelines structured each team's work by:

- providing research questions concerning contexts, representations, and theoretical frameworks;

- establishing the time when to address each question (ex. before, or after the classroom experiment, etc.);
- establishing common concerns to focus on when describing classroom experiments, on the basis of the definition of DF;
- gathering, under the same document, the answers provided by each team to the chosen questions, in a format that could possibly help comparisons.

The guidelines were finally complemented by a final analysis of the cross experiment based on a set of interviews: a senior researcher in each team, who was not directly involved with the experimental work, interviewed the young researchers who carried out the field experiments (Artigue& al., 2007). Interviews followed a specific technique named “interview for explicitation” (Vermesch & Maurel, 1997): young researchers were asked to tell what they had done and how, but they were not directly questioned about the rationale for their actions.

## THE LESSONS DRAWN FROM THE TELMA CROSS-EXPERIMENT

As was expected, the cross-experiment methodology, thanks to the perturbation it introduced in the normal functioning of the research teams, contributed to make visible the invisible, explicit the implicit. The space limitations of this research report do not allow us to enter into the necessary details, but we will try to show some important lessons that we drew from this cross-experimentation regarding both the role played by theoretical frames in design and analysis, and the needs and potentials in terms of coordination of theoretical frames. In the oral presentation, we plan to illustrate these results by using the two particular cases which are provided by the TELMA teams of the two co-authors of this research report: the DIDIREM team which experimented a digital artefact: Arilab, designed by the ITD team and the ITD team which experimented a digital artefact: Aplusix, designed by the Metah French team sharing the same didactical culture as DIDIREM.

The cross-experiment confirmed the conjectured relationship between theoretical frames and the key concern hierarchy, and showed the precise effects of this relationship in the design of the experiments, from the selection of the digital artefact to be experimented, the type of tasks proposed to the students, the diversity of semiotic mediations considered and the role given to these, the granularity in the planning of their management, the respective role given to the teacher and the student, to the attention paid to the distance with institutional and cultural habits. Moreover, it was evidenced that this influence was more or less conscious to the researchers. Familiar constructs were often used in a naturalized way and that was also the case regarding values. For that reason, the reflective interviews introduced in the cross-experimentation methodology were especially productive.

Another important result was that, even if important, the role of theoretical frames and concerns in shaping the design was limited. Answers to the guideline questionnaires and interviews evidenced the existing gap between what the theories offered and the decisions to be taken in the design. A lot of design decisions were determined by usual habits and experience and not under the control of theory. The same occurred in the implementation of the experimental design. Moreover, it clearly appeared that, for a given team, the hierarchy of key concerns was dependant on the moment of the experimentation: for instance concerns which played major role in the design of the experiment were less apparent in the analysis of the experiment. Vice versa, during the analysis phase, researchers often realized that they had underestimated specific needs in the design, and this awareness also contributed to move the concern hierarchy. They also faced unexpected events that were not so unexpected when adopting other theoretical perspectives, for instance those offered by other teams.

More generally, regarding connection and integration issues between theoretical frames, we draw from this experience a number of lessons potentially helpful for future research. We list below three of these.

The necessity of distinguishing, when looking at integration, possibilities and needs between design and a posteriori analysis. The economical and coherence needs of design are different of those of a posteriori analysis. Incorporating two many different theoretical frames can make design quite impossible, but in a posteriori analysis introducing new theoretical frames for instance for explaining unexpected events, producing alternative explanations, is easier and can be an effective support towards theoretical integration. For instance, the cross-experiment made clear that the theory of didactic situations and theory of semiotic mediation, which have a crucial role in design for the DIDIREM and the ITD team respectively, induce to control and anticipate in the design of an experiment is quite different but that each vision has its own coherence and leads the design in a different and potentially productive direction. But we also got the evidence that the theoretical tools of one approach can enrich the a posteriori analysis of the other one.

The fact that the hierarchy of concerns can be exploited for looking at possible theoretical connections in different ways. In TELMA work, similarities in hierarchies were first exploited for establishing connections between theoretical frames and concepts, but contrasted priorities can also been exploited for looking at possible complementarities between theoretical frames.

The fact that progressing in the comparison and connection between theoretical frames needs the development of specific structures and languages making the communication possible. In our case, these structure and languages were provided by the notion of didactical functionality and the language of concerns. They obliged us to approach theories in terms of functionalities and this approach was really productive.

Beyond that, progression needs also the building of some form of collaborative practice supporting the comparison and connection work. Knowledge in this domain as in others cannot only result from readings, explanations and discussions. In our case, the cross-experimentation was asked to play this role, and the results it allowed us to achieve led us to reinvest this methodology in a new and more ambitious European project: the Remath project (Representing Mathematics with Digital Technologies) where the collaboration is extended towards the development of digital artefacts, of a common language for scenarios, and of an integrative platform MathDils. In this project, each team experiments both faliliar and alien digital artefacts in realistic contexts and cross-experiments. Moreover each team experiments both its own ILE and an alien ILE in realistic contexts, and the methodological tools built in TELMA are no longer only used to foster communication per se but also to achieve specific common research goals.

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