Inner-outer preconditioning strategy for 3d inductance extraction coupling with fast multipole method

T. S. Nguyen, Olivier Chadebec, Jean-Michel Guichon, Gérard Meunier, Tung Le Duc

INNER-OUTER PRECONDITIONING STRATEGY FOR 3D INDUCTANCE EXTRACTION COUPLING WITH FAST MULTIPOLE METHOD

T-S. Nguyen, J-M. Guichon, O. Chadebec, G. Meunier, T. Le-Duc

Grenoble Electrical Engineering Laboratory, Grenoble University
Grenoble-INP, Université Joseph Fourier, UMR CNRS 5269, France
Trung-Son.Nguyen@g2elab.grenoble-inp.fr

Keywords: Preconditioning techniques; Krylov subspace methods; PEEC; Fast Multipole Method.

Abstract

This paper presents an efficient preconditioning technique in order to couple Partial Element Equivalent Circuit (PEEC) method with Fast Multipole algorithm (FMM).

1 Introduction

The PEEC approach [1] have proven to be a reliable and fast method for modelling power electronics devices, printed circuit board layouts or EMC filters. However, for complicated three-dimensional structures like industrial applications, the approach is strongly limited due to the fact that it produces a linear system of equation **Ax = b** where the impedance matrix **A** is complex, very large, fully populated, non symmetric and with a bad condition number. Using a direct solver, the matrix storing cost increases in $O(N^2)$ and the computational complexity of solving process increases in $O(N^3)$. To address this problem, an Adaptive Multi-Level Fast Multipole Method (AMLFMM) [2] must be used coupled with an iterative solver like GMRES. Such approach speeds-up the matrix-vector product to O(NlogN) and also avoids the storage of the fully dense matrix. It must be pointed out that only some part of matrix **A** is explicitly known by this approach
However, even if this technique is applied, we may not be able to get a solution because of the non-convergence of GMRES algorithm. Therefore, we need to apply a preconditioning technique in order to ensure the convergence and/or to reduce the number of iterations.
The purpose of this paper is to present a new, original and efficient preconditioning technique dealing with such problems where a small part of matrix **A** can only be known indirectly.

2 PEEC equivalent circuit formulation with FMM

In the case of Magneto-harmonic analysis, the classical PEEC method is derived from the equation governing the total electric field at a point **r** :

$$\frac{\mathbf{J}(\mathbf{r})}{\sigma} + j\omega\frac{\mu_0}{4\cdot\pi}\iiint_v \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}dv = -\nabla\phi(\mathbf{r}) \qquad (1)$$

where **J** is a current density, ϕ is a scalar electric potential, σ is the material conductivity, μ_0 is the vacuum permeability and ω is the working frequency. The conducting volume V is discretized into elementary conductors with constant current density in each of them. If the whole current flowing into each elementary conductor (or branch) is $\mathbf{I_b}$ and associated voltage is $\mathbf{V_b}$, we get a system of equations linking currents vector to voltages vector.

$$\mathbf{Z_b}\mathbf{I_b} = (\mathbf{R_b} + j\omega\mathbf{L_b})\mathbf{I_b} = \mathbf{V_b} \qquad (2)$$

where $\mathbf{Z_b}$ is called the complex impedance matrix, $\mathbf{R_b}$ is a diagonal matrix whose elements are DC resistance of each branch and $\mathbf{L_b}$ is a dense matrix of partial inductances. Expressions to compute each element of both matrices $\mathbf{R_b}$ and $\mathbf{L_b}$ can be found in reference [1]. In order to solve the problem, it remains to add circuit equations (Kirchhoff's circuit law) ensuring the current conservation.
Using a mesh-based analysis [3], where mesh is any loop of branches in the graph representing the circuit, we can transform (2) into a new equation:

$$\mathbf{M}\mathbf{Z_b}\mathbf{I_b} = \mathbf{M}\mathbf{V_b} = \mathbf{V_s} \qquad (3)$$

with $\mathbf{M}^t\mathbf{I_m} = \mathbf{I_b}$ (4) and $\mathbf{M}\mathbf{Z_b}\mathbf{M}^t = \mathbf{Z_m}$ (5), where $\mathbf{Z_m}$ is a complex mesh-based impedance matrix, $\mathbf{I_m}$ is a vector mesh-based currents, **M** is the incidence matrix, $\mathbf{V_s}$ is the vector of source voltages (most part of time equal to 0) and the size of $\mathbf{I_m}$ being the number of fundamental loops (or meshes). Finally, we have:

$$\mathbf{M}\mathbf{Z_b}\mathbf{M}^t\mathbf{I_m} = \mathbf{Z_m}\mathbf{I_m} = \mathbf{V_s} \qquad (6)$$

This approach used in FastHenry [3] is known to reduce the number of unknowns and to lead to a better condition number in comparison with classical modified nodal analysis (MNA).
The solution of (6) in the coupling PEEC/AMLFMM context [3] leads to the storage of the only part of $\mathbf{Z_m}$ corresponding to near field interactions whereas the far field interactions are computed thanks to matrix-vector products accelerated by FMM. As a consequence, the matrix $\mathbf{Z_m}$ is decomposed into a far-field and a near-field (NF) matrix:

$$\mathbf{Z_m} = \mathbf{Z_{m\_near}} + \mathbf{Z_{m\_far}} \qquad (7)$$

Let us notice that to reduce the memory consumption only smaller matrix $\mathbf{Z_{b\_near}}$ is stored and the relation with matrix $\mathbf{Z_{m\_near}}$ is computed via formula (5). This point makes the preconditioning technique more difficult to apply.

3 Inner-outer near-field preconditioning technique

In our approach, the choice of meshes is based on geometric criteria leading to the building of a $\mathbf{Z_m}$ matrix with higher elements located mainly to its diagonal. This first fundamental step leads to a reliable set of equations. However, an efficient preconditioning technique is still needed.

Usually, when an iterative solver is used, the preconditioning is applied to the residual and can be achieved thanks to the solving of another linear system. This technique based on nested solver schemes has been investigated by many authors and has shown a good efficiency in electromagnetism [4].

Let us call **P** the preconditioner and **rp** the preconditioned residual. We have:

$$\mathbf{P}^{-1} \cdot \mathbf{Z_m} \cdot \mathbf{I_m} = \mathbf{P}^{-1} \cdot \mathbf{V_s} \qquad (8)$$

The system needs to be solved to get the preconditioned residual is then:

$$\mathbf{P} \cdot \mathbf{rp} = \mathbf{V_s} - \mathbf{Z_m} \mathbf{I_m} = \mathbf{r} \qquad (9)$$

In order to be efficient, the resolution of the last system (9) must be faster than the initial one (8). Let us notice that the second solving process can lead to a higher number of iterations but the time needed must be smaller.

To do this, as **P** we have chosen an approximation of $\mathbf{Z_{m\_near}}$ matrix called filtered matrix $\mathbf{Z_{m\_near\_filtered}}$. To build it, only most important elements in each row of $\mathbf{Z_{b\_near}}$ are conserved (selected thanks to a coefficient range from 0 to 1). We have then to solve:

$$\mathbf{Z_{m\_near\_filtered}} \cdot \mathbf{rp} = \mathbf{r} \qquad (10)$$

This new system is then solved by a nested GMRES solver. The process named outer-inner preconditioning strategy is resumed in the following figure:

Outer solver GMRes :

- Equation : **Zm.Im = Vm**
- Matrix-vector product: AMLFMM + near product
- Preconditioning : **Inner solver** GMRes
  - Equation : $\mathbf{Z_{m\_near\_filtered}}$**.rp = r**
  - Matrix-vector product : Low near product
  - Preconditioning : block-diagonal (or another Inner solver)

Figure 1: Nested solver for near-field iterative preconditioning

In the inner GMRES solver, the preconditioning strategy used is a classical block-diagonal one. Let us notice that we could apply this idea recursively and nest several GMRES schemes with filtered NF matrix down to diagonal matrix. However, a two-level scheme is already quite efficient.

4 Numerical results

These preconditioning strategies have been used to model an EMC filter in InCa3D software [5]. The discretization of the geometry leads to 4065 complex unknowns. Several preconditioning techniques have been tested and compared:

- Classical block-diagonal preconditioning with maximum 100 elements per block.
- Incomplete LU Factorization [6] with p = 50, τ = 1E-5.
- Our Inner-Outer GMRES scheme.

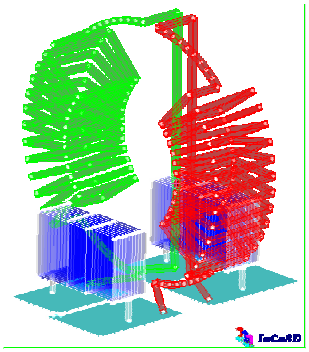

Figure 2: EMC filter structure

Presented example was computed on Intel Core 2 Duo @2.66Ghz with 2 GB memory. Matrix-Vector products and block-diagonal preconditioning was computed by a parallel version of FMM developed in Java.

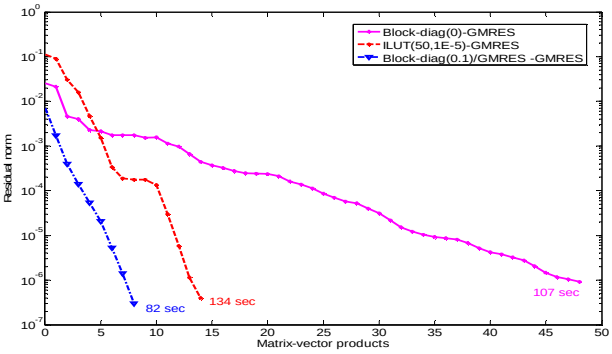

Figure 3: Convergence of main GMRES solver

The ILUT approach is the most costly in memory and computation time. This is due to the fact that $\mathbf{Z_{m\_near}}$ which is quite fully dense has to be explicitly built. Inner-outer GMRES preconditioning technique presents best performances in the context of FMM implementation.

References

[1] A. E. Ruehli, "Equivalent circuit models for three dimensional multiconductor systems" *IEEE Trans. Microw. Theory Techn.*, **vol. 22**, no.3, pp. 216-221, Mar. 1974.

[2] J. Carrier et al, "A fast adaptive multipole algorithm for particle simulations", *SIAM J. Sci. Statist. Comput.*, 1988

[3] M. Kamon et al, "FastHenry: A multipole-accelerated 3-D inductance extraction program," *IEEE Trans. Microwave Theory Tech.*, **vol. 42**, no. 9, pp. 1750-1758, Sept. 1994.

[4] B. Carpentieri et al, "Combining fast multipole techniques and an approximate inverse preconditioner for large electromagnetism calculations," *SIAM J. Sci. Comput.*, **vol. 27**, no. 3, pp. 774-792, 2005.

[5] [Online].http://www.cedrat.com, InCa3D software, Cedrat

[6] Y.Saad, "ILUT: A dual threshold incomplete LU factorization," Numer. Linear Algebra Appl., vol. 1, pp. 387-402, 1994.