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The normalised optimised anisotropic wavelet coefficient (NOAWC) method: An image processing tool for multi-scale analysis of rock fabric

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Abstract. The 2D Anisotropic Wavelet Transform (2DAWT) is an image analysis tool which is able to decipher signals where information obtained from different scales are inter-mixed. Extended from the Optimised Anisotropic Wavelet Coefficient method (OAWC) of Ouillon *et al.* [1995], we present a method which discriminates the objects and groups of objects depending on their area, shape ratio, orientation and position. Illustrated in a synthetic example, we show that this method allows one to distinguish between different sub-populations of objects within a single phase, and quantify the anisotropies of shape, orientation and spatial distribution at different scales (objects, clusters of objects, alignments of objects or clusters). Applied to a natural rock sample (Sidobre granite, Montagne Noire, France), the 2DAWT has permitted us to detect and accurately characterise the different levels of mineral organisation, and thus, to contribute to the understanding of the physical processes, such as crystallisation, fluid migration, deformation, etc... responsible for such organisations.

Introduction

The Anisotropic Wavelet Transform is an efficient mathematical tool for multi-scale analysis of multidimensional signals [Ouillon *et al.*, 1995, Hagelberg and Helland, 1995] since it allows one to decipher different levels of organisation within a given field of observation. Rock fabric is defined as the complete geometrical and spatial configuration of components of a deformed rock [see Hobbs *et al.*, 1976]. Thus, rock fabric analysis requires recognition and texture quantification of each level of mineral organisation. These levels are, from the finest to the largest scale: (i) the mineral itself, (ii) the texture or spatial organisation of minerals, (iii) the spatial variation of texture which defines connex entities, for example clusters of minerals, and finally (iv) the overall structure, such as alignments of clusters, defined by the spatial distribution of entities. The scope of this paper is to present the Normalised Optimised Anisotropic Wavelet Coefficient method (NOAWC) as a relevant tool for 2D multi-scale rock fabric analysis. This method is considered to be a part of an objective and complete description of rock fabrics.

Wavelet Transform Formalism

The efficiency of wavelet theory in image processing was demonstrated by Antoine *et al.* [1992]. A number of

developments in mathematics and theoretical physics related to wavelets are given in detail in Meyer [1992]. Topics such as edge detection and texture discrimination [Mallat, 1992], and structural identification of galaxies [Slezak *et al.*, 1992] are also close to our domain of application.

The 2D continuous Wavelet Transform (WT) helps decipher signals in which information carried by different spatial wavelengths or scales are combined using a battery of filters called wavelets. The filters are derived from a mother function Ψ_σ , which, in the present study, is also called « Anisotropic Mexican hat » (Fig. 1a). Its equation is given by:

$$\Psi_\sigma(\vec{x}) = \Psi_\sigma(x_x, x_y) = (2 - \frac{x_x^2}{\sigma^2} - x_y^2) e^{-\frac{1}{2}(\frac{x_x^2}{\sigma^2} + x_y^2)} \quad (1)$$

The effectiveness of wavelet transform comes from its great flexibility which allows both multi-scale and local analysis. WT analysis depends on the dilation/contraction scale factor a which, in turn, controls the resolution of the analysis and allows multi-scale study, and $\vec{b} = (bx, by)$ the translation vector, particularly suited for a local analysis; variation of \vec{b} represents the “sliding” of the wavelet over the 2D signal (image) $I(\vec{x})$, where $\vec{x} = (x_x, x_y)$ are the Cartesian coordinates of each pixel of the image. In addition, the anisotropic wavelets are able to detect anisotropy of shapes and singularities in any direction and at any scale. Due to the anisotropic shape of the filter itself, the WT analysis is also a function of σ , the shape ratio of the wavelet, and θ , the azimuth of the long axis of the wavelet (Fig. 1a).

The wavelet transform $C(a, \vec{x}, \sigma, \theta)$, of a signal $I(\vec{x})$ is the convolution of $I(\vec{x})$ with the analysing wavelet $\Psi(a, \vec{b}, \sigma, \theta)$:

$$C(a, \vec{x}, \sigma, \theta) = \Psi(a, \vec{x}, \sigma, \theta) \otimes I(\vec{x}) \quad (2)$$

For each analysing wavelet, the signal is transformed by the convolution into a set of coefficients C . This set is called the wavelet image and is represented by a map of coefficients. At a given position $\vec{x} = \vec{b}$, the wavelet coefficient $C(a, \vec{b}, \sigma, \theta)$ contains all the information we need for the present analysis (Fig. 1b, 1c). Due to its intrinsically multi-scale and local properties, the WT analysis allows detection of small structures superimposed onto larger ones. A constant signal will produce null coefficients. In contrast, if the signal presents some irregularity, the wavelet will react by producing locally high wavelet coefficients. The strongest coefficients will point out the best match, in terms of scale, location, shape ratio and orientation, between the analysing filter and the individual particle or group of particles hereafter called objects. A given filter detects only objects that have the same

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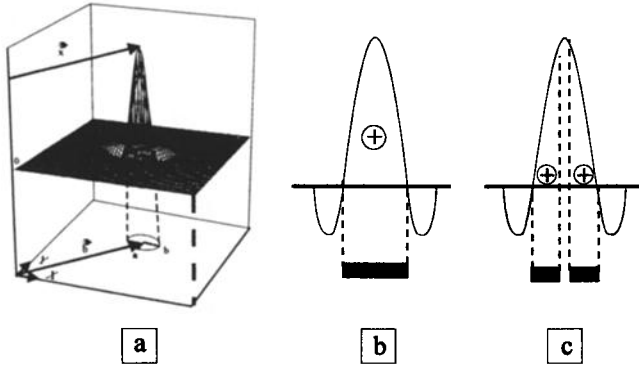


Figure 1. Anisotropic « Mexican hat » and corresponding sections visualising the calculation of the Wavelet Transform Coefficient. (a) Discretised analysing filter $\Psi(a, \vec{b}, \sigma, \theta)$ of resolution $a = 2$, shape ratio $\sigma = 2$, orientation $\theta = 0^\circ$, location $\vec{x} = (32, 32)$ in a 64×64 mesh world; and the corresponding detected object of short axis $a = 2$, long axis $b = 4$, shape ratio $b/a = 2$, elongation $\theta = 0^\circ$, location $\vec{b} = (32, 32)$; (b) Ideal match between the analysing wavelet and corresponding object (segment in bold) producing the maximal theoretical wavelet coefficient; (c) Multi-scale property: strong coefficient reflecting both the match between the analysing wavelet and the objects, and the local density in the space delimited by the positive part of the filter.

geometrical parameters (size, shape ratio, orientation). Hence an objective detection of each organisation level of an image requires to analyse each point of the 2D signal with all the possible discretised a , θ and σ values. This procedure creates a tremendous amount of data. In order to reduce this set of data, a method, extended from the algorithms of Ouillon *et al.* [1995] and Hagelberg *et al.* [1995], has been developed.

The NOAWC method

The NOAWC method is based on the method that was developed for the detection of the multi-scale organisation of linear objects, or faults and study of the hierarchical geometry of faulting [Ouillon *et al.*, 1996]. Because the multi-scale fabric analysis necessitates accurate measurements of weak shape anisotropies, the latter method was improved by introducing a normalisation procedure and a stronger filtering of the data, hence greatly facilitating the interpretation.

In order to obtain a significant calibration, for each analysing wavelet, the coefficient calculated at each point of the image is divided by the theoretical maximum value obtained for a perfect match between the wavelet and the corresponding object. Hence, the value of the normalised coefficient is a local indicator of the quality of the match between the filter and the objects of the image. But also, it is an indicator of the local density in the space delimited by the positive part of the filter (Fig. 1c). Therefore, individual particles, or elementary parts in the sense of Pratt [1978], have strong coefficients whereas larger scale objects made of several particles, those defining the texture and the structure, have weaker coefficients.

Normalisation is a necessary step before any attempt is made to stack the coefficients maps. In practice, at each pixel of the image, we select, among all the previously calculated coefficients, the local optimum filter characterised by the maximal normalised wavelet coefficient. At this step of the procedure, the parameters (a , σ , θ) of the local optimum filter, i.e. corresponding to the set of wavelet which best defines the local signal, are stored in the computer. In the final set of data,

only the «significant» features of the image will be extracted and stored. Significant features will be those for which the normalised wavelet coefficient is larger than a critical value depending on the local density, which itself is function of the size of the filter, and the quality of the match (Fig. 1).

The different magnitudes of the coefficients for each resolution allow one to recognise without ambiguity the different levels of organisation of the image. Thus, in order to extract only the meaningful features at a given resolution, we select the data corresponding to this resolution and to the associated range of coefficient magnitudes. The final extraction of the parameters of the selected filters (\vec{b} , σ , θ) helps to quantify the geometry of all the objects of scale a , whatever their location, shape anisotropy and orientation. The knowledge of the latter parameters allows to compute by various mode the intensity of the fabrics and especially by the evaluation the standard deviation. In summary, the spatial organisation associated with a given scale parameter a can be detected independently of the other scales, and accurately quantified, using the properties of WT (multi-scale and local analysis) and the NOAWC method (detection, selection and quantification).

A theoretical case study

We first consider a binary image (256×256 pixels) representing a set of particles with a bimodal size distribution (Fig. 2a). The large and small particles have different orientation distributions, with average grain elongations differing by about 50° . The large grains form 3 clusters that are elongated at about 90° with respect to the horizontal reference line of the image. The clusters themselves define an alignment at 140° measured counter-clockwise with respect to the reference line.

In our example, resolutions are chosen as multiples of 2 ($a = 2n$, $n = 1, 2, \dots, 10$), and for each resolution, the integration scale σa is performed in between the sampling scale and the image size. For each couple (a , σa), the azimuth θ varies from 0° to 180° by 10° steps. The corresponding WT calculation for all filters, synthesis of the data, recognition and characterisation of the different levels of organisation, are the object of the software developed by Darrozes *et al.* [1997, in press].

Using the latter software, analysis by the NOAWC method shows that significant organisations are detected for the resolution of 2, 4, 12 and 16 pixels (Fig. 2). At $a = 2$ pixels (Figs. 2b), a well-defined preferred orientation is detected at an angle of 60° (Figs. 2h), corresponding to the Shape Preferred Orientation (SPO) of the small grains. At $a = 4$ pixels (Figs. 2c, 2h), the wavelet optimum matches correspond to the large particles for which a SPO at 110° is found. These two families of grains are defined by normalised coefficients greater than 0.65 in profile AB (Figs. 2g) of the resulting NOAWC map (Figs. 2f) which is a staking of all the resolution. For a resolution $a = 12$ pixels, the textured area occupied by the 3 clusters is well detected (Fig. 2d). It may be further characterised by an orientation of $\theta = 90^\circ$ of the clusters (Fig. 2h). Finally, for the analysis with the largest wavelength, $a = 16$ pixel, the resulting image (Fig. 2e) has a simplified geometry which exhibits the long range alignment of 140° of the clusters (Fig. 2h). Due to the dissymmetry of the frequency analysis, edge effects of large filters (Fig. 2e), ever discussed in Ouillon *et al.* [1996] are reduced when stacking of all resolutions is done (Fig. 2f).

Obviously, the NOAWC method requires a calculation time longer, about 3 hours for a complete analysis (Pentium 90

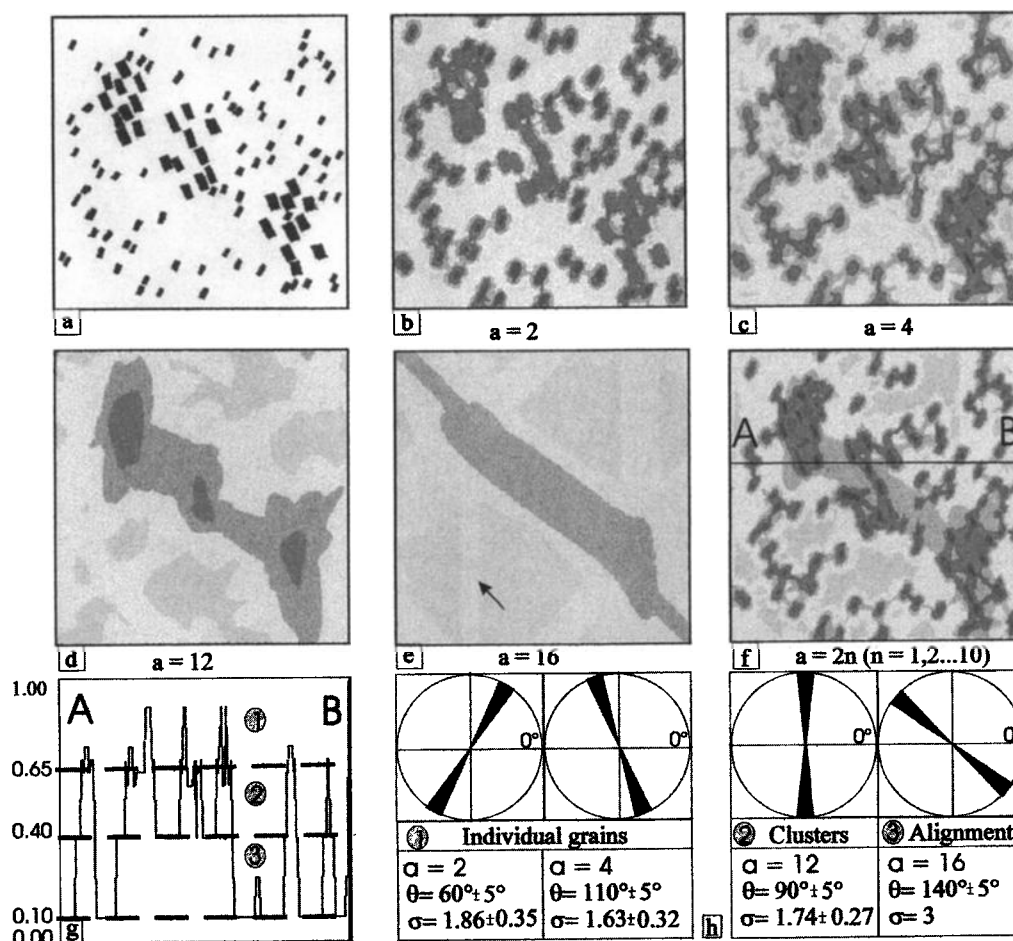


Figure 2. Theoretical case study. (a) Constructed binary image; (b), (c), (d), (e): NOAWC maps obtained, respectively, for the resolutions $a = 2, 4, 12$ and 16 pixels outlining the different levels of organisation: small and large particles, clusters of particles and alignment of clusters. The intensity of the normalised coefficient is represented by a grey scale where the black colour is equal to 1 and the white equal to 0. The arrow point out the edge effects of large filters. (f) Synthetic image corresponding to the stacking of all the previous NOAWC maps, (g) Coefficient intensities along the AB section of Fig. 2f showing the different levels of organisation. Circled numbers: (1) highest coefficients (> 0.65) giving the centroids of the individual grains; (2) coefficients around 0.4 corresponding to the clusters formed by the larger objects; and (3) coefficients around 0.1 pointing to the alignment of clusters; (h) Simplified orientation rose diagrams, for thresholds of resolution $a = 2, 4, 12$ and 16 pixels, and corresponding coefficient intensities, giving respectively the orientation of the small particles, at 60° (in the direct trigonometric frame), large particles at 110° , clusters of objects at 90° and alignments of clusters at 140° .

Mhz), than the current methods such as the intercept-counting method [Launeau *et al.*, 1990], the inertia tensor method [Launeau *et al.*, 1996] or the autocorrelation function [Panozzo-Heilbronner, 1992]. But human intervention is therefore restricted, at the beginning of the program, to implement the input parameters. However, (i) the possibility of sub-fabric detection, (ii) the ability of multi-scale and local analysis, and (iii) the ability of detection and quantification of anisotropies of (1) shape, (2) orientation and (3) spatial distribution, prove that the NOAWC method is particularly well suited for rock fabric studies.

A real case study

The mineral organisation of a late Variscan granite pluton (Sidobre pluton, northern Montagne Noire, France), characterised by a slight anisotropy [Darrozes *et al.*, 1994], has been studied using the NOAWC method. A $20 \text{ cm} \times 20 \text{ cm}$ sample was cut parallel to XZ, i.e. parallel to the magmatic stretching direction and perpendicular to the magmatic flattening plane.

The multi-scale fabric analysis was performed on the K-feldspar crystals of the latter sample that were isolated using the procedure of Launeau and Bouchez [1992]. In the resultant image (256×256 pixels) two sub-populations of K-feldspars could be distinguished, attesting for their complex crystallisation history (Fig. 3a): (1) megacrysts (MFK) centimeters in size, and parallelepipedic in shape, together with (2) an interstitial phase (IFK), millimeters in size and eventually forming small veinlets. The results obtained from the same set of filters as used in the former theoretical study, are depicted in figure 3. Two different SPOs displaying two approximately perpendicular sub-fabrics (Fig. 3c) are detected: (1) for $a = 2$ pixels at a mean angle of $100^\circ \pm 10^\circ$, and (2) for $a = 4$ and $a = 6$ pixels at $10^\circ \pm 6^\circ$. These orientations have also been found by Darrozes *et al.* [1994] who used the intercept counting method of Launeau *et al.* [1990]. However in the present study, new alignments of MFK grains have been identified, for $a = 8$ and $a = 10$ pixels resolution (Figs. 3b, 3d).

The NOAWC method has therefore (i) permitted us to determine the SPO of MFK which is attributed to the magmatic

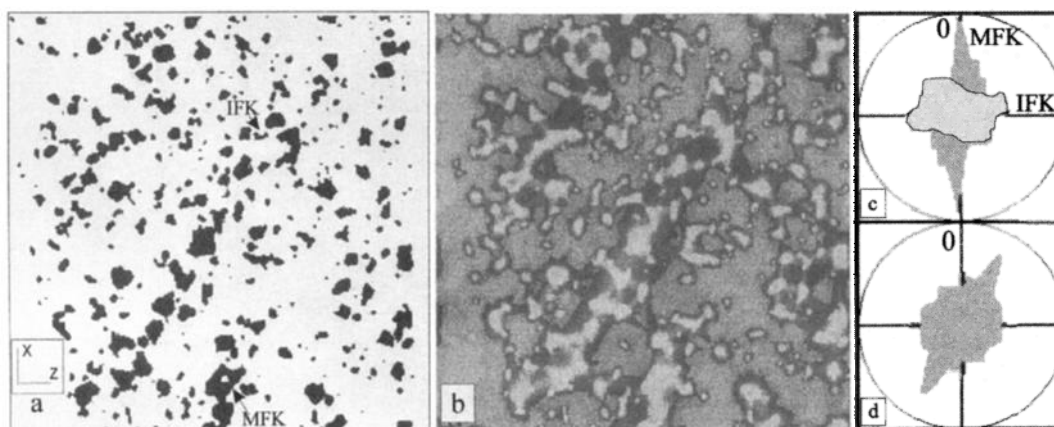


Figure 3. Natural example: study of a 20x20 cm XZ rock section from the Sidobre granite pluton (France). (a) Isolated K-feldspar crystals displaying 2 sub-populations: megacryst K-feldspars (MFK) and interstitial K-feldspar (IFK); (b) synthetic NOAWC map obtained for $a = 2n$ ($n=1, 2, \dots, 10$) resolutions; (c), (d) Orientation rose diagrams displaying the 2 perpendicular sub-fabrics (c) and long range alignments of crystals elongated at 30° , in the XZ frame (d).

flow, (ii) given evidence the perpendicular orientation of the IFK with respect to the magmatic flow, attributed to the late crystallisation process, with infilling microfractures in the presence of a residual melt [see Bouchez *et al.*, 1992] and forming discontinuous and irregular millimetric veinlets, and (iii) to detect grain alignments underlying the structure of the rock. Hence, this new mode of investigation which allows recognition and quantification of the different levels of organisation of a rock fabric, naturally leads to a better understanding of the physical processes responsible for SPO, texture and structure of rocks.

Conclusion

Due to its intrinsic properties, i.e. allowing local and multi-scale analysis, the wavelet transform gives a complete description of the spatial and geometrical configurations of the 2D signal in the full range of scales, from a few pixels to the whole image. We have proven that the NOAWC method is able to detect the various levels of organisation of an image without any preconceived choice of the parameters. Its applicability to geology has been tested on a theoretical example to characterise the shape preferred orientation and spatial distribution of grains, to distinguish between different sub-fabrics of a given phase, and finally to outline organisations at different scales (grain, cluster, alignment of clusters...). Applied to a natural sample, the NOAWC method has further allowed detection and accurate quantification of the Shape Preferred Orientations of two populations of grains, the direction of their alignments, and therefore, to question the processes responsible for such organisations.

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