



**An Integrated Production and Maintenance Planning Model
with time windows and shortage cost**

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An Integrated Production and Maintenance Planning Model with time windows and shortage cost

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An Integrated Production and Maintenance Planning Model with time windows and shortage cost

Abstract: In this paper, we tackle the problem of integrating production and maintenance. Production problem addresses the issue of determining the production lot sizes of various items. Preventive maintenance is carried out in time windows to restore the production line to an 'as-good-as-new' status, and when a production line fails, a minimal repair is carried out to restore it to an 'as-bad-as-old' status. The resulting problem is modelled as a linear mixed-integer program. It takes into account demand shortage and the reliability of the production line. Computational experiments are carried out to show the effectiveness of the integrated model compared to classical separate model for different instances, and the obtained results are analyzed in detail.

Keywords: Production and maintenance planning, Time windows, Shortage cost.

1. Introduction

In the industrial environment, the relationship between production and maintenance has been conflictual in nature. This attitude is perpetuated by the lack of communication regarding the scheduling requirements of each function. Maintenance is generally a secondary process in companies that have production as their core business. Production management often views maintenance in the context of hours or days out of service and fails to realize the strategic importance of incorporating maintenance in the manufacturing planning. On the other hand, management for the maintenance function attempts to impose constraints on production that it deems necessary to achieve complete equipment reliability. The result is that the implementation of an optimal maintenance policy is constrained by the demands of production.

Currently, production planning and maintenance are performed separately. For instance, common ERP modules include production planning module that optimizes the utilization of manufacturing capacity, parts, components and material resources. Maintenance module is a peripheral module little known in fact, it is not part of core ERP application as production planning, finance, HR (Human Resources) Modules. it is considered as computerized maintenance management system (CMMS), and it allows managing preventive and corrective maintenance of all material assets of the company. We notice that each of these modules is used independently with specific data from each service, and no coordination is implemented to reduce production and maintenance costs. Therefore, maintenance planning should be an integral part of the overall business strategy and should be coordinated and scheduled with manufacturing activities. So, we should consider maintenance as integral parts of the production plan rather than as interruptions to that plan. Any violation of the maintenance schedule is treated as a violation of the production plan integrity.

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3 To solve this problem, we develop an integrated production planning and maintenance
4 activities. This integration hedge against often avoidable failures and re-planning occurrences,
5 and minimize loss of demand which is not without consequence for companies. In this area,
6 we propose an integrated model of production and maintenance planning that minimizes
7 production, inventory, set up, demand shortage, preventive and corrective maintenance costs.
8 The particularity of our model is that preventive maintenance actions are planned in time
9 windows, and demand shortage is allowed when the resource capacity is not sufficient to meet
10 all demand.

11
12 The remainder of the paper is organized as follows. In the second section a brief literature
13 review is presented. In the third section, we describe separate production planning and
14 maintenance problem, and we state its mathematical formulation. We illustrate our integrated
15 problem in the fourth section. Computational results are showed in the fifth section, and we
16 end up with conclusion and outlooks in the last section.

26 **2. Brief literature review**

27
28 The problem of joint planning of production and maintenance is a recent problem tackled in
29 research due to its importance in the current highly competitive environment. Several
30 relations exist in different ways between maintenance and production according to Budai and
31 al. (2006). First of all, when planning maintenance takes into account production (e.g. Dekker
32 and Van Rijn. (1996); BÄackert and Rippin. (1985); Cassady and al. (2001), Marquez and al.
33 (2003). Secondly , maintenance can also be seen as a production process which needs to be
34 planned, see e.g. Dijkstra and al, (1994). Shenoy and Bhadury. (1993), Rosheil and Christy
35 (1996), Yan and al. (2004). Finally, the integrated models of production and maintenance
36 which we are interested in our research work.

37
38 Several studies of integrated production planning and maintenance problem were proposed
39 with different approaches, resolution methods, and modeling tools. In the literature, we find
40 models developed with stochastic aspect by using such Markov chains to oversee the state of
41 machine or to determine an optimal buffer level according to the production rate in order to
42 establish an interdependent policy of maintenance and production. In the last decade, large
43 models were stated in this context, some of them treat the control problem of a stochastic
44 manufacturing system with random breakdowns, repairs and preventive maintenance. The
45 objective of the control models is to find the production and the preventive maintenance rates
46 of the machines so as to minimize the total cost of inventory/backlog, repair and preventive
47 maintenance see e.g. Kenné and Boukas (2003), Kenné and Gharbi (2004), Gharbi and Kenné
48 (2005), and Rezg and al. (2005). An extension was proposed by Charlot and al. (2007)
49 considering preventive maintenance activities and differentiates two types of repairs: repairs
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3 with lockout/tagout, associated with accident prevention, and repairs without lockout/tagout.
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5 Other research works deal with models of production systems with buffer inventory. The role
6
7 of this buffer inventory is to satisfy demand when corrective or preventive maintenance is
8
9 carried out. One of the earliest and basis work on this subject is Van der Duyn Schouten and
10
11 Vanneste (1995). In their model the demand rate is constant and equal to d (units/time) and as
12
13 long as the fixed buffer capacity (K) is not reached the installation operates at a constant rate
14
15 of p units/time ($p > d$) and the excess output is stored in the buffer. When the buffer is full,
16
17 the installation reduces its speed from p to d . Upon failure corrective maintenance starts and
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19 the installation becomes as good as new. The decision to start a preventive maintenance
20
21 action is not only based on the condition of the installation, but also on the level of the buffer.
22
23 The criterion to minimize is the average inventory level and the average number of
24
25 backorders. Iravani and Duenyas (2002) extended the above model by assuming a stochastic
26
27 demand and production process. Kyriakidis and Dimitrakos (2006) studied an infinite-state
28
29 generalization of Van der Duyn Schouten and Vanneste (1995). Preventive maintenance
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31 actions can be regularly (after each T time periods) performed and the duration of corrective
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33 and preventive maintenance actions is random see e.g. Chelbi and Ait KAdi(2004), and the
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35 possibility of imperfect production can be included in model for example Zequeria and al.
36
37 (2004). Dhouib and al. (2008) worked on production lines composed of several serial
38
39 machines which are subject to random operation-dependent failures when it has no
40
41 intermediate buffers between adjacent machines. Models of preventive maintenance policy
42
43 with control quality of products for a randomly failing production system producing
44
45 conforming and non-conforming units can be developed e.g. Radhoui and al. (2009). The
46
47 considered system consists of one machine designed to fulfill a constant demand. According
48
49 to the proportion l of non-conforming units observed on each lot and compared to a threshold
50
51 value l_m , one decides to undertake or not maintenance actions on the system.

44
45 A continuous time models with an infinite time horizon were presented such economic
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47 manufacturing quantity (EMQ) models with failure aspects. The study of those models is
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49 based on failures of production system and their impacts on the decisions of the lot sizing, see
50
51 e.g. Groenevelt and al (1992) who consider the effects of stochastic machine breakdowns and
52
53 corrective maintenance on economic lot sizing decisions, and propose to use the EMQ as an
54
55 approximation to the optimal production lot size. Chung (2003) provides a better
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57 approximation to the optimal production lot size, Makis and Fung (1995) proposed a model
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59 for joint determination of the lot size, inspection interval and preventive replacement time for
60
a production facility that is subject to random failure. Srinivasan and Lee (1996) considered
an (S, s) policy, i.e. as soon as the inventory level reaches S , a preventive maintenance
operation is initiated and the machine becomes as good as new. After the preventive

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3 maintenance operation, production resumes as soon as the inventory level drops down to or
4 below a prespecified value s , and the facility continues to produce items until the inventory
5 level is raised back to S . If the facility breaks down during operation, it is minimally repaired
6 and put back into commission.
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10 Unlike the above mentioned continuous time models, the discrete time models assume that
11 the planning horizon is finite and divided into discrete periods for which demand is given and
12 may vary between periods. A few works exist in this area, they concern aggregate production
13 planning problems known as multi items capacitated lot sizing problems at which, we
14 integrate decision variables modeling preventive and corrective maintenance. To our
15 knowledge, the only works developed within this framework are proposed by Weinstein and
16 Chung (1999), Aghezzaf and al. (2007), and Aghezzaf and Najid (2008). Weinstein and
17 Chung (1999) presented a three part-model to resolve the conflicting objectives of system
18 reliability and profit maximization. An aggregate production plan is first generated, and then a
19 master production schedules is developed to minimize the weighted deviations from the
20 specified aggregate production goals. Finally, work-center loading requirements, determined
21 through rough cut capacity planning, are used to simulate equipment failures during the
22 aggregate planning horizon. Aghezzaf and al. (2007) studied a production system that
23 subjects to random failures. They assume that any maintenance action carried out on the
24 system, in a period, reduces the system's available production capacity during that period.
25 The objective is to find an integrated lot-sizing and preventive maintenance strategy of the
26 system that satisfies the demand for all items over the entire horizon without backlogging, and
27 which minimizes the expected sum of production and maintenance costs. An extension of the
28 above work is treated by Aghezzaf and Najid (2008). They developed models which take into
29 consideration the parameters of reliability of the system and consider parallel production
30 lines. Our motivation by this paper is to provide an extension of these models to a more
31 complete and more flexible model in relation to the date of preventive maintenance tasks. The
32 aim of our study is to establish a joint production and maintenance planning problem at the
33 tactical level to minimize the total costs associated with production and maintenance. Before
34 describing our problem, we will define both of aggregate production and maintenance
35 planning.
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53 **3. Statement of the separate problem**

54 **3.1 Maintenance problem**

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56 Throughout their operational life, complex industrial systems are subject to corrective
57 operations and preventive maintenances to keep them in working order. Preventive
58 maintenance is a schedule of planned maintenance actions aimed at the prevention of
59
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breakdowns and failures. The primary goal of preventive maintenance is to prevent equipment failure before it actually occurs. Corrective maintenance is defined as maintenance work which involves the repair or replacement of components which have failed or broken down.

Maintenance problem is to answer the question: when do we perform preventive maintenance task, ie, what is the periodicity of preventive maintenance?

Several policies have been proposed to address this issue, the readers can refer to Wang (2002) for a detailed review of these policies. Initially, in our policy the preventive maintenance actions are performed at fixed time intervals kT_{PM} ($k=1, 2 \dots$), and corrective maintenance actions are carried out at failures occurs. Then, we have to determine the optimal length of preventive maintenance period T_{PM}^* which minimizes the expected maintenance cost per unit time (example, see figure 1).

Parameters:

C_p : Preventive maintenance cost.

C_r : Corrective maintenance cost.

T_{PM} : Preventive maintenance period (periodicity).

T_{PM}^* : Optimal preventive maintenance period.

$r(t)$: Failure rate function.

$n_1 = \left\lfloor \frac{T}{T_{PM}^*} \right\rfloor$: Number of preventive maintenance during the horizon.

$NB(t)$: Expected number of failures during T_{PM} .

$$NB(t) = \int_0^t r(u) du$$

$C_{exp}'(t_1, t_2)$: Expected corrective maintenance cost between $[t_1, t_2]$

$$C_{exp}'(t_1, t_2) = C_r * \int_{t_1}^{t_2} r(u) du$$

$C_{exp}(t)$: Expected maintenance cost for a period t .

$$C_{exp} = C_p + C_r * NB(t)$$

$E(T_{PM})$: Expected cost per unit time of maintenance Gertsbakh, (2000).

$$E(t) = \frac{C_{exp}(t)}{t} \quad (1)$$

CT : Expected total cost of maintenance throughout a finite time horizon.

[Insert figure 1 here]

We consider in this paper a finite time horizon of length T . Once, the length of preventive maintenance period is known, we compute the expected total maintenance cost CT throughout the horizon which is based on the expected maintenance cost per preventive maintenance period $C_{exp}(T_{PM}^*)$ and $C_{exp}^c(T, n_I T_{PM}^*)$ (example, see figure 2)¹.

[Insert figure 2 here]

The mathematical formulation of the expected total maintenance cost CT throughout the horizon is given below:

$$CT = \begin{cases} n_I \times C_{exp}(T_{PM}^*) & \text{If } \frac{T}{T_{PM}^*} \text{ is integer} & (2) \\ n_I \times C_{exp}(T_{PM}^*) + C_{exp}^c(T, n_I T_{PM}^*) & \text{else} & (3) \end{cases}$$

We implement an algorithm to compute preventive maintenance period and all the costs described above including the calculation of the expected total maintenance cost CT in a finite horizon. This cost is important to compare a separate study of the problem with our integrated one in computational results in the following section. The algorithm is stated as follows:

Algorithm

Initialization:

Parameters of the used failure distribution;

Preventive maintenance cost;

Corrective maintenance cost;

Main program:

for $t = 1$: to Number_period *do*

Computation of expected number of failures in each period t ($NB(t)$).

Computation of expected maintenance (preventive and corrective) cost in each period t ($C_{exp}(t)$).

Computation of expected maintenance cost per unit time in each period t (equation (1)).

End for

Determination of the optimal result $[(T)_{MP}^*, E(T_{MP}^*)]$.

Computation of expected maintenance cost per preventive maintenance period $(C_{exp}(T_{PM}^*))$

¹ Note that if $\frac{T}{T_{PM}^*}$ is an integer value, CT will dependent only on $C_{exp}(T_{PM}^*)$.

Computation of expected total maintenance cost throughout the horizon (equation (2) or (3))

3.2 Aggregate production planning

A production plan establishes resource requirements to perform processing from raw material to finished products, in order to satisfy customers in the most efficient and most economical possible way. In other words, decisions to produce are made in the best report between the financial objective and the one of customer satisfaction.

The basis of our work is the multi items capacitated lot sizing problems which are the main production planning problems with setups between production lots. Because of these setups, it is often too costly to produce a given product in every period. On the other hand, generating fewer setups by producing large quantities to satisfy future demands results in high inventory holding costs. Thus, the objective is to determine the periods where production should take place, and the quantities to be produced, in order to satisfy demand while minimizing production, setup and inventory holding costs. Other costs might also be considered, examples are backorder cost, backlogging cost, shortage cost etc. In our work, we will consider shortage cost. To our knowledge, there are few works that deal with lots sizing problems with demand shortage. Sandbothe and Thompson (1990) and Aksen and al. (2003) addressed a single item lot sizing problem with constant capacity and shortage cost. Absi and kedad (2008) dealt with a multi item capacitated lot sizing problem with shortage cost, and an extension to a multi-item capacitated lot-sizing problem with setup times , safety stock and demand shortage Absi and kedad (2009). For more information about multi items capacitated lot sizing problems, the reader can refer to a detailed review presented by Karimi and al. (2003).

Let define the parameters and the mathematical formulation of the multi items capacitated lot sizing problem with demand shortage denoted (MCLSP-SC) below:

Index:

- P : set of item.
 T : set of period.
 i : Items.
 t : Periods.

Model parameters:

- d_{it} : Demand of item i to satisfy during period t.
 f_{it} : Set-up cost of producing one unit of item i in period t.
 P_{it} : Fixed cost of producing one unit of item i period t.

h_{it} : Variable cost of holding one unit of item i by the end of period t .
 φ_{it} : Unit cost for requirements not met regarding demand for item i in period t .
 ρ_i : Processing time for each item i .
 $\theta_p(t)$: Capacity consumed by preventive maintenance action in period t .
 $\theta_r(t)$: Capacity consumed by corrective maintenance action in period t .
NB : Vector of N (Number of period) elements contains the expected number of failures
 $NB(t)$ in each period t of horizon if we don't perform any preventive maintenance action
 (figure 5).

$$NB(t) = \int_0^t r(u) du$$

$K(t)$: Available capacity in period t .

$$K'(t) = \begin{cases} K(t) - \theta_p(t) - \theta_r(t) \times NB(t) & \text{if PM is performed} \\ K(t) - \theta_r(t) \times NB(t) & \text{otherwise} \end{cases}$$

$$d_{tN}^i = \sum_{k=t}^N d_{ik}$$

Decision variables:

y_{it} : Binary set up variable of item i in period t .

x_{it} : Quantity of item i produced in period t .

I_{it} : Inventory of item i at the end of period t .

r_{it} : Demand shortage for item i in period t .

δ_t : Binary preventive maintenance variable.

$$\delta_t = \begin{cases} 1 & \text{if preventive maintenance action is carried out in the beginning of period } t. \\ 0 & \text{Otherwise.} \end{cases}$$

MCLSP-SC:

$$\text{Minimize } Z_1 = \sum_{i \in P} \sum_{t \in H} f_{it} y_{it} + p_{it} x_{it} + h_{it} I_{it} + \varphi_{it} r_{it} \quad (4)$$

Subject to:

$$x_{it} + r_{it} + I_{i,t-1} - I_{it} = d_{it} \quad \forall i \in P, \forall t \in H \quad (5)$$

$$\sum_{i \in P} \rho_i x_{it} \leq K'(t) \quad \forall i \in P, \forall t \in H \quad (6)$$

$$x_{it} - d_{tN}^i y_{it} \leq 0 \quad \forall i \in P, \forall t \in H \quad (7)$$

$$r_{it} \leq d_{it} \quad \forall i \in P, \forall t \in H \quad (8)$$

$$x_{it}, I_{it}, r_{it} \geq 0 \quad \forall i \in P, \forall t \in H \quad (9)$$

$$y_{it} \in \{0, 1\} \quad \forall i \in P, \forall t \in H \quad (10)$$

$$I_0 = 0$$

The objective function (4) minimizes the sum of the set up, holding, production, demand shortage costs over the whole N-period horizon. Constraint (5) is the inventory balance equation. Constraint (6) is the capacity constraint where preventive maintenance actions and expected number of breakdowns in each period are known. Constraint (7) relates the continuous production variables to the binary setup variables. Constraint (8) expresses that quantity lost of item i in period t must be less than demand of item $i \in P$ in period t . The constraints (9) and (10) express non-negativity and integrality constraints.

The multi items capacitated lot sizing problem often has been studied in literature. Solution methods of the problem can be classified into three main categories. The first is exact methods e.g Barany et al. (1984), Eppen and Martin (1987), the second category is common-sense or specialized heuristics e.g Maes and Van Wassenhove (1988), Kirca and Kokten (1994,) Karni and Roll (1982) and the third category belong to mathematical programming-based heuristics e.g. Thizy and Van Wassenhove (1985), Trigeiro(1987), Diaby et al. (1992). There are few references dealing with lot-sizing problems with shortage costs. . Absi and kedad (2008) used inequalities within a branch-and-cut framework to find near optimal solutions, and Absi and kedad (2009) developed a Lagrangian relaxation of the capacity constraints to obtain lower and upper bounds for their problem. To the best of our knowledge, there are the first papers that deal with shortage costs for the multi-item capacitated lot-sizing problem.

Let $Cost_{prod,maint}$ the total cost of maintenance and production in the separate study, it will be the sum of expected total cost of maintenance throughout a finite time horizon CT (determined in section 3.1) and the optimal cost obtained from MCLSP-SC model Z_1^* .

$$Cost_{prod,maint} = CT + Z_1^*$$

4. Integrated production planning and maintenance problem

Consider a planning horizon H of length $T = N \times \tau$ covering N periods of fixed length τ , and a set of item $i \in P$ to be produced on a capacitated production line. During each period $\forall t \in H$, a demand d_{it} of item $i \in P$ should be satisfied. Items are produced on a production line with known capacities given in unit time, and processing time expressed in unit time per item. Furthermore, we allow demand shortage unfulfilled due to insufficient capacity, and we will assign a high unit cost for each item lost in each period.

If in separate study the optimal length of preventive maintenance period is $T_{PM}^* = n^* \times \tau$. It means that preventive maintenance tasks are performed periodically in the beginning of period's $t = 1, n^* + 1, 2n^* + 1, 3n^* + 1, 4n^* + 1 \dots$. Our aim is not to plan these preventive maintenance actions at the beginning of those periods, but to plan them in time windows

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3 $[(pn^* - k) \times \tau, (pn^* + k) \times \tau]$. In other words, a preventive maintenance task will be
4 performed at the earliest in the beginning of the period $t = pn^* + 1 - k$ or at the latest in the
5 beginning of the period $t = pn^* + 1 + k$, and will complete within the period in which it
6 started, with $p = 1, 2, \dots, \left\lfloor \frac{T}{T_{PM}^*} \right\rfloor - 1$ (number of preventive maintenance actions),
7 and k is chosen to avoid the overlaps between time windows, so:
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$$13 \quad k = \begin{cases} \left\lfloor \frac{n^* - 1}{2} \right\rfloor & \text{if } n \text{ is an even number} \\ \frac{n^* - 1}{2} & \text{else} \end{cases}$$

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20 Moreover, we assume that each preventive or corrective maintenance action carried out on the
21 production line consumes capacity units, at the beginning of the planning horizon the
22 production line is in «as good as new (AGAN), the production line is considered here as a
23 complex system and the failure rate is an overall rate of the whole line. When a preventive
24 maintenance is carried out, the production line is restored to AGAN status and when a
25 production line fails, a minimal repair is carried out to restore it to “as bad as old” (ABAO)
26 status. Finally we assume that the expected failures increase with elapsed time since the last
27 preventive maintenance. Figure 3 shows an example of integrated production planning and
28 maintenance. Figure 3 shows an example of integrated production planning and
29 maintenance.
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35 [Insert figure 3 here]

36
37 The proposal mathematical program models the problem of determining optimal
38 integrated production and maintenance plan in a production system. Indeed, the model
39 determines the optimal production plan and optimal date of preventive maintenance
40 actions taking into account the reliability of the production line, knowing that the
41 failure rate increases from the last preventive maintenance task. The mathematical
42 formulation of our problem denoted (MCLSP-TW-SC) is below. We keep the same
43 parameters as in Section 3.2, and we add a new binary decision variable z_{jt} .
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$$50 \quad z_{jt} = \begin{cases} 1 & \text{if in period } t, \text{ the last preventive maintenance action ended in period } j. \\ 0 & \text{Otherwise.} \end{cases}$$

51
52
53 The variable z_{jt} must equal one when the last preventive maintenance was occurred in period
54 j , and we are now in period t . To ensure this, we define z_{jt} as:

$$55 \quad z_{jt} = (1 - \delta_t)(1 - \delta_{t-1}) \dots (1 - \delta_{j+1}) \delta_j$$

56
57
58 This is a nonlinear expression, but can be solved by introducing the following three linear
59 constraints below:
60

$$\begin{aligned}
 z_{jt} &\leq 1 - \delta_k & \forall t \in H, j = 1, \dots, t-1, k = j+1, \dots, t \\
 z_{jt} &\leq \delta_j & \forall t \in H, j = 1, \dots, t-1 \\
 z_{jt} &\geq \sum_{k=j+1}^t (1 - \delta_k) + \delta_j - (t-j) & \forall t \in H, j = 1, \dots, t-1
 \end{aligned}$$

MCLSP-TW-SC:

$$\begin{aligned}
 \text{Minimize } Z &= \sum_{i \in P} \sum_{t \in H} f_{it} y_{it} + p_{it} x_{it} + h_{it} I_{it} + \varphi_{it} r_{it} + \\
 &\sum_{t=2}^N (C]_{pr}(t) \delta_t + C_r \sum_{j=1}^{t-1} [NB(t-j+1) z_{jt}) + C_{pr}(1) \delta_1 \quad (11)
 \end{aligned}$$

Subject to:

$$x_{it} + r_{it} + I_{i,t-1} - I_{it} = d_{it} \quad \forall i \in P, \forall t \in H \quad (12)$$

$$\left\{ \begin{aligned}
 \sum_{i \in P} \rho_i x_{it} + \theta_{pr}(t) \delta_t &\leq K(t) - \theta_r(t) \sum_{j=1}^{t-1} NB(t-j) z_{jt} & t = 2..N \\
 \sum_{i \in P} \rho_i x_{it} + \theta_{pr}(t) \delta_t &\leq K(t) & t = 1
 \end{aligned} \right. \quad (13)$$

$$\sum_{i \in P} \rho_i x_{it} + \theta_{pr}(t) \delta_t \leq K(t) \quad t = 1$$

$$x_{it} - d_{tN}^i y_{it} \leq 0 \quad \forall i \in P, \forall t \in H \quad (14)$$

$$r_{it} \leq d_{it} \quad \forall i \in P, \forall t \in H \quad (15)$$

$$\sum_{t=pn+1-k}^{pn+1+k} \delta_t = 1 \quad p = 1, 2, \dots, \left\lfloor \frac{T}{T_{PM}} \right\rfloor - 1 \quad (16)$$

$$\delta_t + \delta_{t-1} \leq 1 \quad t = 1, 2, \dots, N-1 \quad (17)$$

$$z_{jt} \leq 1 - \delta_k \quad \forall t \in H, j = 1, \dots, t-1, k = j+1, \dots, t \quad (18)$$

$$z_{jt} \leq \delta_j \quad \forall t \in H, j = 1, \dots, t-1 \quad (19)$$

$$z_{jt} \geq \sum_{k=j+1}^t (1 - \delta_k) + \delta_j - (t-j) \quad \forall t \in H, j = 1, \dots, t-1 \quad (20)$$

$$x_{it}, I_{it}, r_{it} \geq 0 \quad \forall i \in P, \forall t \in H \quad (21)$$

$$y_{it}, z_{jt}, \delta_t \in \{0, 1\} \quad \forall i \in P, \forall t \in H, j = 1, \dots, t-1 \quad (22)$$

$$I_0 = 0, \delta_1 = 1$$

The objective function (11) minimizes the sum of the set up, holding, production, demand shortage, and maintenance (preventive and corrective) costs over the whole N-period horizon.

Constraint (12) is the inventory balance equation. Constraint (13) is the capacity constraint

that considers preventive and corrective maintenance. Constraint (14) relates the continuous production variables to the binary setup variables. Constraint (15) expresses that quantity lost of item i in period t must be less than demand of item $i \in P$ in period t . The constraint (16) ensures that one maintenance is carried out in the interval $[(pn + 1 - k), (pn + 1 + k)]$. The constraint (17) ensures that two preventive maintenance actions cannot be carried out successively. Constraints (18)-(20) force the decision variable z_{it} to 0 or 1. The constraints (21) and (22) express non-negativity and integrality constraints.

[Insert figure 4, 5 here]

It is time now to define how we calculate cost $C_{pr}(t)$ and capacity $\theta_{pr}(t)$ consumed when a preventive maintenance action is planned. From Figure 4, we observe that if a preventive maintenance is performed at a given period t , the maintenance cost in this period t is the sum of preventive and corrective maintenance costs. The corrective maintenance cost in period t is the expected number of failures in that period t multiplied by the cost of one corrective maintenance action. Notice that if no preventive maintenance action was performed, the maintenance cost in period t is the corrective maintenance cost. The same reasoning can be applied for the capacity consumed by maintenance actions in a given period t when preventive maintenance or not was carried out. Let be the expression of maintenance cost and capacity consumed by maintenance action in a given period t :

$$C_{pr}(t) = C_p + C_r * NB(1) \quad (23)$$

$$\theta_{pr}(t) = \theta_p(t) + \theta_r(t) * NB(1) \quad (24)$$

If preventive maintenance is planned at any period t , the failure rate is renewed. So the expected number of failures in this period t is the first element of vector NB which is $NB(1)$. This justifies equations (23), and (24).

If we eliminate constraints (16)-(20), our problem is reduced to a separate study of multi capacitated lot sizing problem with shortage cost (MCLSP-SC) where preventive maintenance periodicity and expected number of breakdowns in each period are known. Then, we can assimilate the (MCLSP-TW-SC) to one with set-up time (MCLSP-SC-ST) which is NP hard Absi and kedad (2008).

5. Computational results

In this section, we discuss the results of some computational experiments carried out to test our integrated model. The tests aim to compare our integrated model to separate one. Remind that the separate model is the study done in section 3.1 and 3.2. We created five sets of randomly generated instances. We consider a following planning horizon composed of T

production periods of fixed length $\tau = 1$ to produce a set of items on production line each with an available capacity. The production, set-up, and holding costs are respectively 10, 25, and 5. Four parameters are considered for the analysis:

Problem dimension: The problem dimensions represented by the number of products N and the number of periods T . We use three different combinations $(N, T) \in \{(3, 12); (6, 18); (12, 24)\}$

Production capacity utilization: The capacity required, in each period, is then computed as if lot-for-lot solutions were implemented. The capacity is then obtained by dividing the later result by the target average utilization of capacity ρ . The factor ρ is set to 0.85, 0.95 and 1.1 corresponding respectively to situations with moderately loose, tight and too tight capacity constraints.

Demand pattern: the demand of each product at each period is generated randomly from the interval [20,100].

Shortage cost: the shortage cost is fixed for all products for all periods. It is generated randomly from the interval [50,100].

All problem tests are generated with Weibull distribution of production line. Let be $f(t)$, and $r(t)$ respectively its probability density and failure rate functions: The shape and the scale parameters are respectively $\beta = 3$ and $\lambda = 4$.

$$f(t) = \left(\frac{\beta}{\lambda}\right) \left(\frac{t}{\lambda}\right)^{\beta-1} \exp\left(-\frac{t}{\lambda}\right)^{\beta}$$

$$r(t) = \left(\frac{\beta}{\lambda}\right) \left(\frac{t}{\lambda}\right)^{\beta-1}$$

[Insert table 1 here]

Table 1 shows the expected number of failures as a function of system's age. The cost of preventive maintenance action is set to $C_p = 28$, and the cost of minimal repair action is given by $C_r = 35$. The capacity lost, when a preventive maintenance task and minimal repair action are carried out, is respectively $\theta_p = 0.067K(t)$, and $\theta_r = 0.33K(t)$.

[Insert figure 6 here]

[Insert figure 7 here]

[Insert figure 8 here]

Figures 6, 7, and 8 show the expected total cost of maintenance strategy as a function of the preventive replacement cycle T . We illustrate optimal preventive maintenance cycle and the

total expected cost of maintenance for whole planning horizon associated to the optimal cycle in table 2.

[Insert table 2 here]

We got a periodicity $n = 3$ periods ($T_{PM}^* = 3\tau$), and $k=1$. So, we can say that the time windows are $[3p, 3p+2]$ with $p = 1 \dots \left\lfloor \frac{T}{T_{PM}^*} \right\rfloor - 1$. We will show computational results in the tables 3–5 obtained for each set of instances. All tests were run on a Pentium 4 (3 GHz) with 1 Gb of RAM. We used standard MIP software (XpressMP) with the solver default settings.

[Insert table 3 here]

[Insert table 4 here]

[Insert table 5 here]

To assess the quality of our model solved by XpressMP, we considered four parameters: number of products, number of periods, shortage cost, and capacity tightness. For both series of results, we provide:

- #BestSol: it can be instances that could be solved to optimality (*value) and the computational times vary from 0.15 seconds to 5 minutes, or instances which the best integer solution (value) is got after 20 min of computations.
- Gab (%): for the instances that could not be solved to optimality, the average relative gap value obtained between the best integer solution and the best lower bound.
- C1: total cost of separate problem.
- C2: total cost of integrated problem.

Now, we compare the results obtained with separated and integrated models. The results show, when capacity is moderately loose, the separate problem gives better result for some instances and same results for others, the reason is that the problem is considered as uncapacitated lot sizing problem. Notice that all instances are solved to optimality, the computational time varies from 0.15 seconds to 10 seconds for all instances. When capacity is tight, the integrated problem provides better or same results. The computational times vary from 2 minutes to 5 minutes to solve to optimality for all instances with $(N, T) = (3, 12)$, see tables 3, 4 and 5, and instances with $(N, T) = (6, 18)$ in separate case when shortage cost equals to 65 and 75, see table 3 and 4. When the capacity is too tight, for all instances the integrated problem gives better results. The computational times vary from 1 to 3 minutes to solve to optimality for all instances with $(N, T) = (3, 12)$ (tables 3, 4 and 5).

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3 As a conclusion, the integrated model gives better solution than the separate model for
4 different shortage cost and when the capacity is too tight.
5
6

7 8 **6. Conclusion**

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10 A joint production and maintenance planning model for a production system subject to
11 random failures has been proposed. This model takes, explicitly, into account the reliability
12 parameters of the system and its capacity in the development of the optimal production and
13 maintenance planning. Time windows are added to reduce demand shortage in period of high
14 demand and to give more flexibility to preventive maintenance actions. The computational
15 results show the effectiveness of our model in a more practical and realistic case when the
16 capacity is too tight, which means that the capacity is not sufficient to meet all demands. The
17 Optimization solver Xpress MP can solve to optimality problem with small instances, and for
18 larger ones we observe that it is enable to solve them to optimality, in particular when
19 capacity is too tight. Therefore, we think about solving problem with large instances using
20 relaxation method, and meta-heuristic in future.
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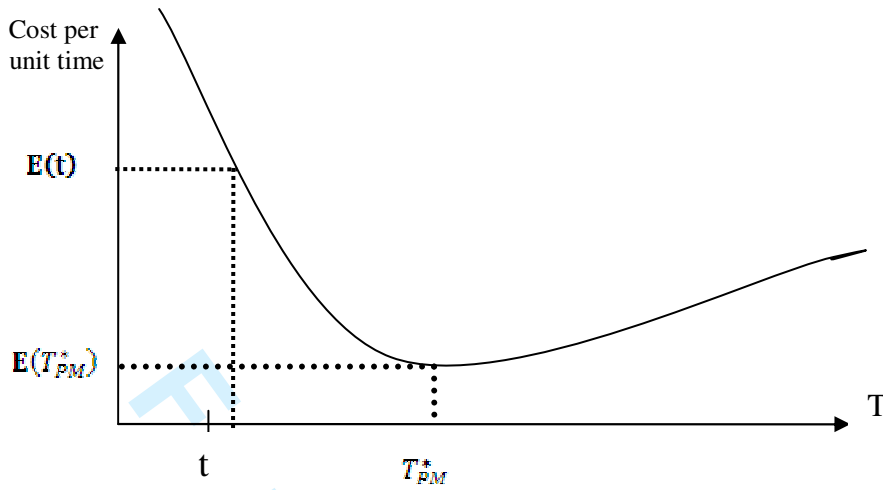


Figure 1: Expected maintenance cost per unit time of the system.

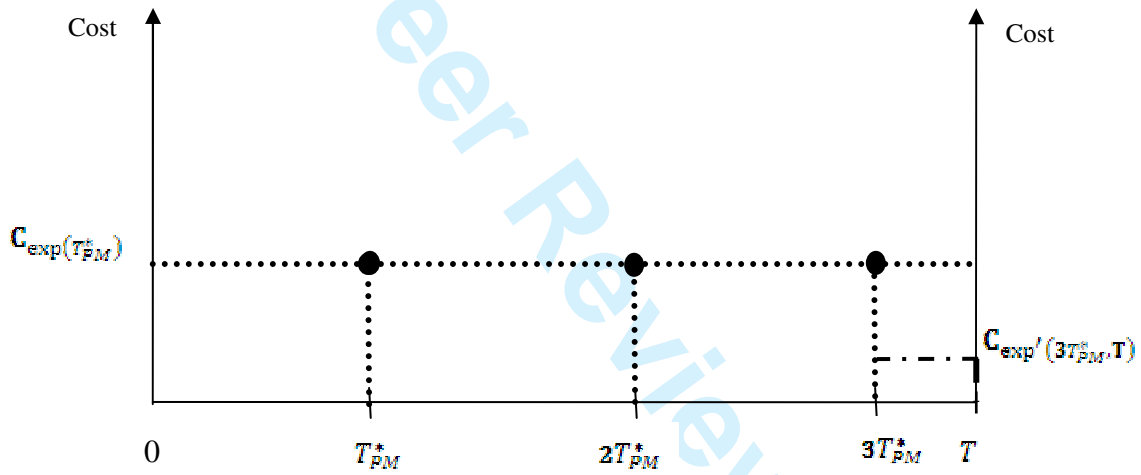


Figure 2: Expected maintenance cost per preventive maintenance period.

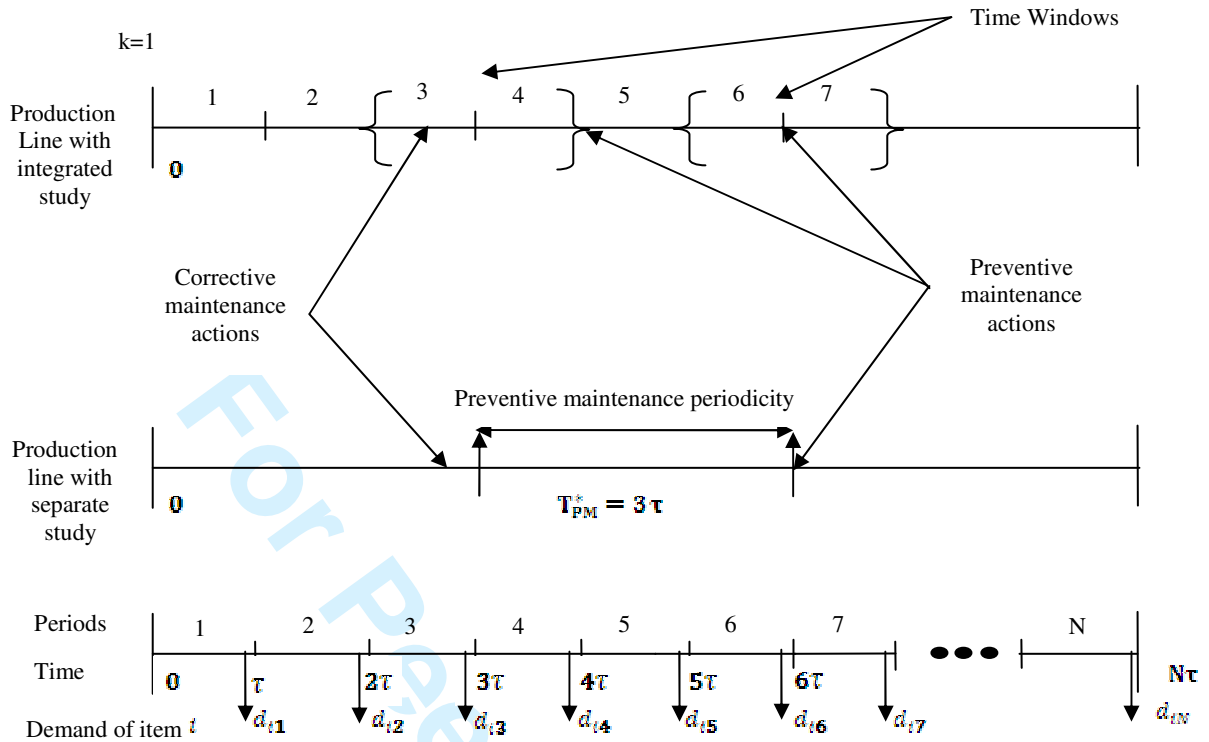


Figure 3: An integrated production and maintenance plan

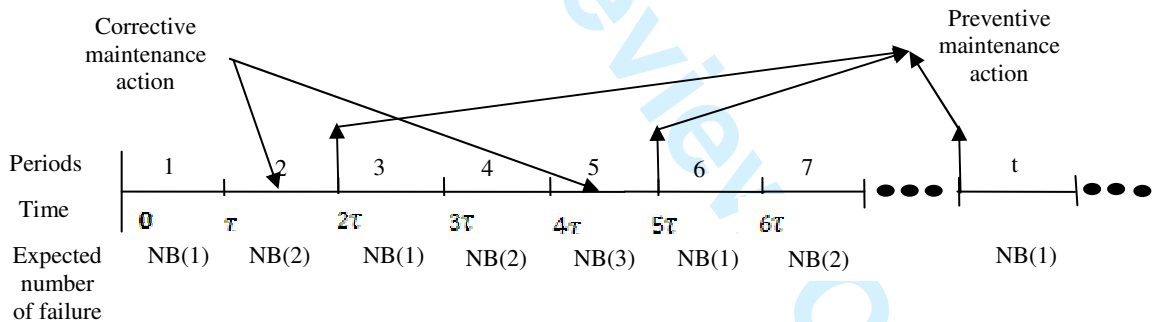


Figure 4: Expected number of failures according to preventive maintenance actions

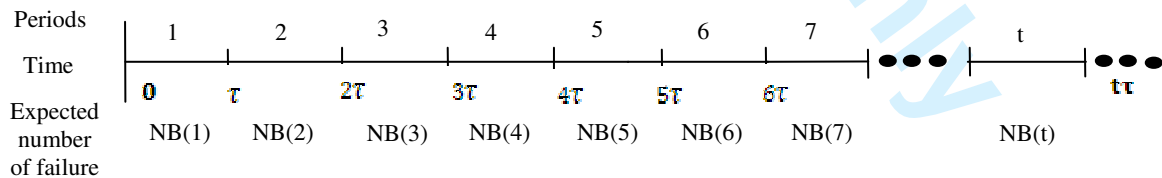


Figure 5: Expected number of failures without preventive maintenance actions.

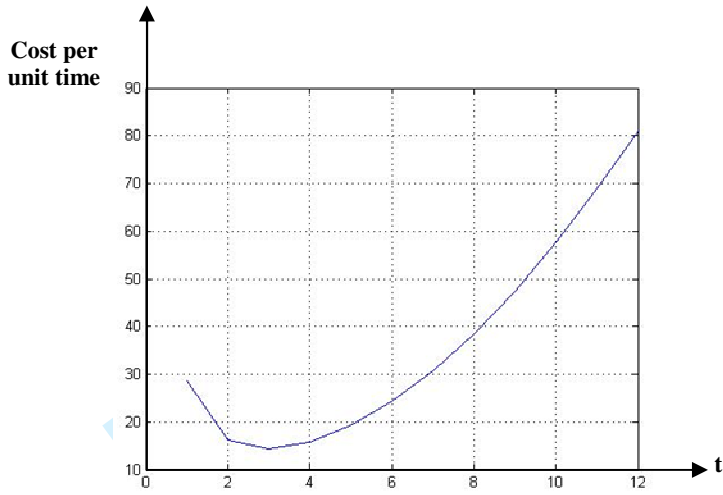


Figure 6: Average maintenance cost per unit time with T=12

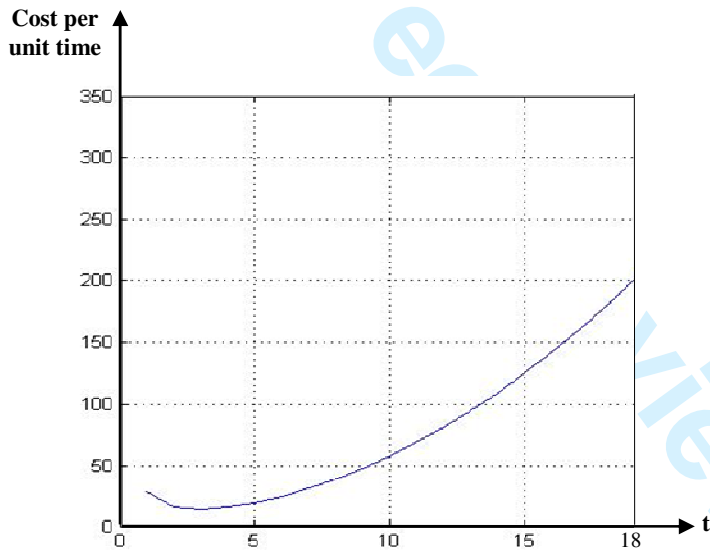


Figure 7: Average maintenance cost per unit time with T=18

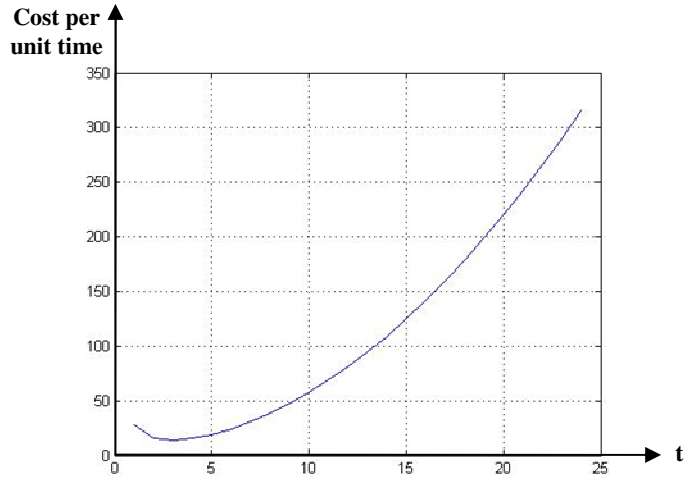


Figure 8: Average maintenance cost per unit time with $T=24$

Age	Expected number of failures	Age	Expected number of failures
$[0, \tau]$	0.0157	$[12\tau, 12\tau]$	7.3282
$[1\tau, 2\tau]$	0.1095	$[13\tau, 14\tau]$	8.5470
$[2\tau, 3\tau]$	0.2970	$[14\tau, 15\tau]$	9.8595
$[3\tau, 4\tau]$	0.5782	$[15\tau, 16\tau]$	11.2657
$[4\tau, 5\tau]$	0.9532	$[16\tau, 17\tau]$	12.7657
$[5\tau, 6\tau]$	1.4220	$[17\tau, 18\tau]$	14.3595
$[6\tau, 7\tau]$	1.9845	$[18\tau, 19\tau]$	16.0470
$[7\tau, 8\tau]$	2.6407	$[19\tau, 20\tau]$	17.8282
$[8\tau, 9\tau]$	3.3907	$[20\tau, 21\tau]$	19.7032
$[9\tau, 10\tau]$	4.2345	$[21\tau, 22\tau]$	21.6720
$[10\tau, 11\tau]$	5.1720	$[22\tau, 23\tau]$	23.7345
$[11\tau, 12\tau]$	6.2032	$[23\tau, 24\tau]$	25.8907

Table 1: Expected number of failures of the system for $T= 12, 24$.

	$T= 12$	$T=18$	$T= 24$
Optimal preventive maintenance periodicity (T_{PM}^*)	3	3	3
Expected total cost of maintenance	57.0318	85.5477	114.0635

Table 2: Expected total cost of maintenance of the system for $T= 12, 18, 24$.

Item	Period	Total cost	Shortage cost = 65					
			Capacity tightness		Tight		Too tight	
			Moderately loose		Tight		Too tight	
			#BestSol	Gap (%)	#BestSol	Gap (%)	#BestSol	Gap (%)
3	12	C1	*22527	0	*24265	0	*40822	0
		C2	*22621	0	*22436	0	*39071	0
6	18	C1	*62367	0	*79652	0	129797	0.017
		C2	*62367	0	79652	0.15	129687	0.09
12	24	C1	*174927	0	220042	0.04	346727	0.04
		C2	*175012	0	219977	0.12	346697	0.1

Table 3: Summary solution with shortage cost equals to 65.

Item	Period	Total cost	Shortage cost = 75					
			Capacity tightness		Tight		Too tight	
			Moderately loose		Tight		Too tight	
			#BestSol	Gap (%)	#BestSol	Gap (%)	#BestSol	Gap (%)
3	12	C1	*23447	0	*24982	0	*47802	0
		C2	*23541	0	*23046	0	*46784	0
6	18	C1	*67297	0	*84652	0	136222	0.035
		C2	*67297	0	84652	0.11	135911	0.21
12	24	C1	*173227	0	217387	0.02	371422	0.03
		C2	*173312	0	217387	0	370986	0.1

Table 4: Summary solution with shortage cost equals to 75.

Item	Period	Total cost	Shortage cost = 95					
			Capacity tightness		Tight		Too tight	
			Moderately loose		Tight		Too tight	
			#BestSol	Gap (%)	#BestSol	Gap (%)	#BestSol	Gap (%)
3	12	C1	*22947	0	*28092	0	*51647	0
		C2	*23041	0	*26145	0	*48231	0
6	18	C1	*64097	0	86132	0.03	160427	0.025
		C2	*64097	0	86132	0.1	160034	0.23
12	24	C1	*172737	0	223212	0.025	428437	0.05
		C2	*172822	0	223212	0.1	427724	0.16

Table 5: Summary solution with shortage cost equals to 95.