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AMR METHODS IN SOLIDS MECHANICS FOR PELLET-CLADDING INTERACTION MODELLING

LAURÉLINE BARBIÉ^{*,†}, ISABELLE RAMIÈRE^{*}, FRÉDÉRIC LEBON[†]
AND JÉRÔME SERCOMBE^{*}

^{*} CEA-Cadarache, DEN/DEC/SESC/LSC
13108 St Paul Lez Durance Cedex, France
e-mail: {laureline.barbie;isabelle.ramiere;jerome.sercombe}@cea.fr

[†] Laboratoire de Mécanique et d'Acoustique, UPR 7051, CNRS
31 chemin Joseph-Aiguier
13402 Marseille cedex 20, France
e-mail: {barbie;lebon}@lma.cnrs-mrs.fr

Key words: Adaptive Mesh Refinement (AMR) techniques, Local Defect Correction (LDC) method, Solids mechanics, Elastic behaviour, Pellet-Cladding Interaction (PCI)

Abstract: In this study, an Adaptive Mesh Refinement (AMR) method is applied in order to conduct reliable solids mechanics studies with acceptable computing times when local phenomena are under consideration. The Local Defect Correction (LDC) method is performed, which enables us to validate the AMR philosophy for homogeneous linear elastic behaviour. The test case under study outcomes from an industrial situation: the pellet-cladding interaction in pressurized water reactors. A simplified model focused on the cladding response is used.

1 INTRODUCTION

For now, AMR methods are widely used in fluid mechanics but not yet very applied in solids mechanics. However, when localized phenomena are under consideration, these techniques are very efficient. For that reason, the goal of our work is to extend AMR techniques to tridimensional non-linear solids mechanics (non-linear constitutive equations, contact, etc.).

This work is applied to a specific industrial test case: the pellet-cladding interaction (PCI) in pressurized water reactors. PCI is characterized by concentrated stress on the cladding, and the problem is fully tridimensional. As computing complete 3D simulations is limited by current computer capacities, AMR techniques are full of promise.

As a first step, we only consider the 2D response of the cladding, supposed to have a linear elastic behaviour. The effect of the pellet is then represented thanks to the boundary conditions.

2 ADAPTIVE MESH REFINEMENT APPROACH

2.1 General principle

Adaptive Mesh Refinement (AMR) methods are devoted to take into account localized phenomena and significant variations of scale. To guarantee the stability and the uniformity of the mesh, our approach is based on hierarchical multigrid architecture [1]. The regular structured AMR meshes are then generally not adapted to the local phenomena of the physical problem but located around these phenomena.

We consider a set of nested domains Ω_l , $0 \leq l \leq l^*$, containing the local phenomena of interest [2], such that $\Omega_0 = \Omega$ (original physical domain) and $\Omega_l \subset \Omega_{l-1}$. Nested structured regular sub-grids G_l are defined on each Ω_l , such that $h_l < h_{l-1}$. Then G_0 corresponds to the initial coarse grid defined on the original physical domain.

The core of a two-grid AMR process is composed by two steps:

1. A prolongation step from the coarse grid G_{l-1} to the fine grid G_l , which consists in defining the boundary conditions on the fine grid.

For $l \neq 0$ the boundary conditions on $\partial\Omega_l$ are defined by:

- on the part of the boundary common with $\partial\Omega$, a restriction of the boundary conditions defined on $\partial\Omega$.
- on the other part of the boundary, a projection of the former coarser solution obtained on G_{l-1} .

2. A restriction step from the fine grid G_l to the coarse grid G_{l-1} , which consists in correcting the solution on the coarse grid.

These two steps are repeated until the convergence of the coarse solution.

A local multigrid process is the generalization of the two-grid method. The two-grid algorithm is then repeated recursively on more sub-grids, as we can see figure 1 where S is the solver (which may be different on each grid), P the prolongation step and R the restriction step.

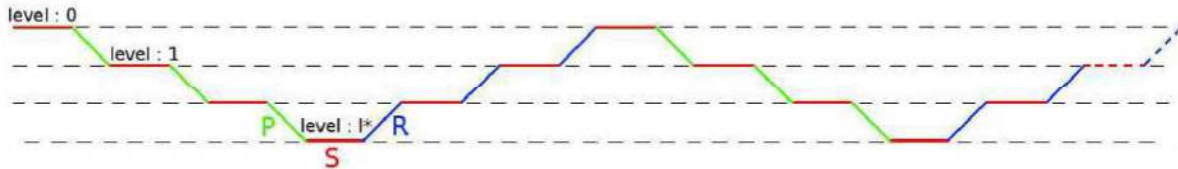


Figure 1: Local multigrid iteration represented by V-cycles

2.2 Local Defect Correction (LDC) method

This method, introduced by Hackbusch [3], consists in using a local defect correction in the restriction step of the AMR algorithm. We define the sets A_l (nodes of the grid G_l strictly included in the fine grid G_{l+1}) and \hat{A}_l (nodes of G_l such that the discretization scheme on these nodes only implies nodes of A_l).

If u_l^k is the displacement vector at level l and iteration k , L_l is the stiffness matrix at level l and f_l^k is the loadings vector at level l and iteration k , the restriction step writes:

- Compute the restricted solution $\tilde{u}_l^k(x) = (R_{l+1}^l u_{l+1}^k)(x) \quad \forall x \in A_l$, where R_{l+1}^l is the restriction operator (or fine-to-coarse operator),
- Compute the local defect $r_l^k(u)(x) = (f_l^0 - L_l \tilde{u}_l^k)(x) \quad \forall x \in \hat{A}_l$,
- Solve the corrected problem on G_l with $f_{\Omega_l}^k = f_{\Omega_l}^0 - \chi_{\hat{A}_l} r_l^k(u)$, where χ is the characteristic function.

In our case, we use an interpolation operator as restriction operator.

Due to the definition of A_l and \hat{A}_l , we have to use rather wide grids so that the areas where the restriction is applied are not too small. In our case, supposing that the boundary conditions are reliable, we preset $A_l = \hat{A}_l$.

3 INDUSTRIAL TEST CASE

3.1 Introduction to Pellet-Cladding Interaction (PCI)

The test case under study outcomes from an industrial situation, the pellet-cladding interaction in pressurized water reactors [4].

During the irradiation, the fuel pellet swells and the cladding creeps and shrinks inducing contact. Furthermore, fuel pellet is submitted to a non-isothermal loading during irradiation, which leads to thermal gradient in the pellet and, consequently, to high stresses which usually exceed the tensile strength of the material. As the pellet is fragmented (figure 2(a)), the contact between fuel pellet and cladding is discontinuous. This phenomenon is very localized (about micrometers for a structure about 10 millimeters of diameter).

Moreover, as the axial dimension of the fuel pellet is finite, the thermal gradient associated to fuel fragmentation leads to close the pellet-cladding gap first at the inter-pellet plane (hourglass shape of the pellet, figure 2(b)). This phenomenon is also very localized (about 0,1 millimeters for a structure of 14 millimeters of height).

Then to model PCI in PWRs, it is necessary to account for various non-linear physical phenomena which take place in the fuel pellet and the cladding, furthermore on a wide range of scales [5]. Modelling precisely this phenomenon is important, as it concerns the integrity of the cladding which is the first confinement barrier for the irradiated fuel. That's why research and development on this item are still undertaken worldwide in order

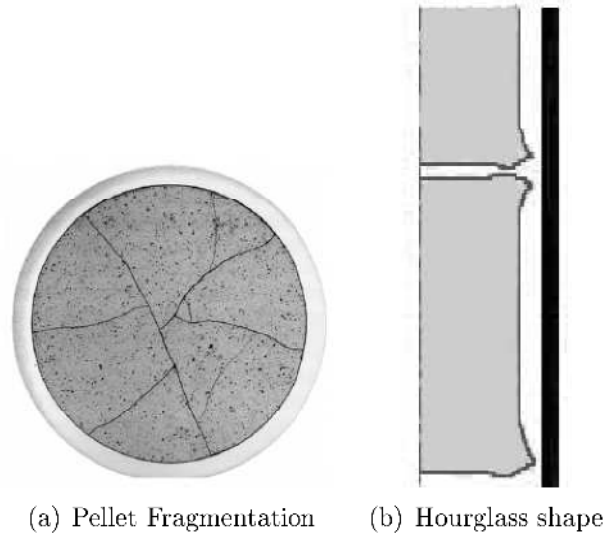


Figure 2: Pellet behaviour during irradiation

to improve the understanding of the mechanisms possibly leading to PCI failure, as well as to qualify a PCI resistant rod design.

Complete 3D simulations are currently limited because of the required unstructured and irregular mesh, inducing an ill-conditioned system with an important number of degrees of freedom. AMR approach seems then well suited for this application.

In this study, a simplified model focused on the cladding response is used. Its behaviour is supposed elastic and the contact with the pellet is represented by a discontinuous pressure on the internal radius.

3.2 2D modelling

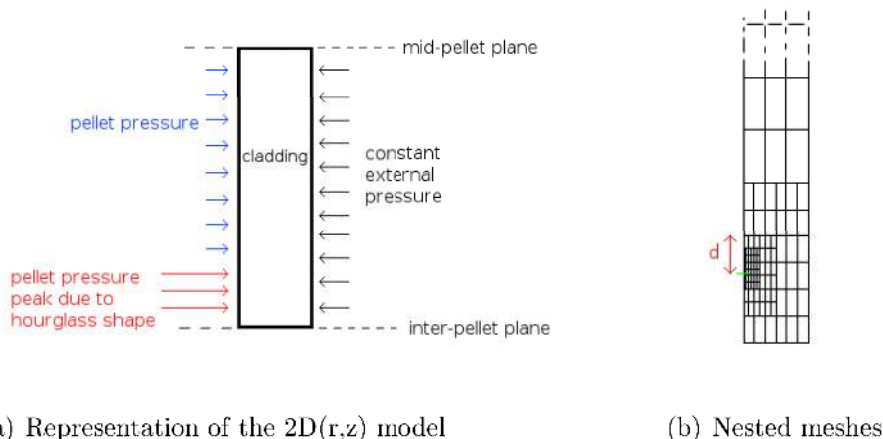
As first modelling, we consider separately the two phenomena occurring in the PCI and described before: the Hourglass phenomenon and the pellet fragmentation. Each of these phenomena can be completely described by 2D models as a first approximation.

The meshes used to obtain reference solutions are adapted to the problem geometry and the pressure discontinuity, whereas those used in the AMR approach are regular structured meshes, not adapted to the singularity but localised around it. For the two types of approaches, a Q1 FEM solver is used. In this article, ‘classical FEM’ denotes the FEM applied on meshes adapted to the physics of the problem while ‘AMR approach’ denotes the FEM applied within an AMR process on nested regular meshes. In the AMR approach, between two consecutive nested grids, we choose to divide the mesh step by two in each direction.

3.2.1 Hourglass shape phenomenon: 2D(r,z) modelling

Modelling hypothesis

Our model is axisymmetric and the hourglass shape of the fuel pellet is represented by a pressure peak on the internal radius of the cladding. Thanks to symmetrical considerations, we only consider a half-pellet height of the cladding (figure 3(a)).



(a) Representation of the 2D(r,z) model

(b) Nested meshes

Figure 3: Hourglass shape phenomenon

To validate our approach, we first compute a reference solution adapted to the discontinuity, obtained with a fine uniform mesh (with a step of 5 micrometers while the discontinuity is 160 micrometers from the inter-pellet plane).

Results of the AMR approach

For the AMR approach, we use nested meshes whose an exemple is given on figure 3(b). The interest of this approach is that these meshes are not adapted to the geometry of the discontinuity. We compare the results obtained with the reference mesh and with the AMR algorithm applied on nested meshes. For the AMR approach, the pressure gap is imposed from the closest upper node of the physical discontinuity.

We compare the Von Mises stress on the cladding, as presented on the figure 4. The Von Mises stress fields are very similar, which confirms our approach accuracy. To further appreciate the performance of our LDC method, we study several error norms (L_2 norm, L_∞ norm or max norm) calculated from the reference solution. For each simulation, it is the final coarse solution (obtained at convergence) that is considered in the error calculation. The AMR errors will be represented according to the initial distance between the singularity and its approximation (represented by d on the figure 3(b)).

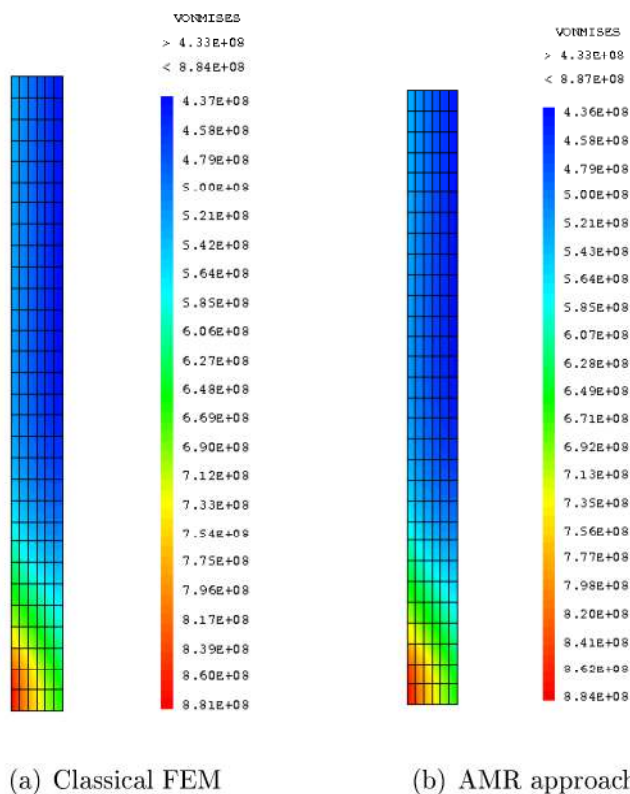


Figure 4: Von Mises stresses for the 2D(r,z) model, interpolated on the coarser grid

We remind that the norms are:

$$\|v\|_{L_2} = \left(\int_{\Omega} v^2 d\omega \right)^{1/2} \quad \text{and} \quad \|v\|_{L_{\infty}} = \max_{\Omega} (v)$$

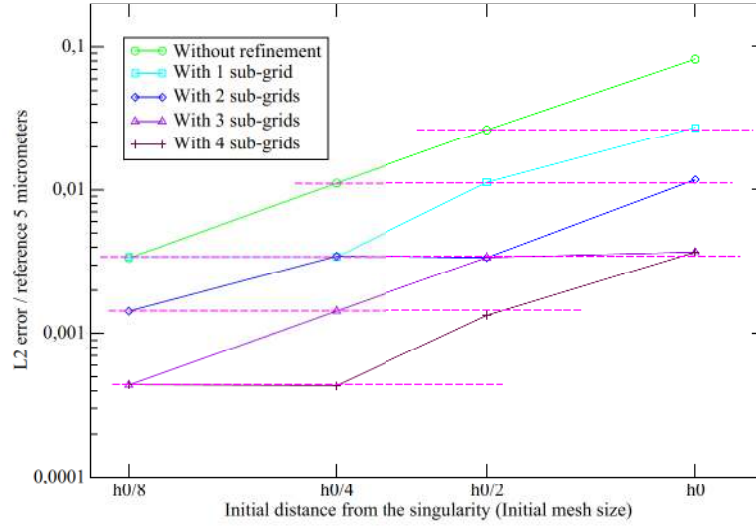
Figure 5 confirms that the two norms are equivalents, as we observe the same behaviour on the two graphics.

We observe that without refinement (0 sub-grid), the error decreases linearly with the singularity approximation: a first-order accuracy is then obtained. The loss of an order of convergence (the classical Q1 FEM is second-order accurate for the L_2 -norm) is due to the use of meshes non-adapted to the discontinuity.

Moreover, whatever the number of sub-grids used, for the same finer grid size we obtain the same error after convergence: then the method is of first-order with respect to the finest grid [6].

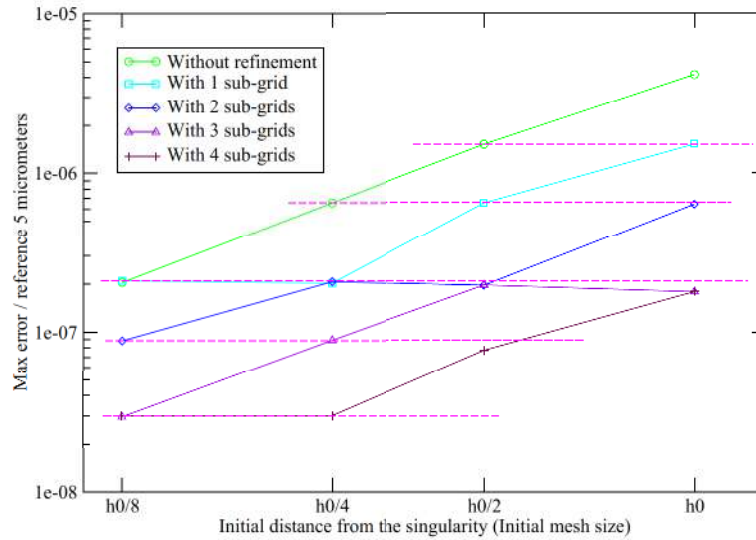
Lastly, we can observe a plateau for some sub-grids. This plateau is due to the fact that in this case, between two sub-grids, even if the mesh size is divided by two, the singularity approximation is not improved.

LDC algorithm applied to 2D(r,z) model



(a) L_2 error norm

LDC algorithm applied to 2D(r,z) model



(b) L_∞ error norm

Figure 5: Results for the 2D(r,z) model

As we observe the same error for a global fine grid than for nested local sub-grids with the same finest mesh size, we can conclude that our AMR approach is validated for the 2D linear elastic behaviour.

3.2.2 Contact with a fractured pellet: $2D(r,\theta)$ modelling

Modelling hypothesis

We focus here on the mid-pellet plane (in order to use generalized plane strains hypothesis). It does not allow to account hourglass shape but it enables us to describe fuel crack opening.

Here, we assume that the fuel pellet is initially fragmented in eight symmetrical parts, as often observed after base irradiation [7]. Thanks to symmetrical considerations, we only represented a sixteenth of the cladding (figure 6). As the fractures are open at the external radius of the pellet, there is no contact between the pellet and the cladding in the fracture plan. Then, in our symmetrical representation, under the fuel pellet fracture, no pressure is applied. For the AMR approach, the pressure gap is imposed from the closest upper node of the physical discontinuity.

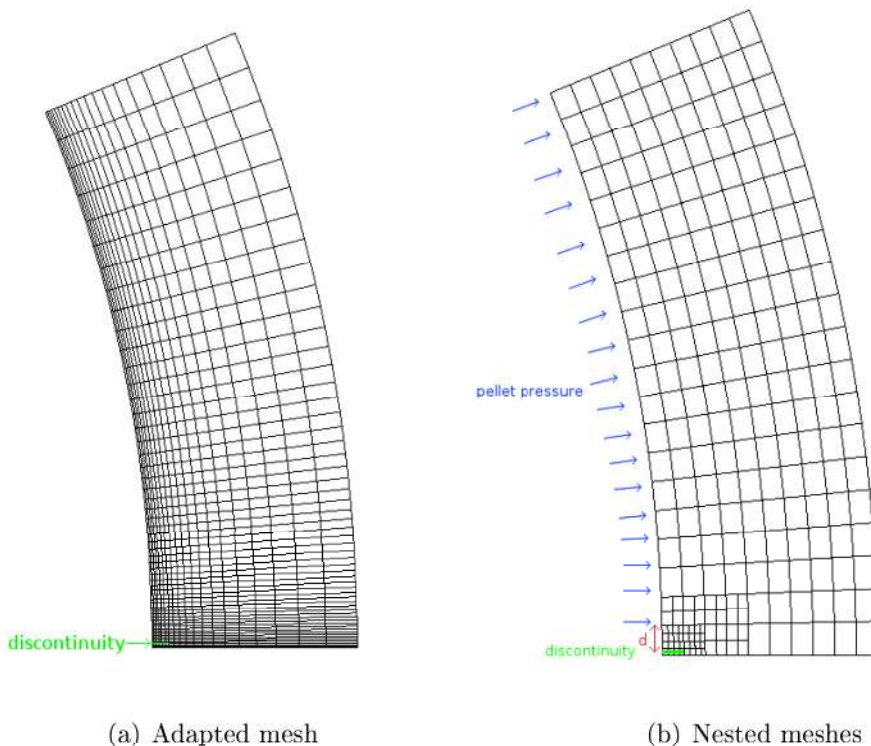


Figure 6: Contact with fractured pellet

We study the influence of the singularity position. We study a pressure gap of 8 and 10 micrometers, which is very localized as the cladding has an internal diameter of 8 millimeters and a thickness of 0,6 millimeters.

Singularity thickness of $8 \mu\text{m}$

Due to the circular geometry, our meshes are approximations of the global domain studied.

The reference solution is adapted to the discontinuity, which implies a really non-uniform mesh (cf. example figure 6(a) whose mesh step varies from $5\mu\text{m}$ to $50\mu\text{m}$). The AMR nested meshes are not strictly uniform but are as regular as possible (cf. figure 6(b)). Figure 7 validates the AMR approach as we obtain almost the same result on a global fine grid than on local nested grids with the same finest discretization step.

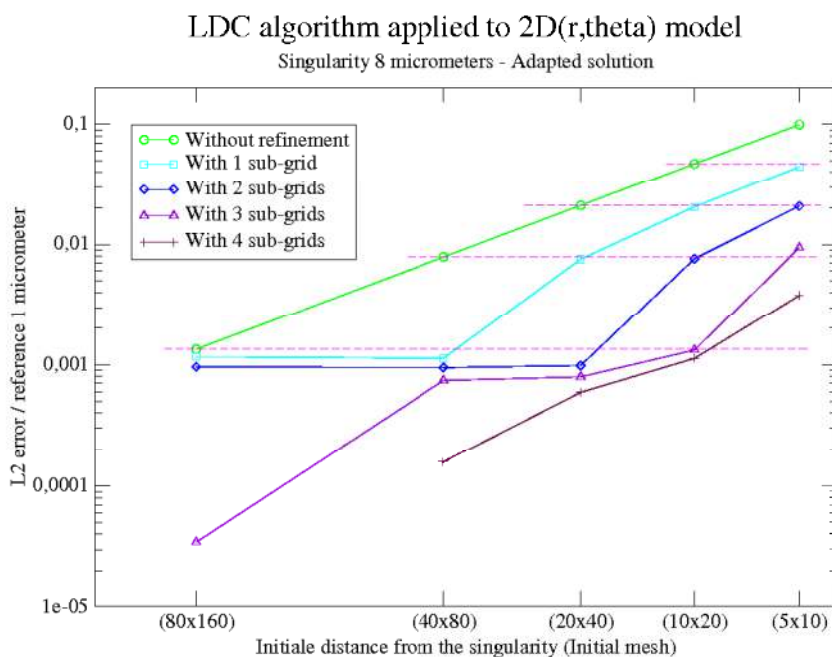


Figure 7: Results for the 2D(r,θ) model, singularity thickness of $8\mu\text{m}$

Singularity thickness of $10 \mu\text{m}$

We study the case where the singularity approximation is fastly improved between two mesh sizes (see figure 8 between mesh (40x80) and mesh (80x160)).

When the error decreases abruptly, the correction on local grid could be not sufficient. Thus, when this phenomenon comes on the second or third sub-grid, we see on the figure 8 that the correction is not as effective as if it happens on the first sub-grid. This weakness could be explained by the fewer zones where the correction is done. However, the AMR algorithm still brings partial correction.

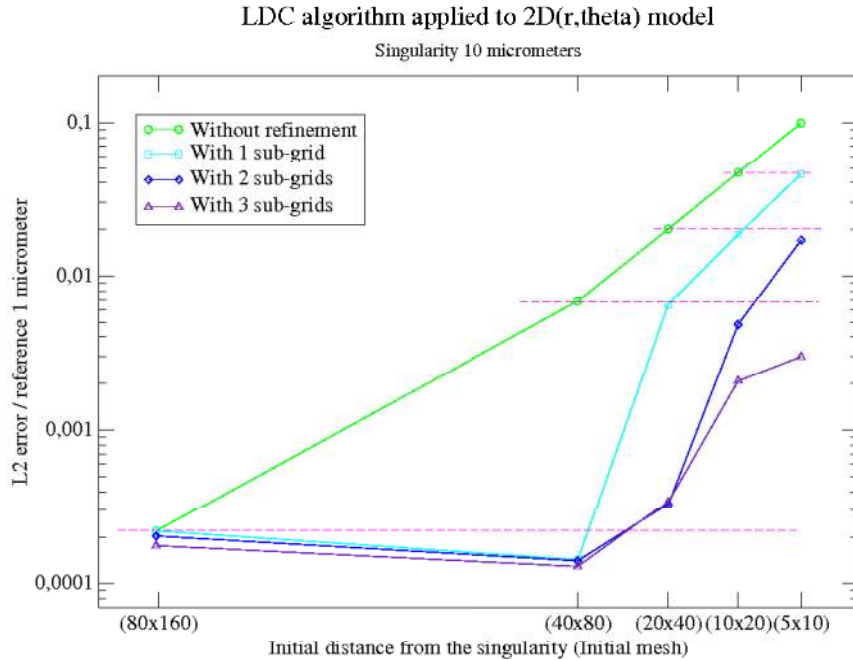


Figure 8: Results for the 2D(r,θ) model, singularity thickness of $10\mu\text{m}$

3.2.3 CPU time and memory space required for each approach

We also compare CPU time required by the simulation with regard to the precision obtained (see figure 9), the results without refinement being FEM computations on a non-adapted mesh (without any sub-grid).

The more the precision expected is constraining, the more the AMR approach is advantageous. In the same way, the more the precision expected is constraining, the more the use of several sub-grids on the AMR approach is interesting.

AMR techniques are also interesting for saving memory space, as on each sub grid we only use a few number of nodes distributed equitably.

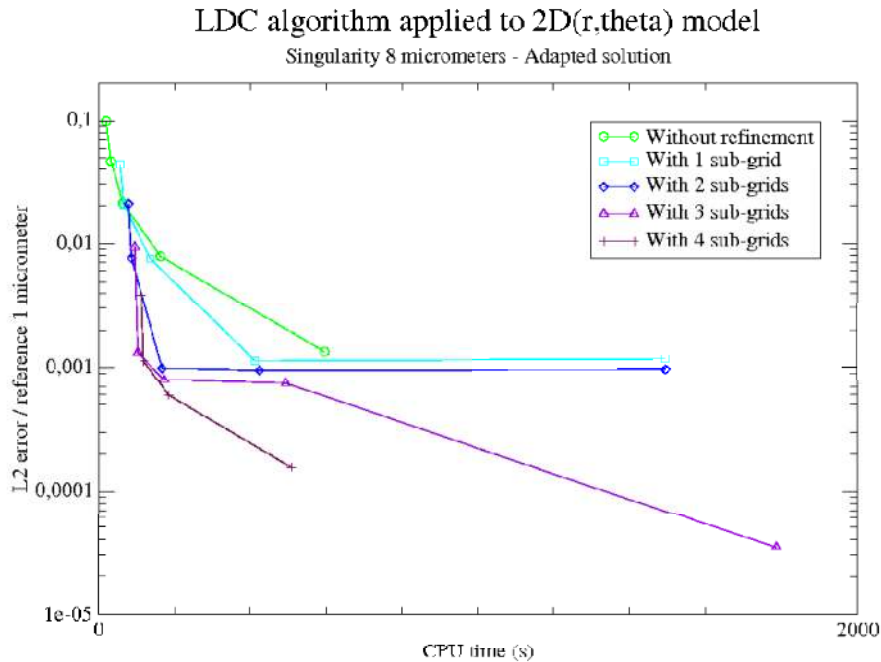


Figure 9: CPU time needed for the 2D(r,θ) model

4 CONCLUSION AND PERSPECTIVES

To conclude, this work was a first step to validate AMR techniques for solids mechanics problems.

For the 2D(r,z) model, the geometry of the problem is very simple and can be represented perfectly by rectangular meshes. In this case, we obtained strictly the same result for global fine grid or AMR approach with a finest grid with the same mesh size.

For the 2D(r,θ) model, the geometry of the problem is not so simple but AMR approach is still efficient.

Using AMR techniques allows us to save CPU time and memory space during computation, which could allow to compute problems whose computation is impossible using classical FEM approaches, such as 3D PCI modelling. Meshes are not adapted to the singularity but we saw here that we obtain satisfactory results (validated in 2D).

However, there are some limitations, especially when the approximation varies abruptly between two consecutive nested meshes for example.

Now, the validation of this technique has to be extended to solids mechanics specificities, such as material non-linearity, contact...

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