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Three-particle correlations in QCD parton showers

Redamy Pérez-Ramos,¹ Vincent Mathieu,^{1,2} and Miguel-Angel Sanchis-Lozano¹

¹*Departament de Física Teòrica and IFIC, Universitat de València - CSIC
Dr. Moliner 50, E-46100 Burjassot, Spain*

²*Service de Physique Nucléaire et Subnucléaire, Université de Mons-UMons
Place du Parc 20, B-7000 Mons, Belgium*

Three-particle correlations in quark and gluon jets are computed for the first time in perturbative QCD. We give results in the double logarithmic approximation and the modified leading logarithmic approximation. In both resummation schemes, we use the formalism of the generating functional and solve the evolution equations analytically from the steepest descent evaluation of the one-particle distribution. We thus provide a further test of the local parton hadron duality (LPHD) and make predictions for the LHC.

The observation of quark and gluon jets has played a crucial role in establishing Quantum Chromodynamics (QCD) as the theory of strong interaction within the Standard Model of particle physics. The jets, narrowly collimated bundles of hadrons, reflect configurations of quarks and gluons at short distances. Powerful schemes, like the Double Logarithmic Approximation (DLA) and the Modified Leading Logarithmic Approximation (MLLA), which allow for the perturbative resummation of soft-collinear and hard-collinear gluons before the hadronization occurs, have been developed over the past thirty years (for a review see [1]). One of the striking predictions of perturbative QCD (pQCD), which follows as a consequence of Angular Ordering (AO) within the MLLA and the Local Parton Hadron Duality (LPHD) hypothesis [2], is the existence of the hump-backed shape [1] of the inclusive energy distribution of hadrons, later confirmed by experiments at colliders. Thus, the study of inclusive observables like the inclusive energy distribution and k_{\perp} spectra of hadrons [3] have shown that the perturbative stage of the process, which evolves from the hard scale or leading parton virtuality $Q \sim E$ to the hadronization scale Q_0 , is dominant. In particular, these issues suggest that the hadronization stage of the QCD cascade plays a subleading role and therefore, that the LPHD hypothesis is successful while treating one-particle inclusive observables.

The study of particle correlations in intrajet cascades, which are less inclusive observables, focuses on providing tests of the partonic dynamics and the LPHD. In [4], this observable was computed for the first time at small x in MLLA for particles staying close to the maximum of the one-particle distribution. In [5], the previous solutions were extended, at MLLA, to all possible values of x by solving the QCD evolution equations exactly. This observable was measured by the OPAL collaboration in the e^+e^- annihilation at the Z^0 peak, that is for $\sqrt{s} = 91.2$ GeV at LEP [6]. Though the agreement with predictions presented in [5] was improved, a discrepancy still subsists pointing out a possible failure of the LPHD for less inclusive observables. However, these measurements were repeated by the CDF collaboration in $p\bar{p}$ collisions at the Tevatron for mixed samples of quark and

gluon jets [7]. The agreement with predictions presented in [4] turned out to be rather good, specially for particles having very close energy fractions ($x_1 \approx x_2$). A discrepancy between the OPAL and CDF analysis showed up and still stays unclear. Therefore, the measurement of the two-particle correlations at higher energies at the LHC becomes crucial. Furthermore, going one step beyond, in this letter we give predictions for the three-particle correlations inside quark and gluon jets. This observable and the two-particle correlations can be measured in equal footing at the LHC so as to provide further verifications of the LPHD for less inclusive observables.

A generating functional $Z(E, \Theta; \{u\})$ can be constructed [1] that describes the azimuth averaged parton content of a jet of energy E with a given opening half-angle Θ ; by virtue of the exact AO (MLLA), which satisfies an integro-differential system of evolution equations. In order to obtain *exclusive* n -particle distributions $D_A^{(n)}(k_i, E)$ one takes n variational derivatives of Z_A over $u(k_i)$ with appropriate particle momenta, $i = 1 \dots n$, and sets $u \equiv 0$ after wards; *inclusive* distributions are generated by taking variational derivatives around $u \equiv 1$. Let us introduce the n -particle differential correlations for $A = G, Q, \bar{Q}$ jets as,

$$A_{1\dots n}^{(n)}(z) \equiv \frac{x_1}{z} \dots \frac{x_n}{z} D_A^{(n)} \left(\frac{x_1}{z} \dots \frac{x_n}{z}, \ln \frac{zQ}{Q_0} \right), \quad (1)$$

together with $A_{1\dots n}^{(n)} \equiv A_{1\dots n}^{(n)}(1)$ for later use; x_i corresponds to the Feynman energy fraction of the jet taken away by one hadron “ i ”. For instance, for three-particle correlations $n = 3$, the observable to be measured reads $\mathcal{C}_{A_{123}}^{(3)} = \frac{A_{123}^{(3)}}{A_1 A_2 A_3}$. The production of three hadrons is displayed in Fig.1 after a quark or a gluon (A) jet of energy E with half opening angle Θ_0 and virtuality $Q = E\Theta_0$ has been produced in a high energy collision. The kinematical variable characterizing the process is given by the transverse momentum $k_{\perp} = zE\Theta_1 \geq Q_0$ (or $(1-z)E\Theta_1 \geq Q_0$) of the first splitting $A \rightarrow BC$. The parton C fragments into three offspring such that three hadrons of energy fractions x_1, x_2 and x_3 can be triggered from the same cascade following the condition $\Theta_0 \geq \Theta_1 \geq \Theta_2 \geq \Theta_3$, which arises from exact

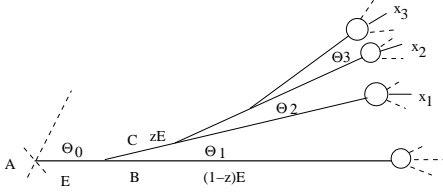


FIG. 1: Three-particle yield and angular ordering inside a high energy jet.

AO in MLLA [1]. We make use of variables, $\ell = \ln \frac{z}{x_i}$, $y = \ln \frac{x_j E \Theta_1}{Q_0}$, $\ell_i = \ln \frac{1}{x_i}$, $y_j = \ln \frac{x_j E \Theta_0}{Q_0}$, $\eta_{ij} = \ln \frac{x_i}{x_j}$, $Y = \ell_i + y_j + \eta_{ij} = \ln(Q/Q_0)$ with $\eta_{ij} = \ln(x_i/x_j)$ and $\lambda = \ln(Q_0/\Lambda_{QCD})$. The two variables entering the evolution equations are z and Θ_1 , such that $x_1 \leq z \leq 1 \Rightarrow 0 \leq \ell \leq \ell_1$. Accordingly, the anomalous dimension related to the coupling constant can be parametrized as

$$\gamma_0^2(Q^2) = 2N_c \frac{\alpha_s(Q^2)}{\pi}, \quad \gamma_0^2(\ell + y) = \frac{1}{\beta_0(\ell + y + \eta_{ij} + \lambda)},$$

where $\beta_0 = \frac{1}{4N_c} (\frac{11}{3}N_c - \frac{4}{3}n_f T_R)$, with $T_R = 1/2$ and n_f the number of light quark flavors. From AO and the initial condition at threshold $x_3 E \Theta_0 \geq x_3 E \Theta_1 \geq x_3 E \Theta_3 \geq Q_0$, one has the bounds $\frac{Q_0}{x_3 E} \leq \Theta_1 \leq \Theta_0 \Rightarrow 0 \leq y \leq y_3$ for the integrated evolution equations. The evolution equations satisfied by (1) are derived from the MLLA master equation for the generating functional $Z_A(E, \Theta; u(k_i))$. For three-particle correlations, one takes the first $\frac{\delta Z_A}{\delta u(k_1)}$, second $\frac{\delta^2 Z_A}{\delta u(k_1) \delta u(k_2)}$, and finally third $\frac{\delta^3 Z_A}{\delta u(k_1) \dots \delta u(k_3)}$ functional derivatives of $Z_A(E, \Theta; u(k_i))$ over the probing functions $u(k_i)$ so as to obtain the differential system of evolution equations:

$$\hat{Q}_{\ell y}^{(3)} = \frac{C_F}{N_c} \gamma_0^2 G^{(3)} - \frac{3}{4} \frac{C_F}{N_c} \gamma_0^2 \left(G_{\ell}^{(3)} - \beta_0 \gamma_0^2 G^{(3)} \right), \quad (2)$$

$$\begin{aligned} \hat{G}_{\ell y}^{(3)} &= \gamma_0^2 G^{(3)} - a \gamma_0^2 \left(G_{\ell}^{(3)} - \beta_0 \gamma_0^2 G^{(3)} \right) \\ &+ (a-b) \gamma_0^2 \left[\left(\hat{G}_{12}^{(2)} G_3 + \hat{G}_{13}^{(2)} G_2 + \hat{G}^{(2)23} G_1 \right)_{\ell} \right. \\ &- \left. \beta_0 \gamma_0^2 \left(\hat{G}_{12}^{(2)} G_3 + \hat{G}_{13}^{(2)} G_2 + \hat{G}^{(2)23} G_1 \right) \right] \\ &+ (a-c) \gamma_0^2 \left[(G_1 G_2 G_3)_{\ell} - \beta_0 \gamma_0^2 G_1 G_2 G_3 \right], \end{aligned} \quad (3)$$

where $\hat{A}_{ij}^{(2)} = A_{ij}^{(2)} - A_i A_j$ and $\hat{A}^{(3)} = A^{(3)} - A_1 A_2 A_3 - \hat{A}_{12}^{(2)} A_3 - \hat{A}_{13}^{(2)} A_2 - \hat{A}_{23}^{(2)} A_1$. The subscripts ℓ and y in the equations (2) and (3) denote $\partial/\partial \ell$ and $\partial/\partial y$ respectively. The first terms of the equations in (2) and (3) are of classical origin and therefore, universal. Corrections $\propto -\frac{3}{4}$, a , $(a-b)$ and $(a-c)$, which are $\mathcal{O}(\sqrt{\alpha_s})$ suppressed, better account for energy conservation at each vertex of the splitting process, as compared with the DLA $\mathcal{O}(1)$. The hard constants are obtained after integration over the regular part of the DGLAP splitting functions [1] as performed in [4, 5]. In the equation for the gluon initiated jet (3), the first and second constants $a(n_f = 3) = 0.935$

and $b(n_f = 3) = 0.915$ were obtained in the frame of the single inclusive distribution and two-particle correlations respectively [4]. The third constant $c(n_f)$ appearing for the first time in this frame reads

$$c(n_f) = \frac{1}{4N_c} \left[\frac{11}{3} N_c + \frac{4}{3} n_f T_R \left(1 - 2 \frac{C_F}{N_c} \right)^3 \right]^{n_f=3} \stackrel{=}{=} 0.917.$$

The equation (3) is self-contained and can be solved iteratively by setting $G^{(3)} = C_{G_{123}}^{(3)} G_1 G_2 G_3$ and $G_{ij}^{(2)} = C_{G_{ij}}^{(2)} G_i G_j$ in the left and right hand sides of (3). Accordingly, the solution of (2) is also obtained by setting $Q^{(3)} = C_{Q_{123}}^{(3)} Q_1 Q_2 Q_3$ and $Q_{ij}^{(2)} = C_{Q_{ij}}^{(2)} Q_i Q_j$ in the l.h.s. of (2) and $G^{(3)} = C_{G_{123}}^{(3)} G_1 G_2 G_3$ in the r.h.s. of the same equation such that the iterative solutions can be written in the compact form

$$\begin{aligned} C_{A_{123}}^{(3)} - 1 &= \left(C_{A_{12}}^{(2)} - 1 \right) F_{A_{12}}^{(2)} + \left(C_{A_{13}}^{(2)} - 1 \right) F_{A_{13}}^{(2)} \\ &+ \left(C_{A_{23}}^{(2)} - 1 \right) F_{A_{23}}^{(2)} + \frac{N_c^2}{C_A^2} F_{A_{123}}^{(3)}. \end{aligned} \quad (4)$$

The MLLA two-particle correlators $C_{A_{12}}^{(2)}$ will be taken from [5] for the computation of $C_{A_{123}}^{(3)}$. Moreover,

$$F_{G_{ij}}^{(2)} = 1 + \frac{1 - b \Psi_{\ell} + \xi_1^{ij} - \epsilon_1}{2 + \Delta_{12} + \Delta_{13} + \Delta_{23} + \epsilon_1}, \quad (5)$$

$$F_{G_{123}}^{(3)} = \frac{1 - c \Psi_{\ell} + \xi_1^{12} + \xi_1^{13} + \xi_1^{23} - \epsilon_1}{2 + \Delta_{12} + \Delta_{13} + \Delta_{23} + \epsilon_1} \quad (6)$$

and for the quark jet

$$F_{Q_{ij}}^{(2)} = 1 + \frac{\tilde{\xi}_1^{ij} - \tilde{\epsilon}_1}{3 + \Delta_{12} + \Delta_{13} + \Delta_{23} - a \Psi_{\ell} + \tilde{\epsilon}_1}, \quad (7)$$

$$F_{Q_{123}}^{(3)} = \frac{C_{G_{123}}^{(3)} (1 - a \Psi_{\ell}) + \tilde{\xi}_1^{12} + \tilde{\xi}_1^{13} + \tilde{\xi}_1^{23} - \tilde{\epsilon}_1}{3 + \Delta_{12} + \Delta_{13} + \Delta_{23} - a \Psi_{\ell} + \tilde{\epsilon}_1}, \quad (8)$$

where $\Psi_{\ell} = \psi_{1,\ell} + \psi_{2,\ell} + \psi_{3,\ell} = \mathcal{O}(\gamma_0)$ and $\psi = \ln[G(\ell, y)]$. Higher order corrections arising from the solution of the system of equations (2,3) have been neglected in (4). In this case, $G(\ell, y)$ is the inclusive energy distribution, which will be inserted from the steepest descent method presented in [5]. The other functions appearing in (5,6) are $\Delta_{ij} = \gamma_0^{-2} (\psi_{i,\ell} \psi_{j,y} + \psi_{i,y} \psi_{j,\ell}) = \mathcal{O}(1)$ and

$$\zeta_{\ell} = \frac{\dot{C}_{G_{123},\ell}^{(3)}}{\dot{C}_{G_{123}}^{(3)}} = \mathcal{O}(\gamma_0^2), \quad \zeta_y = \frac{\dot{C}_{G_{123},y}^{(3)}}{\dot{C}_{G_{123}}^{(3)}} = \mathcal{O}(\gamma_0^2),$$

$$\chi_{\ell}^{ij} = \frac{\dot{C}_{G_{ij},\ell}^{(2)}}{\dot{C}_{G_{ij}}^{(2)}} = \mathcal{O}(\gamma_0^2), \quad \chi_y^{ij} = \frac{\dot{C}_{G_{ij},y}^{(2)}}{\dot{C}_{G_{ij}}^{(2)}} = \mathcal{O}(\gamma_0^2),$$

$$\xi_1^{ij} = \gamma_0^{-2} \left(\chi_{\ell}^{ij} \Psi_y + \chi_y^{ij} \Psi_{\ell} \right) = \mathcal{O}(\gamma_0),$$

$$\epsilon_1 = \gamma_0^{-2} (\zeta_{\ell} \Psi_y + \zeta_y \Psi_{\ell}) = \mathcal{O}(\gamma_0),$$

with $\zeta = \ln \dot{C}_{G_{123}}^{(3)}$, $\chi = \ln \dot{C}_G^{(2)}$. The set of functions appearing in (7,8) is obtained from the previous by replacing $\zeta \rightarrow \tilde{\zeta}$, $\chi \rightarrow \tilde{\chi}$, $\xi \rightarrow \tilde{\xi}$, $\dot{C}_{G_{ij}}^{(2)} \rightarrow \dot{C}_{Q_{ij}}^{(2)}$ and $\dot{C}_{G_{ij}}^{(3)} \rightarrow \dot{C}_{Q_{ij}}^{(3)}$

where the dotted $\dot{C}_{A_{ij}}^{(2)}$ and $\dot{C}_{A_{ij}}^{(3)}$ are the DLA solutions of the two- and three-particle correlators; that is why this solution is said to be iterative. The DLA two-particle correlators are taken from [8] and the DLA expression for $\dot{C}_{A_{ij}}^{(3)}$ can be obtained from (4) by setting all MLLA $\mathcal{O}(\gamma_0)$ corrections to zero:

$$\dot{C}_{A_{ij}}^{(2)} - 1 = \frac{N_c}{C_A} \frac{1}{1 + \Delta_{ij}}; \quad (9)$$

$$\begin{aligned} (\dot{C}_{A_{123}}^{(3)} - 1) - (\dot{C}_{A_{12}}^{(2)} - 1) - (\dot{C}_{A_{13}}^{(2)} - 1) - (\dot{C}_{A_{23}}^{(2)} - 1) & \quad (10) \\ &= \frac{N_c}{C_A} \frac{(\dot{C}_{A_{12}}^{(2)} - 1) + (\dot{C}_{A_{13}}^{(2)} - 1) + (\dot{C}_{A_{23}}^{(2)} - 1)}{2 + \Delta_{12} + \Delta_{13} + \Delta_{23}} \\ &+ \frac{N_c^2}{C_A^2} \frac{1}{2 + \Delta_{12} + \Delta_{13} + \Delta_{23}}. \end{aligned}$$

The solutions have the following simple physical interpretation: the first term ($= -1$) in the l.h.s. translates the independent or decorrelated emission of three hadrons in the shower. After inserting the two-particle correlator with color factor $\propto \frac{N_c}{C_A}$ (9) in the l.h.s. of (10), terms $\propto \frac{N_c}{C_A}$ correspond to the case where two partons are correlated inside the same subjet, while the other one is emitted independently from the rest. Next, replacing (9) in the r.h.s. of (10), one obtains a contribution $\propto \frac{N_c^2}{C_A^2}$ describing the independent emission of two partons inside the same subjet. The last term $\propto \frac{N_c^3}{C_A^3}$ involves three particles strongly correlated inside the same partonic shower as depicted in Fig.1. This term corresponds indeed to the genuine cumulant correlation $F_{A_{123}}^{(3)}$, first obtained in this letter for this observable.

The evaluation of (4), which is expressed in terms of the logarithmic derivatives of the single inclusive distribution $\ln[G(\ell, y)]$, will be performed using the steepest descent method to determine $G(\ell, y)$ [5, 8]. Thus, the MLLA logarithmic derivatives were written in [5] in the form: $\psi_{i,\ell} = \gamma_0 e^{\mu_i} + a\gamma_0 f_1(\mu_i, \nu_i) + \beta_0 \gamma_0 f_2(\mu_i, \nu_i)$, $\psi_{i,y} = \gamma_0 e^{-\mu_i} + a\gamma_0 \tilde{f}_1(\mu_i, \nu_i) + \beta_0 \gamma_0 \tilde{f}_2(\mu_i, \nu_i)$, where (μ_i, ν_i) are expressed as functions of the original variables (ℓ, y) by inverting the non-linear system of equations [8]:

$$\begin{aligned} \frac{y_i - \ell_i}{\ell_i + y_i} &= \frac{(\sinh 2\mu_i - 2\mu_i) - (\sinh 2\nu_i - 2\nu_i)}{2(\sinh^2 \mu_i - \sinh^2 \nu_i)}, \\ \frac{\sinh \nu_i}{\sqrt{\lambda}} &= \frac{\sinh \mu_i}{\sqrt{\ell_i + y_i + \lambda}}. \end{aligned}$$

In particular, this method allows for the estimation of the observable for particles with energies near the maximum or hump of the one-particle distribution $|\ell - \ell_i| \ll \sigma \propto Y^{3/2}$, which applied to the three-particle correlations, will appear in a forthcoming paper. For instance, at DLA one has $\Delta_{ij} = 2 \cosh(\mu_i - \mu_j)$ with such a parametrization of the logarithmic derivatives of the inclusive spectrum. Close to the hump one has $\Delta_{ij} \simeq (\ell_i - \ell_j)^2$, thus the correlations are expected to be quadratic as a function of $(\ell_i - \ell_j)$ and to have a maximum for particles with the

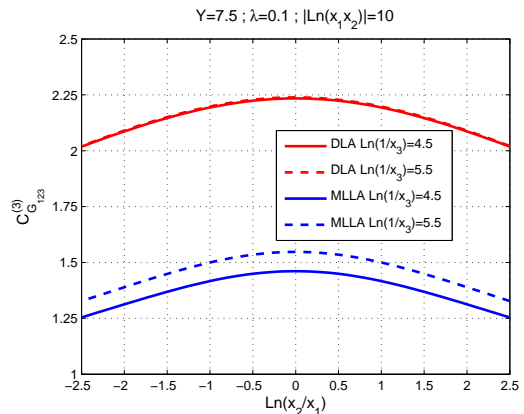


FIG. 2: Gluon jet 3-particle correlator as a function of $\ln(x_2/x_1)$ for fixed $|\ln(x_1 x_2)|$ and $\ln(1/x_3)$.

same energy $x_i = x_j$. In this frame, the role of MLLA corrections should be expected to be larger than for the two-particle correlations. Indeed, higher order corrections increase with the rank of the correlator, which is known from the Koba-Nielsen-Olesen (KNO) problem for intra-jet multiplicity fluctuations [9]. For the 2-particle correlations for instance one has $\propto -b(\psi_{1,\ell} + \psi_{2,\ell})$ and for the three-particle correlator one has the larger correction $\propto -c(\psi_{1,\ell} + \psi_{2,\ell} + \psi_{3,\ell})$.

Finally, we perform theoretical predictions for three-particle correlations for the LHC in the limiting spectrum approximation ($Q_0 \approx \Lambda_{QCD}$). We display the MLLA solutions (4) of the evolution equations (2) and (3). The correlators are functions of the variables ℓ_i, y_i and the virtuality of the jet $Q = E\Theta_0$. After setting $y_i = Y - \ell_i$ with fixed $Y = \ln(Q/Q_0)$ in the arguments of the solutions (4), the dependence can be reduced to the following: $\mathcal{C}_{G_{123}}^{(3)}(\ell_1, \ell_2, \ell_3, Y)$ and $\mathcal{C}_{Q_{123}}^{(3)}(\ell_1, \ell_2, \ell_3, Y)$.

In Figs. 2 and 3, the DLA (10) and MLLA (4) three-particle correlators for $A = G$ and $A = Q, \bar{Q}$ are displayed respectively, as a function of the difference $(\ell_1 - \ell_2) = \ln(x_2/x_1)$ for two fixed values of $\ell_3 = \ln(1/x_3) = 4.5, 5.5$, fixed sum $(\ell_1 + \ell_2) = |\ln(x_1 x_2)| = 10$ and, finally, fixed $Y = 7.5$ (virtuality $Q = 450$ GeV and $\Lambda_{QCD} = 250$ MeV), which is realistic for LHC phenomenology [5]. The representative values $\ell_3 = \ln(1/x_3) = 4.5, 5.5$ ($x_3 = 0.011, x_3 = 0.004$) have been chosen according to the range of the energy fraction $x_i \ll 0.1$, where the MLLA scheme can only be applied.

In Figs. 4 and 5, the DLA (10) and MLLA (4) three-particle correlators for $A = G$ and $A = Q, \bar{Q}$ are depicted, in this case as a function of the sum $(\ell_1 + \ell_2) = |\ln(x_1 x_2)|$ for the same values of $\ell_3 = \ln(1/x_3) = 4.5, 5.5$, for $x_1 = x_2$ and $Y = 7.5$. As expected in both cases, the DLA and MLLA three-particle correlators are larger inside a quark than in a gluon jet. Of course, these plots will be the same and the interpretation will apply to all possible permutations of three particles (123). As

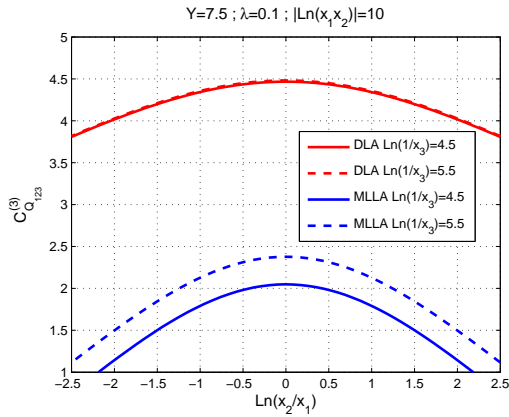


FIG. 3: Quark jet 3-particle correlator as a function of $\ln(x_2/x_1)$ for fixed $|\ln(x_1 x_2)|$ and $\ln(1/x_3)$.

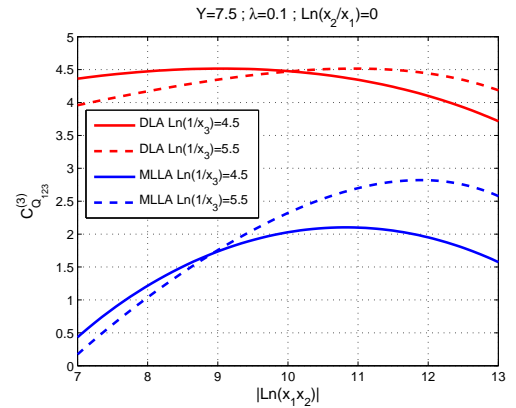


FIG. 5: Quark jet 3-particle correlator as a function of $|\ln(x_1 x_2)|$ for $x_1 = x_2$ and $\ln(1/x_3)$.

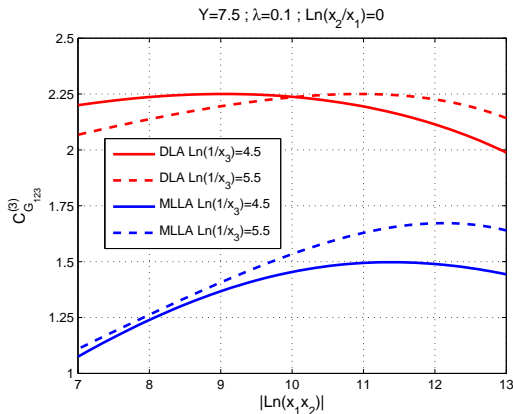


FIG. 4: Gluon jet 3-particle correlator as a function of $|\ln(x_1 x_2)|$ for $x_1 = x_2$ and $\ln(1/x_3)$.

remarked above, the difference between the DLA and MLLA results is quite important pointing out that overall corrections in $\mathcal{O}(\sqrt{\alpha_s})$ are large. Indeed, the last behavior is not surprising as it was already observed on the treatment of multiplicity fluctuations of the third kind given by [10]

$$\frac{\langle n(n-1)(n-2) \rangle_Q}{\langle n \rangle_Q^3} = 4.52 [1 - (2.280 - 0.018n_f)\sqrt{\alpha_s}].$$

For instance, for one quark jet produced at the Z^0 peak of the e^+e^- annihilation ($Q = 45.6$ GeV), one has $\alpha_s = 0.134$. Replacing this value into the previous formula for the quark jet multiplicity correlator, one obtains a variation from 4.52 (DLA) to 0.83 (MLLA). Because of this, DLA has been known to provide unreliable predictions which should not be compared with experiments. From Fig.2 and Fig.3, the correlation are observed to be the strongest when particles have the same energy and to decrease when one parton is harder than the others. Indeed, in this region of the phase space two competing constrains should be satisfied: as a consequence of

gluon coherence and AO, gluon emission angles should decrease and on the other hand, the convergence of the perturbative series $k_\perp = x_i E \Theta_i \geq Q_0$ should be guaranteed. That is why, as the collinear cut-off parameter Q_0 is reached, gluons are emitted at larger angles and destructive interferences with previous emissions occur. Moreover, the observable increases for softer partons with x_3 decreasing, which is for partons less sensitive to the energy balance. In Fig.4 and Fig.5 the MLLA correlations increase for softer partons, then flatten and decrease as a consequence of soft gluon coherence, reproducing for three-particle correlations, the hump-backed shape of the one-particle distribution. Because of the limitation of the phase space, one has $\mathcal{C}^{(3)} \leq 1$ for harder partons.

In this letter we provide the first full pQCD treatment of three-particle correlations in parton showers and a further test of the LPHD within the limiting spectrum approximation. We give the first analytical predictions of this observable in view of forthcoming measurements by ATLAS, CMS and ALICE at the LHC.

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