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# A ply scale non-local fibre rupture criterion for CFRP woven ply laminated structures

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We propose here a model to describe the rupture of woven ply laminated structures. The validity scope of this model depends on the 'diffused damage' phase up to the first intra-laminar macro-crack only (first ply failure model). The model is based on a previous continuum damage approach [Hochard C, Aubourg PA, Charles JP. Modelling of the mechanical behaviour of woven fabric CFRP laminates up to failure. Compos Sci Technol 2001;61:221–30] and a new non-local fibre rupture criterion. This non-local criterion uses a ply scale characteristic volume of ply thickness and in plane area depending on the meso-structure of the ply. In the case of the balanced woven ply studied here, a simple circular area is chosen and identified on a test showing a stress gradient. The local shear damage and inelastic strain evolution laws and the non-local fibre rupture criterion have been integrated in Abaqus. Numerical results compared to tests for woven ply laminated structures show the efficiency of the proposed approach.

*Keywords:* Woven laminate; Non-local criterion; Size effect; Damage mechanics; Structural computation

## 1. Introduction

Rupture in composite material is due to numerous mechanisms acting at different scales. Models to describe the evolution of the overall mechanisms are complex (see [2] for an overview). Woven ply laminates [1] show a good resistance to delamination and are not sensitive to transverse rupture due to weaving. Furthermore it was observed from the experimental tests that the first rupture of a ply leads to the rupture of the laminate and the structure. So, a first ply failure model in plane stresses is generally sufficient to describe the rupture of any woven ply laminate. In that work, we use the model initially proposed in [1] which have the following characteristics:

- It describes the progressive diffuse damage induced by small cracks parallel to the fibres and the brittle behaviour in the fibre direction.
- It takes into account the couplings between traction, compression and shear on the evolution of the damage and the inelastic strain.

The model was identified with classical tensile tests. The brittle elastic behaviour in the fibre direction was described by a local criterion identified with a  $[0^\circ]$  test.

In order to validate it on complex structures, this first model was implemented in a FEM code (see [3]) and numerical simulations were compared to experiments for plates with an open hole submitted to tensile loading. For laminates showing a high level of damage ( $[\pm 45^\circ]$  for example [4]) the comparison was good but in the case of sequences showing brittle behaviour ( $[0^\circ]$ ), the simulations underestimated the rupture. In fact, the local criteria used to describe the fibre rupture do not work when the stress field is not uniform. This fact was pointed out by

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Whitney and Nuismer [5] who developed several criteria (point-wise, average stress) for a quasi-isotropic UD laminate. Based on this kind of criteria, many other researchers proposed modifications, for example to detect the critical region in the individual plies (see [6]) or to take into account the non-linear behaviour for woven plies (see [7]). These criteria can be used in finite element approaches like in [8] or in [9]. Nevertheless, these classical criteria based on a characteristic length are difficult to use for complex structures (multi-perforated plate for example) and any lay-up laminates, in particular in a finite element approach. Moreover, the rupture zone is not a surface (the thickness multiply the characteristic length) but a volume, in particular for woven ply laminates for which the damage zone observed is of rather large size. We propose in this work a non-local criterion based on a ply scale characteristic volume which is easy to implement in a finite element approach. One of the sizes of this volume corresponds to the thickness of the ply and the area in the plane depends on the meso-structure of the ply. In the case of the balanced woven ply studied here, we have chosen a circular area the diameter of which was identified and validated on the different tests. The identified diameter of the circular area is close to the size of the yarn. This criterion based on a ply scale characteristic volume was integrated in Abaqus. This non-local fibre rupture criterion associated to the local shear progressive damage model allows describing the rupture of a large class of laminates and structures. Numerical results compared to tests for woven ply laminated structures show the efficiency of the proposed approach. It should be noted that this non-local approach allows describing the size effect observed experimentally (plates with an open hole with different ratios).

## 2. The woven ply model

Thanks to the yarn weaving, the ply is insensitive to transverse cracks. The carbon fibres are much more rigid than epoxy resin ( $\times 15$ ) and have a brittle behaviour. Due to this fact, no inelastic strains are possible in the fibre direction and can only develop in shear. The local model of the woven ply with balanced warp and fill yarns is first recalled [1]. This model is an extension of the model proposed in [10] for unidirectional plies to woven plies. The shear behaviour is first introduced with different non-linear phenomena, plasticity and damage which are coupled with fibre stretching. The damage kinematics used are based on the following three internal damage variables ( $d_1$ ,  $d_2$ ,  $d_{12}$ ) with the brittle fracture of fibres in the warp and fill directions, respectively. The gradual development of the damage  $d_{12}$  depends on the shear load as well as the tensile load, which generate small cracks parallel to the fibre directions. Under the assumption of plane stresses and small perturbations, we can write the strain energy of the woven ply in the form of Eq. (1).

$$E_D = \frac{1}{2} \left[ \frac{\langle \sigma_1 \rangle_+^2}{E_1^0(1-d_1)} + \frac{\langle \sigma_1 \rangle_-^2}{E_1^0} - 2 \frac{\nu_{12}}{E_1^0} \sigma_1 \sigma_2 + \frac{\langle \sigma_2 \rangle_+^2}{E_2^0(1-d_2)} + \frac{\langle \sigma_2 \rangle_-^2}{E_2^0} + \frac{\sigma_{12}^2}{G_{12}^0(1-d_{12})} \right], \quad (1)$$

where  $\langle \cdot \rangle_+$  is the positive part of  $\cdot$  and  $\langle \cdot \rangle_-$  is the negative part. The tension and compression energy are split in order to describe the unilateral features due to the opening and closing of the micro-defects. From this potential, thermodynamic forces associated with the tension and shear internal variables  $d_i$  ( $i = 1$  and  $2$ ) and  $d_{12}$  are defined:

$$Y_{d_i} = \frac{\partial E_D}{\partial d_i} = \frac{\langle \sigma_i \rangle_+^2}{2E_i^0(1-d_i)^2} \quad \text{and} \quad Y_{d_{12}} = \frac{\partial E_D}{\partial d_{12}} = \frac{\sigma_{12}^2}{2G_{12}^0(1-d_{12})^2}. \quad (2)$$

The evolution of the internal variables depends on these thermodynamic forces and more precisely on their maximum values during the history of the loading. In traction, the development of  $d_1$  and  $d_2$  is discontinuous in order to represent the brittle behaviours according to the warp and the fill directions. It is reminded that the models are integrated for each ply and that all variables are kept for each ply.

### 2.1. Shear behaviour

To investigate the behaviour of the materials, a simple tensile test is performed on  $[\pm 45^\circ]$  laminate. The stress behaviour is not exactly pure shear but for this material (large contrast of rigidity between the two constituents), strains in fibre directions are small so that the state in the ply can be considered as pure shear. The experiments are shown for values expressed in the laminate frame in Fig. 1. It is shown in Fig. 1 that plasticity and damage occurs during straining. By plasticity, we want to model inelastic strain that remains after unloading. By damage, we introduced the loss of rigidity. Here, we do not model the hysteresis loop that is mainly due to friction between

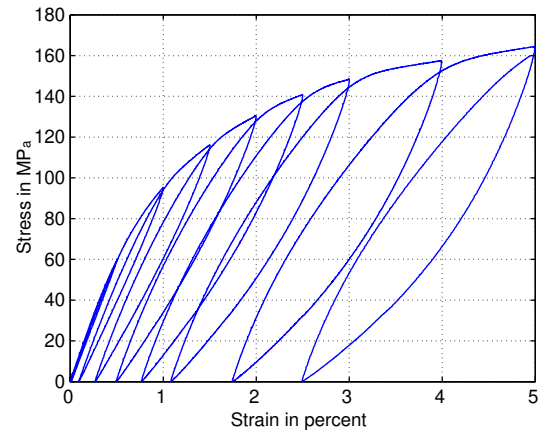


Fig. 1. Behaviour in shear of the woven ply.

the matrix and the fibre. To take into account the tension/shear coupling during the development of  $d_{12}$ , we define the equivalent thermodynamic force and the maximum value of this force during the history of the loading:

$$Y = \alpha_1 Y_{d_1} + \alpha_2 Y_{d_2} + Y_{d_{12}}, \quad (3)$$

where  $\alpha_1$  and  $\alpha_2$  are the tension/shear coupling constants. To identify clearly the coupling constants, a tensile test on an alternative sequence has to be done. It should be noted that this equivalent force which governs the development of the progressive damage variable  $d_{12}$  does not depend on the compression stresses in the warp and fill directions. As for the unidirectional plies [10], a linear law with respect to the square root of  $Y$  is chosen to describe the damage variable development:

$$d_{12} = \frac{\sqrt{Y} - \sqrt{Y_0}}{\sqrt{Y_c} - \sqrt{Y_0}}, \quad (4)$$

where the constant parameters  $Y_0$  and  $Y_c$  correspond to the threshold and the critical value of the development of  $d_{12}$  which varies from 0 to 1. The shear strains are modeled by a simple plastic non-linear hardening law. For simplicity, this part of the model is not detailed here (see [3] for details).

## 2.2. Fibre directions

When a tensile test is performed with a  $[0^\circ]$  manufactured specimen, the behaviour can easily be extracted in the fibre direction. The behaviour is purely elastic until brittle rupture occurs, mainly due to fibre rupture. Even if the traction in the fibre direction creates small cracks in the resin parallel to the fibres and mainly oriented along the orthogonal direction to the traction, these cracks do not influence the global behaviour. One way to model such a rupture is to use the criterion defined by:

$$d_i = 0 \text{ if } Y_{d_i} < Y_{if} \text{ else } d_i = 1. \quad (5)$$

In expression (5),  $Y_{1f}$  and  $Y_{2f}$  are the parameters which define the ultimate forces in the warp and fill directions. For structural tests showing stress gradient, the phenomenon of brutal rupture is always encountered in the same condition as for the specimen. It is assumed that when the rupture of the ply in the fibre direction is encountered, the whole laminate collapses immediately due to the high level of stress necessary to the ply rupture. Due to the stress concentration on the crack tip of the laminate, the crack propagates rapidly in the structure, at least for static loadings. This tends us to stop any computations when the criterion (5) is encountered. This type of consideration is in accordance with the many experiments that we have carried out on structures.

## 3. Non-local criterion

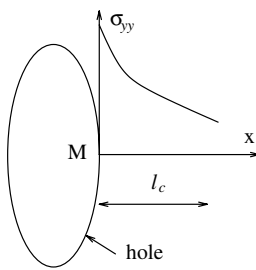
In paper [3] the authors show that using criterion (5) to describe the ply rupture in the fibre direction leads to

underestimate the rupture force by a factor 2 for zero degree laminated structures (plates with an open hole). This fact was also pointed out by Whitney and Nuismer [5] who developed several criteria (point-wise, average stress). These criteria consist to introduce a characteristic length associated for example to a critical average stress over the length. So, with this type of non-local criteria, even if the critical stress is reached locally, the ply does not break. A possible explanation to justify this type of non-local approach is that the first rupture of a fibre does not lead immediately to the rupture of the ply. This remark is in accordance with acoustic observations. The rupture of the ply is more probably associated to the rupture of a certain number of fibres over a characteristic volume which depends on the meso-structure (thickness of the ply, size of a yarn). So, we introduce in this paper a characteristic volume at the scale of the ply to take into account this meso-structure effect. We first recall the experiments (detailed in Ref. [11]) showing different stress gradients in order to calibrate the non-local models to woven plies.

### 3.1. Experiments

To enhance and appreciate the different characteristics of the non-local criteria, a series of tensile tests on different structures are done. The different structures are such that the material is submitted to different stress distributions. Such tests were done by Whitney et al. for quasi-isotropic laminates [5]. We thus reproduced this type of analysis but by modulating the geometry and the radius of an elliptical hole in the plate, as well as size effects ( $\times 1$ ,  $\times 2$ ,  $\times 4$ ,  $\times 6$ ) for the plate with a circular open hole. The  $\times 2$  means that in-plane dimensions are multiplied by a factor 2. The different specimens are shown in Fig. 2 and the experimental rupture force are summed-up in Table 1.

The sequence chosen is a quasi-isotropic laminate  $[+45, 90, 0, -45]_s$ . This type of sequence limits the non-linear phenomena in the structure. For the simulations, the material is assumed to behave linearly ( $E = 33$  GPa,  $\nu = 0.33$ ) until rupture occurs, this assertion was verified with full field measurements (see [11] for details). The experimental force at rupture is introduced in the FE models to obtain the distribution of stress in the section. The non-local criterion used is given by (6):



$$\frac{1}{l_c} \int_0^{l_c} \sigma_{yy}(\mathbf{OM} + \rho \mathbf{x}) d\rho = \sigma_c. \quad (6)$$

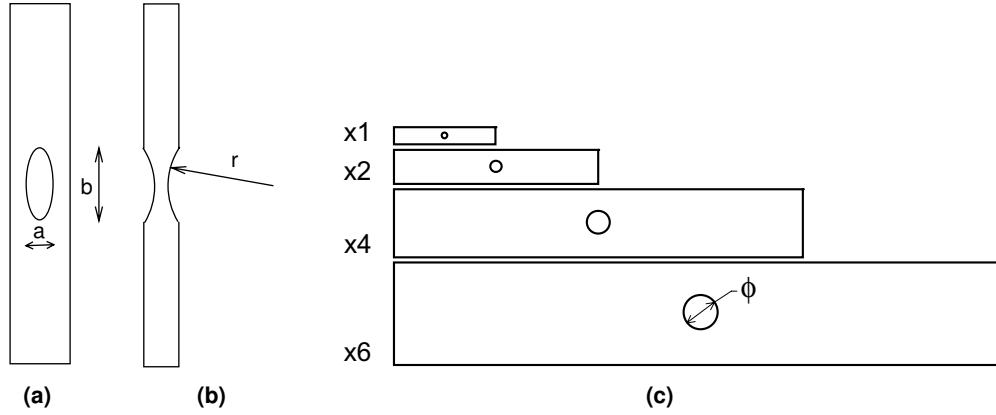


Fig. 2. Specimen shape.

Table 1  
Comparison of experimental force at rupture with predicted criteria on structure

Specimen	Force at rupture (N)	Local criteria (N)	Error (%)	Non-local (N)	Error (%)
<b>Ellips <math>b = 80</math> mm</b>	<b>42424</b>	<b>38990</b>	<b>8</b>	<b>43405</b>	<b>2.3</b>
Ellips $b = 50$ mm	42082	36220	13.9	41100	2.3
Ellips $b = 30$ mm	39044	31725	18.75	37840	3
<b>Ellips <math>b = 15</math> mm</b>	<b>32550</b>	<b>23510</b>	<b>27.7</b>	<b>33225</b>	<b>2</b>
Radius $r = 60$ mm	38200	33665	11.87	37330	2.28
Radius $r = 88$ mm	38550	34995	9.22	38270	0.7
Radius $r = 150$ mm	39550	36485	7.75	39750	0.5
<b>Radius <math>r = 300</math> mm</b>	<b>40300</b>	<b>37565</b>	<b>6.79</b>	<b>40415</b>	<b>0.2</b>
Radius $r = 600$ mm	39600	38095	3.8	40800	3
$\times 6$	78060	70530	9.6	80300	2.8
$\times 4$	56576	47020	16.89	55770	1.4
$\times 2$	32550	23510	27.7	33225	2
$\times 1$	18054	11755	34.9	19040	5.4

The average stress is a function of the distance from the hole and is drawn in Fig. 3 for the different sequences in bold in Table 1, it is seen on that figure that all three curves of the mean stress coincide at one point. From these three specimens chosen, we are able to identify an internal length and an ultimate stress. Finally, for the local criterion, the errors on the rupture force vary from 4% to 30% whereas for this non-local criterion they vary from 0.2% to 5%. One can also note that this non-local criterion makes it possible to describe a scale effect observed in experiments. We carried out tests on perforated plates of which all dimensions in the plan are multiplied by a ratio which varies from 1 to 6 (Table 1). Contrary to experimental measurements, the rupture force estimated by the local criterion are connected by the scale factor (the rupture force estimated for the test  $\times 6$  is six times larger than that estimated for the test  $\times 1$ , in experiments this ratio is only 4.3, ratio obtained with the non-local criterion).

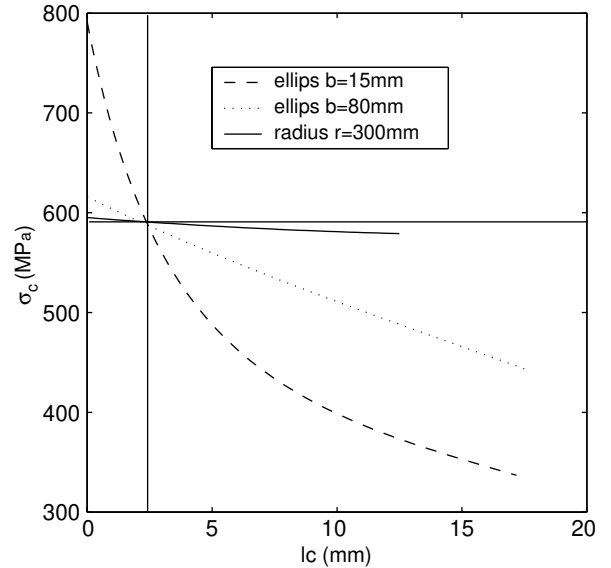


Fig. 3. Mean stress versus the distance from the free boundary for the three different structures chosen for the identification.

### 3.2. Non-local criterion based on a ply scale characteristic volume

We propose here a non-local criterion based on a ply scale characteristic volume. One of the sizes of this volume corresponds to the thickness of the ply and the area  $S$  in the plane depends on the meso-structure of the ply. In the case of the balanced woven ply studied here, we chosen a circular area the diameter of which was identified and validated on the different tests of the previous section. The identified diameter of the circular area is close to the size of the yarn. This simple criterion can be easily apply everywhere in the structure and for any lay-up laminate. In place of using the local associated force defined by (2), we use the mean of this force over the volume. In fact, we replace the criterion (5) by the following:

$$d_i = 0 \text{ if } \overline{Y_{d_i}} < Y_{if} \text{ else } d_i = 1, \quad (7)$$

with,

$$\overline{Y}_{d_i} = \frac{1}{V} \int_V Y_{d_i} dv, \text{ with, } V = h \times S. \quad (8)$$

In (8),  $h$  is the ply thickness and  $S$  the characteristic surface.

### 3.3. Implementation

This criterion based on a ply scale characteristic volume was integrated in Abaqus. The implementation of the non-local criteria cannot be performed in a material routine at every gauss point. The criterion is evaluated in a post-computation relative to the incremental calculation. The criterion is computed with stresses coming from the total increment for all the points in the surface  $S$ . The algorithm breaks down into in three main steps.

- (1) For the first step, look at all the points in the surface  $S$ .
- (2) Average the associated force in the zone using Eq. (8).
- (3) Check the criterion (7).

The criterion is computed in all the gauss points in the structure in order to have an intrinsic criterion. The non-local criterion implemented is slightly enriched with the fact that the criterion is tested for points at least at a certain distance of the border (the radius of the circular area, see Fig. 4). The size of the mesh has to be smaller than the zone where the average of the associated force is done. For the cases studied here, we observed that, even in the high stress gradient zones, from ten gauss points in the zone the mean value does not vary any more.

### 3.4. Validation on plates with a hole for different sequences

In this section, criterion (7) is used to predict the rupture of a plate with an open hole. The plate has the dimension of  $160 \times 50 \times 1.4$  mm, that represents a four plies laminate

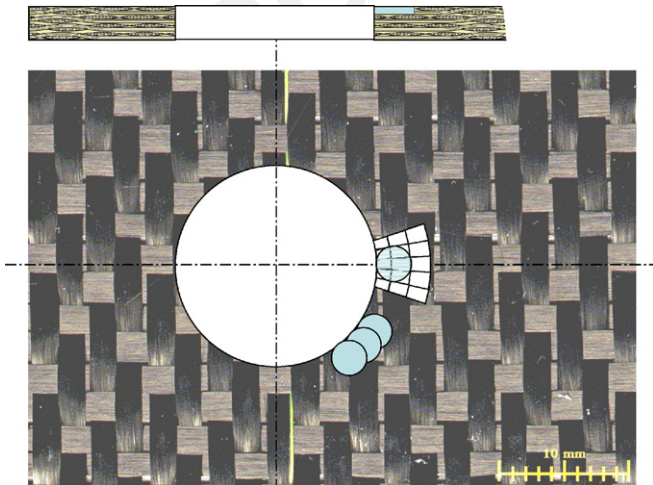


Fig. 4. Characteristic volume for the non-local criterion.

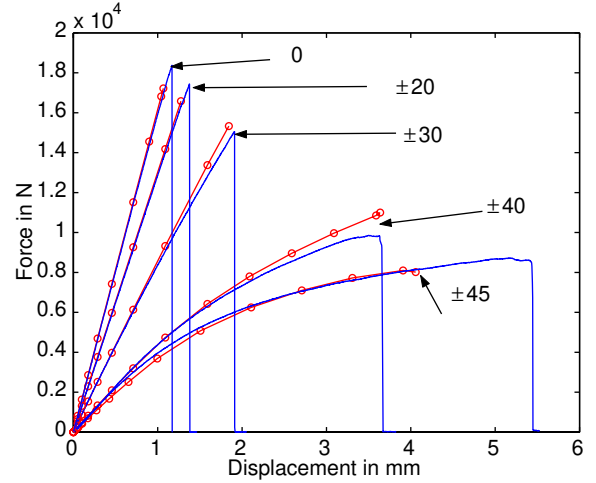


Fig. 5. Comparison of experimental and numerical prediction for different stacking sequences but with a non-local criterion, G963/913, 'o' mark are for computations.

and the hole has a diameter of 15 mm. We compare in the Fig. 5 the experimental force as a function of the displacement with those predicted by our model. For the  $[0^\circ]$ ,  $[\pm 22.5^\circ]$  and  $[\pm 30^\circ]$  laminates, the rupture is observed experimentally to be due to fibre rupture. In the computation for these three structures, the calculations are stopped because of criterion (7). It is seen that the non-local criteria predicted correctly the force of rupture for the different stacking sequences.

## 4. Conclusions

The rupture of woven ply laminates is less complex to model than UD ply laminates. Shear damage and inelastic strain evolution laws have been associated to a simple non-local ply scale criterion based on a characteristic volume. This approach is adapted to structure computation and has been implemented in a FEM code. This approach gives good correlations with experiments on woven ply laminated structures. In the case of static loadings, this first ply failure model is sufficient to describe the rupture of structures. The non-local criterion based on the characteristic volume has been tested in the case of quasi-isotropic UD laminates (see [12]). In this case, the diameter of the circular area is close to the case of woven ply. Another extension concerns the extension to fatigue loading (see [4]).

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