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Extended black hole cosmologies in de Sitter space

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ABSTRACT

We generalize the superposition principle for time-symmetric initial data of black hole spacetimes to (anti-)de Sitter cosmologies in terms of an eigenvalue problem $\Delta_g \phi = \frac{1}{8}(R_g - 2\Lambda)\phi$ for a conformal scale ϕ applied to a metric g_{ij} with constant three-curvature R_g . Here, $R_g = 0, 2$ in the Brill-Lindquist and, respectively, Misner construction of multihole solutions for $\Lambda = 0$. For de Sitter and anti-de Sitter cosmologies, we express the result for $R_g = 0$ in incomplete elliptic functions. The topology of a black hole in de Sitter space can be extended into an infinite tower of universes, across the turning points at the black hole and cosmological event horizons. Superposition introduces binary black holes for small separations and binary universes for separations large relative to the cosmological event horizon. The evolution of the metric can be described by hyperbolic system equations with curvature-driven lapse function, of alternating sign at successive cosmologies. The computational problem of interacting black hole-universes is conceivably of interest to early cosmology when Λ was large and black holes were of mass $< \frac{1}{3}\Lambda^{-1/2}$, here facilitated by a metric which is singularity-free and smooth everywhere on real coordinate space.

1. Introduction

The multihole solutions of Brill-Lindquist and Misner (Misner 1963; Brill & Lindquist 1964; Cook 2001) arise out of a superposition principle in the Hamiltonian constraint of the Einstein equations for time-symmetric initial data. Here, the Schwarzschild line-element in coordinates (r, θ, φ) is transformed into an isotropic line-element described by a conformal factor $\phi(r)$, giving rise to a conformally flat spacetime in vacuum. This construction is remarkable, in allowing for multihole solutions with different extended topologies. To a black hole binary, the Brill-Lindquist solution attributes a three-sheet topology, whereas the Misner solution attributes a two-sheet topology to the same.

The embedding of black holes in (anti-)de Sitter space is well-known for a single black hole with one-sheet topology. De Sitter space is of interest in view of considerable observational evidence for a small but distinctly positive cosmological constant in the Λ CDM model

(de Bernardis et al. 2000; Hanany et al. 2000) with an expected improved uncertainty by the recently launched *Planck* satellite (ESA-SCI 2005). The stability and thermodynamics of the cosmological event horizon in the black hole-de Sitter space is relevant to its potential role to early cosmology (Davies 1987; Chambers 1997).

Here, we consider the problem of extending space beyond the cosmological event horizon. We approach this problem in terms of the turning points at extrema of the circumference in the one black hole-de Sitter space. The result opens the possibility for novel topologies on scales that reach beyond the visible universe. Space of finite volume further facilitates representation in a metric that is smooth everywhere. This is of direct interest to introducing spectral methods for calculating wave-templates for gravitational-wave observatories LIGO and Virgo as they are gradually improving their sensitivity.

To begin, we recall the Brill-Lindquist line-element of a single Schwarzschild black hole Brill & Lindquist (1964); Cook (2001). The event horizon of the black hole represents a turning point in the embedding in two asymptotically flat sheets,

$$ds^2 = -\tanh^2(\lambda/2)dt^2 + 4E^2 \cosh^4(\lambda/2)ds_D^2, \quad (1)$$

over the donut $ds_D^2 = d\lambda^2 + \frac{dx^2}{1-x^2} + (1-x^2)d\varphi^2$ with $-\infty < \lambda < \infty$. Here, we transform the spherical coordinates (ρ, θ, φ) , $x = \cos \theta$ of Brill-Lindquist by $\rho = \frac{1}{2}Ee^\lambda$. The Möbius invariance $\lambda \leftrightarrow -\lambda$ comes with opposite signs in the lapse function $N = \tanh(\lambda/2)$ on either sheet, wherein the horizon surface corresponds to the simple zero $N = 0$.

By Liouville's theorem, spacetime singularities are inherent to any black hole spacetime which is asymptotically regular. They can be moved away into the complex plane as (1) illustrates by mapping $\rho = -\frac{1}{2}E$ to $\lambda = \pi i \pmod{2\pi}$, whereby the metric is analytic everywhere for all real and finite λ , wherein $\lambda = \pm\infty$ represent coordinate singularities associated with asymptotic infinity on each sheet.

The black hole singularity is not directly accessible by observation, whether in real or complex coordinate space. Even an observer in free fall onto a black hole never reaches the central singularity when considered in real coordinates, as in the Schwarzschild line element. Upon approaching the event horizon, time-at-infinity, t , becomes arbitrarily large, which signals evaporation of the black hole by Hawking radiation. The observer hereby traces a shrinking event horizon, never to penetrate it, during which time the black hole singularity diminishes in strength. Note that this result only uses evaporation of a black hole in a finite time-at-infinity, regardless of the details of the luminosity of Hawking radiation. Hawking radiation hereby introduces invariance to the cosmic censorship conjecture, by treating singularities in real and complex coordinate space on equal footing. The same arguments shows that (1) defines a non-traversable wormhole.

In de Sitter space, coordinate singularities associated with the infinite extend of space are avoided. The result for multihole solutions in de Sitter space, then, is a metric which is *everywhere* nonsingular and finite on the coordinate cover of the visible universe with no need for compactification. The cosmological event horizon represents an additional length scale, which can be of interest in considering black hole binaries with large separations. It introduces an additional turning point which opens a window for novel large scale extensions beyond.

In §2, we derive the formalism for constructing multihole initial data in (anti-)de Sitter space. In §3 we give some illustrative examples for binary black holes and binary cosmologies. In §4, we propose a hyperbolic system of equations for their evolution based on the 3+1 Hamiltonian equations of motion (Arnowitt, Deser & Misner 1962), where hyperbolicity generally facilitates stable numerical implementation (e.g., van Putten & Eardley (1996); Nagy, Ortiz, & Reula (2004); Calabrese, Hinder & Husa (2006) and references therein) by ensuring a real dispersion relation and hence stability whenever the Courant-Friedrichs-Lewy condition (Courant, Friedrichs & Lewy 1967) is satisfied. An outlook is included in §5.

2. A superposition principle in (anti-)de Sitter space

The line-element of a Schwarzschild black hole of mass m in a de Sitter space with cosmological constant Λ is

$$ds^2 = -(1 - 2m/r - \frac{1}{3}\Lambda r^2)dt^2 + \frac{dr^2}{1 - 2m/r - \frac{1}{3}\Lambda r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2. \quad (2)$$

When $\Lambda m^2 < \frac{1}{9}$, a black hole of mass m can exist within the larger cosmological event horizon defined by Λ with a black hole horizon at coordinate radius $r = r_1$ and an event horizon at $r = r_2 > r_1$ corresponding to the roots of the redshift factor $N = 0$, $N^2 = 1 - 2m/r - \frac{1}{3}\Lambda r^2$, in (2). The roots are shown in Fig. (1) with the bifurcation point at $\Lambda m^2 = \frac{1}{9}$, including the non-physical real root $r_3 < 0$. The line-element (2) can be transformed to an isotropic line-element

$$ds^2 = -N^2 dt^2 + \phi^4 (d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2 \theta d\varphi^2) \quad (3)$$

according to the conditions

$$\pm \frac{dr}{\sqrt{1 - 2m/r - \frac{1}{3}\Lambda r^2}} = \phi^2 d\rho, \quad r = \phi^2 \rho. \quad (4)$$

For $\Lambda > 0$ the three real roots $r_3 < 0 < r_1 < r_2$ of N satisfy $r_1 = 2m(1+\epsilon)$, $r_{2,3} = \pm\sqrt{\frac{3}{\Lambda}}(1\mp\delta)$, where $\epsilon = \frac{4}{3}\Lambda m^2 + O(\epsilon^2)$ denotes the horizon surface of the black hole and $\delta = m\sqrt{\frac{\Lambda}{3}} + O(\delta^2)$.

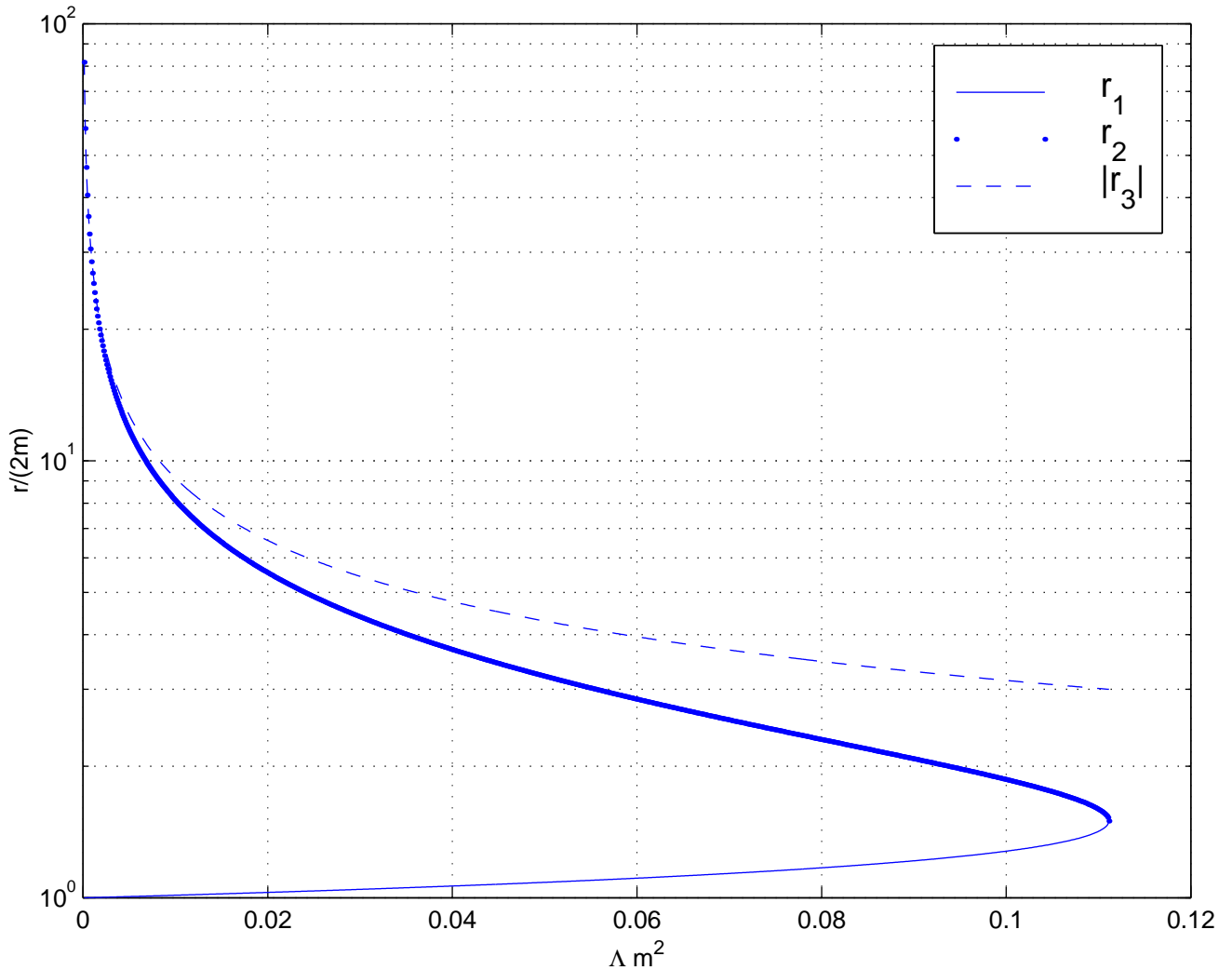


Fig. 1.— Shown are the three real roots of the redshift factor in the Schwarzschild-de Sitter space as a function of $\Lambda m^2 > 0$. They define the coordinate location of the horizon of the black hole (r_1) and of the cosmological event horizon (r_2). The roots bifurcate into a pair of real roots at $\Lambda m^2 = 1/9$. The maximum mass of a black hole in a de Sitter space is hereby $1/3\Lambda^{-1/2}$ (Podolský 1999). Shown is further the numerical root $r_3 < 0$.

The solution to (4) can be expressed following a logarithmic transformation $d\lambda = d\rho/\rho$. For $\Lambda > 0$, the Möbius transformation $t = 2m/r$ (with no reference to time intended) gives

$$\lambda = \left(\int_{t_2}^{t_1} - \int_{t_2}^t \right) \frac{d\tau}{\sqrt{\tau^2 - \tau^3 - 4\Lambda m^2/3}} = 2 \frac{F(\pi/2, m_1) - F(\phi, m_1)}{\sqrt{t_1 - t_3}} \quad (5)$$

in terms of the three roots $t_i = 2m/r_i$ ($t_3 < 0 < t_2 < t_1$), $m_1 = \frac{t_1 - t_2}{t_1 - t_3}$, $\sin \phi = \sqrt{\frac{(t_1 - t_3)(t - t_2)}{(t_1 - t_2)(t - t_3)}}$, where $F(\phi, m_1)$ denotes the incomplete elliptic function of the first kind (Abramowitz & Stegun 1968). For $\Lambda < 0$, the only real root is r_2 . We proceed with $r = \frac{6m}{1-12v}$,

$$\lambda = \int_{v_2}^v \frac{dP}{\sqrt{4P^3 - P/12 + 1/216 + \Lambda m^2/12}}, \quad (6)$$

giving rise to the Weierstrass elliptic function $v(\lambda) = P(\lambda, 1/12, -1/216 - \Lambda m^2/12)$ (Abramowitz & Stegun 1968).

In the resulting conformally flat approach, the resulting extension of de Sitter black hole spacetime is shown in Figs. (2-3). We note that the singular limit $\Lambda = 0$ reduces to the familiar expressions (e.g. Abrahams & Price (1996))

$$\lambda = \ln \left(\frac{1}{m} [r - m + \sqrt{r^2 - 2mr}] \right), \quad r = \rho \left(1 + \frac{m}{2\rho} \right)^2, \quad \rho = \frac{1}{4} (\sqrt{r} + \sqrt{r - 2m})^2. \quad (7)$$

We interpret the result as follows.

Theorem. *The superposition principle for time-symmetric data in the Hamiltonian energy constraint generalizes to (anti-)de Sitter space for eigenfunctions of the Laplace operator of a scaled metric with constant 3-curvature.*

Proof. We recall the conformal decomposition of the Ricci tensor for $h_{ij} = \phi^4 g_{ij}$, whereby ${}^{(3)}R_{ij}(h) = {}^{(3)}R_{ij}(g) + C_{ij}$ with $C_{ij} = -2\phi^{-1}[D_i D_j \phi + g_{ij} \Delta \phi] + 2\phi^{-2}[3D_i \phi D_j \phi - g_{ij} D^p \phi D_p \phi]$. Following a reduction of the Ricci tensor from four to three dimensions, the Einstein equations for time-symmetric data give

$${}^{(3)}R(h) = {}^{(3)}R(g) - 8\phi^{-1} \Delta_g \phi = 2\Lambda. \quad (8)$$

If the scaled metric g_{ij} produces a constant curvature R_g , the ϕ are eigenfunctions of the associated Laplace operator,

$$\Delta_g \phi = \frac{1}{8} (R_g - 2\Lambda) \phi. \quad (9)$$

We can construct solutions by superposition

$$\phi = \sum \mu_i \phi_i, \quad (10)$$

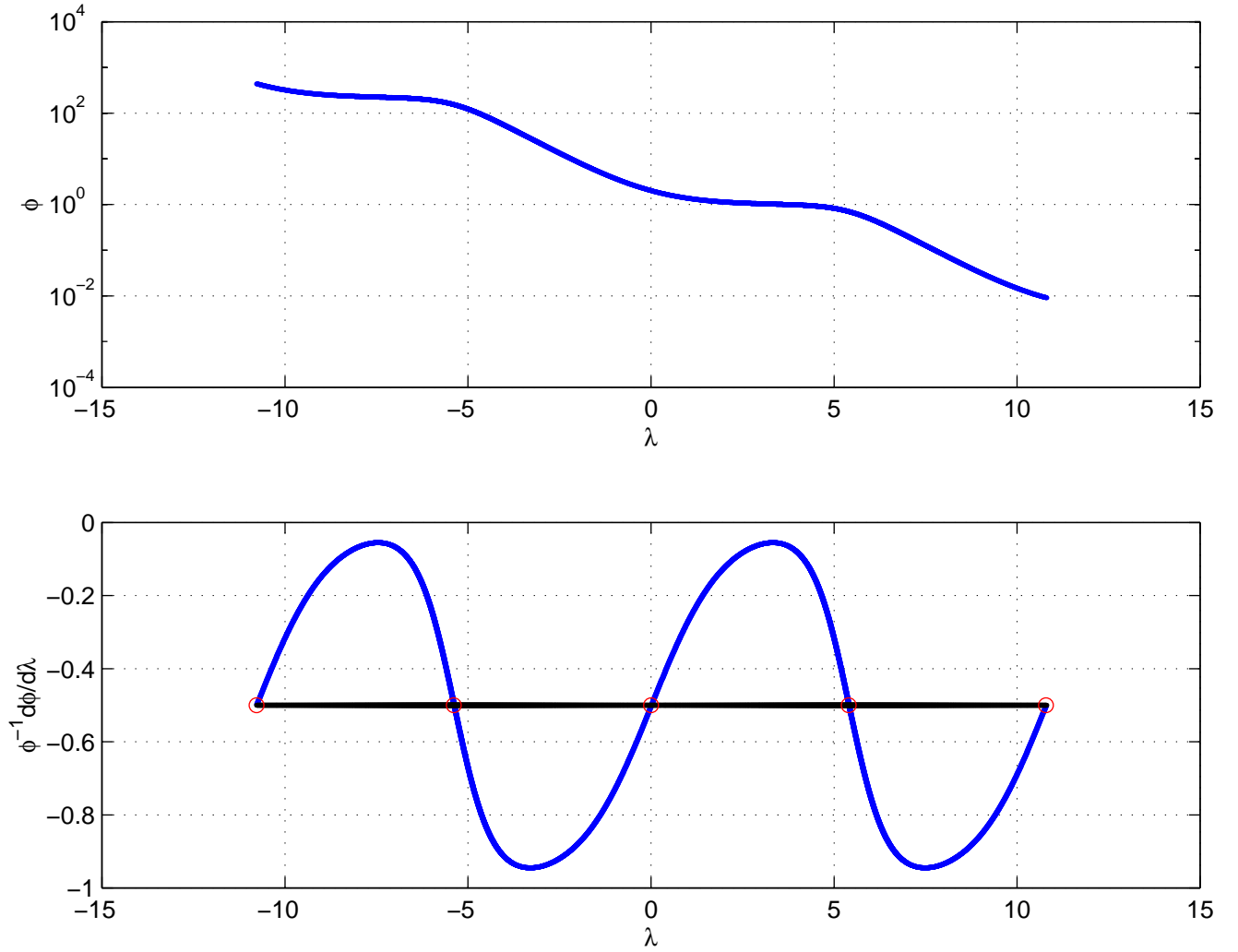


Fig. 2.— For a single black hole in de Sitter space, the scale ϕ is periodic in the radial coordinate $-\infty < \lambda < \infty$, representing a “tower” of cosmologies by successive continuations at the turning points defined by the black hole and cosmological event horizons. The results shown are computed for $m = 1$ and $\Lambda = 0.001$, and may be contrasted with $\phi = (1 - e^{-\lambda})^{1/2}$ in singular limit $\Lambda = 0$.

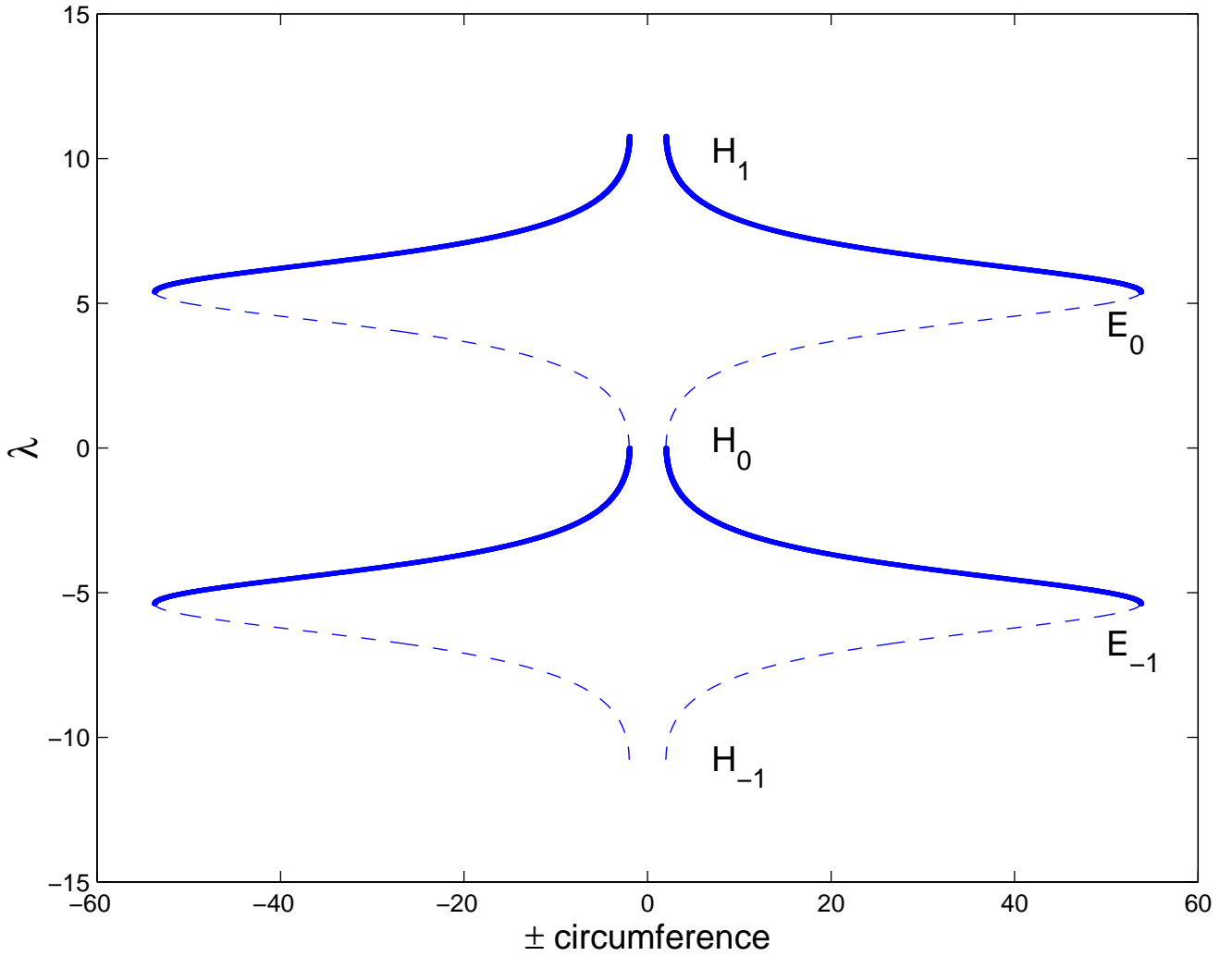


Fig. 3.— The extension of black hole-de Sitter cosmology of a single black hole-de Sitter spacetime can be realized by joining the eigenfunction solutions at successive turning points (solid-to-dashed transitions), representing black hole and cosmological event horizons (H_i, E_i). The results shown are computed for $m = 1$ and $\Lambda = 0.001$.

$\mu_i > 0$, $\sum \mu_i = 1$, of different eigenfunctions ϕ_i to the same eigenvalue, e.g., those translated in any one of the homogeneous directions. The Brill-Lindquist case corresponds to the flat metric $g_{ij} = \delta_{ij}$ with $R_g = 0$ and three homogeneous directions, whereas the Misner case corresponds to the donut metric of (1) with $R_g = 2$ and one homogeneous direction, both with $\Lambda = 0$. In case of the former, we have the coordinate density

$$\phi_p^2 = \frac{r}{\rho}, \quad \rho = \sqrt{(x_1 - p_1)^2 + (x_2 - p_2)^2 + (x_3 - p_3)^2}, \quad (11)$$

associated with black holes at p (and $r = 0$ in the corresponding Schwarzschild coordinates), where the singular limit $\Lambda \rightarrow 0$ recovers $\phi = 1 + \frac{m}{2\rho}$. \square .

The eigenvalue problem (10) suggests an association to a boson field with frequency $\omega = \frac{1}{2}\sqrt{\Lambda}$ in the tangent bundle of de Sitter space. However, this association does not reflect scale invariance in the Riemann tensor, in that it depends only on the log of ϕ .

A cosmological event horizon at finite distance alters the spectrum of eigenfunctions. Whereas the eigenfunctions $r^{-(l+1)}P_l(x)$ to $\Delta\phi = 0$ give convergence and divergence at asymptotic infinity on the two sheet embedding of the Schwarzschild black hole in the Brill-Lindquist case $\Lambda = 0$, the eigenfunctions of $\Delta\phi = -\frac{1}{4}\Lambda\phi$ for $\Lambda > 0$ are periodic across the black hole and cosmological event horizons in Figs. 2-3 and are hereby necessarily bounded and regular everywhere.

A cosmological event horizon also alters the Hilbert space of radiation states at large distances from a black hole, and hence the details of Hawking radiation (Kanti, Grain & Barrau 2005; Zeng et al. 2008). This is particularly pertinent during inflation, when Λ is large. The evaporation time of black holes should remain finite, however, whereby the strong form of the cosmic censorship conjecture described in the introduction should continue to hold whenever $m^2\Lambda < \frac{1}{9}$.

A construction similar to (5-6) may be pursued for the Misner two-sheet embedding of two black holes, based on eigenfunctions $\Delta_D\phi = \frac{1}{4}(1 - a^2\Lambda)\phi$, where D refers to the donut line-element in (1) and a refers to the Misner length scale. A detailed consideration, however, falls outside of the present discussion. It may also be generalized to extra dimensions on the basis of the Schwarzschild-de Sitter solution (Tangherlini 1963; Gao 2004). In 4+1, for example, the transformation (5-6) produces a trigonometric expression.

3. Extended black hole-de Sitter cosmologies

For $0 < m^2\Lambda < \frac{1}{9}$, we construct a few binary black hole spacetimes with different separations as shown in Fig. (4). For small separations, the results are very similar to a

Brill-Lindquist spacetime inside the cosmological event horizon surrounding the binary. For large separations, however, the cosmological event horizon splits into two, leading to a binary of two universes. Here, the second universe lives in the “tower” of the first, on the sheet beyond its cosmological event horizon.

The dynamical evolution of an extended black hole-de Sitter universe with multiple black holes and cosmological event horizons offers a new route to gravitational radiation from the early universe. Here, the waves are generated by the dynamics of primordial black holes, as well as by multipole moments of cosmological event horizons. The spectrum of relic waves may hereby be extended in the infrared, below which what may be expected from mergers of primordial black holes alone. It poses a novel computational challenge for numerical relativity, i.e., to compute these relic waves at the present epoch from an initial distribution of primordial black hole-de Sitter universes at the prior to or at the onset of inflation.

4. Hyperbolicity with curvature-driven lapse

Spacetime can be described by a foliation in spacelike hypersurfaces, given by a 3+1 decomposition in terms of h_{ij} (Thorne, Price & McDonald 1986)

$$ds^2 = -N^2 dt^2 + h_{ij} (dx^i + \beta^i dt)(dx^j + \beta_j dt) \quad (12)$$

with lapse function N and shift functions β^i . The N and β^i govern the flow of coordinates in the foliation of hypersurfaces of constant coordinate time t . In what follows, we shall restrict our attention to normal coordinates described by vanishing shift functions, $\beta^i = 0$.

In the Hamiltonian equations of motion with vanishing shift function, the three-metric h_{ij} evolves according to $\partial_t h_{ij} = -2NK_{ij} = -NK_i^n h_{nj} - h_{im} NK_j^m$. Integration over finite time-intervals Δt gives the product

$$h_{ij}(t_i) = \Pi_i e^{-K_i^m N \Delta t} h_{mn}(t_0) e^{-K_j^n N \Delta t}, \quad (13)$$

We propose time-evolution with curvature-driven lapse function, i.e.,

$$D_t N = -K, \quad D_t K_{ij} = -D_{ij} N + \mathcal{K}_{ij} N, \quad (14)$$

where $D_{ij} = D_i D_j - R_{ij}$ and $\mathcal{K} = K K_{ij} - 2K_i^m K_{jm}$. The gauge condition $D_t N = -K$ is *curvature-driven* and is distinct from the product of curvature and lapse function in the harmonic slicing condition $\partial_t N = -N^2 K$ in Eqs. (69)-(77) of Abrahams et al. (1997); see also Brown (2008) for a recent review.

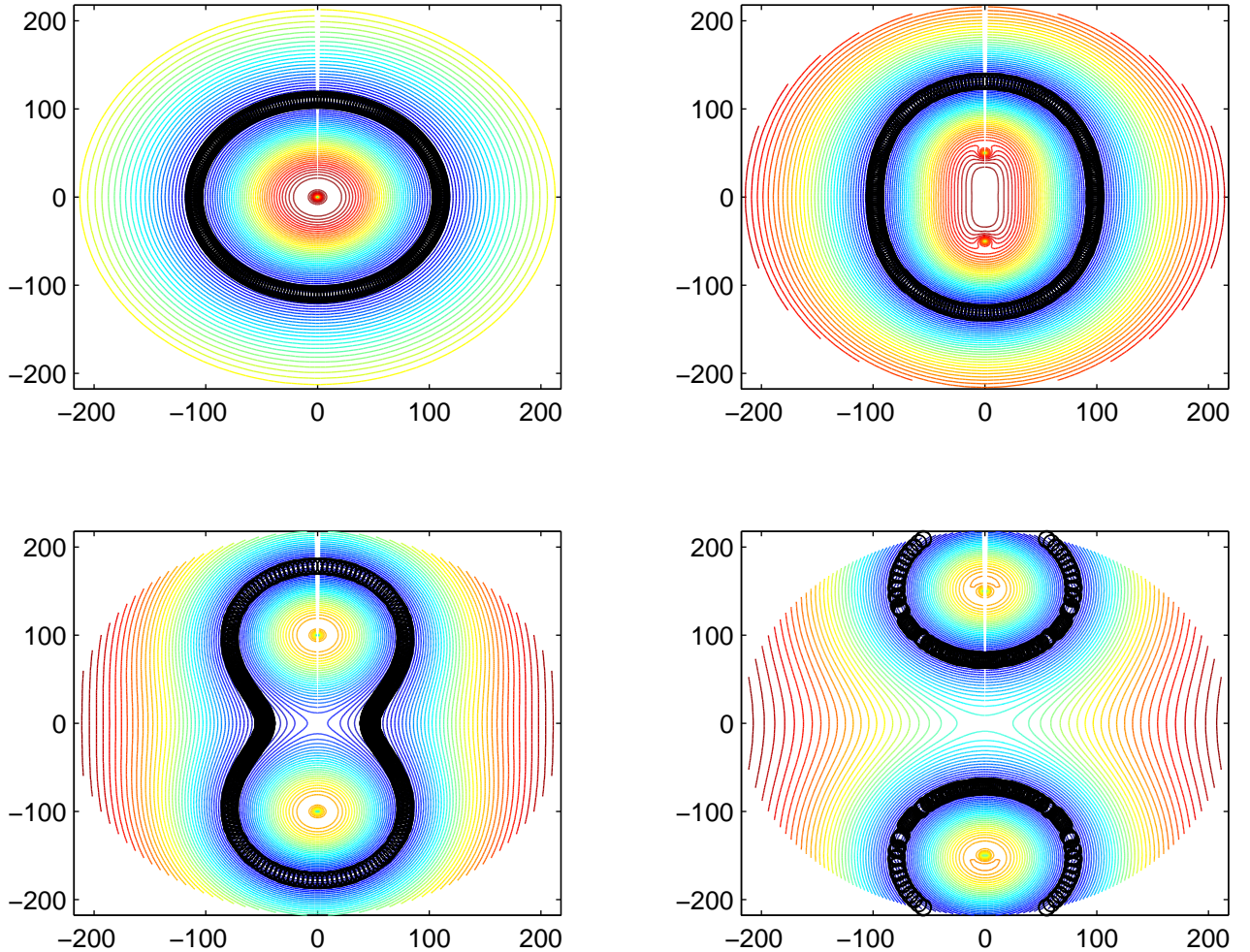


Fig. 4.— Initial data of binaries constructed by the generalized superposition principle starting from zero separation (*left top*). Shown are isocurves of constant $J = |\partial\phi/\partial\lambda + 1/2|$. For small separations, the result is a black hole binary inside a slightly deformed cosmological event horizon $J = 0$ (thick dark line). As the separation increases and becomes large, a binary of two universes forms, wherein the second lives on a sheet adjacent to the first and beyond its cosmological event horizon corresponding to a dashed sheet in Fig. (3) (*right below*). There is an infinity of black hole and cosmological event horizons at exponentially increasing coordinate distances and within the two black holes (not shown).

Theorem. *The 3+1 Hamiltonian evolution equations are hyperbolic with curvature-driven lapse function $\partial_t N = -K$.*

Proof. It suffices to consider the problem of small amplitude wave-motion about flat spacetime, e.g., the asymptotically flat region with $h_{ij} = \delta_{ij}$ with $N = \pm 1$ at large distances. Here, we have

$$\partial_t N = -K, \quad \partial_t^2 h_{ij} = -2R_{ij} + 2D_i D_j N, \quad \partial_t^2 K = \Delta K. \quad (15)$$

We recall that (Wald 1984)

$$R_{ij} = -\frac{1}{2}\Delta\delta h_{ij} + \frac{1}{2}\partial_i\partial^e\bar{\delta}h_{ej} + \frac{1}{2}\partial_j\partial^e\bar{\delta}h_{ei} \quad (16)$$

where $\bar{h}_{ij} = \delta h_{ij} - \frac{1}{2}\delta_{ij}\delta h$, where $\delta h = h^{ij}\delta h_{ij}$ refers to the trace of the metric perturbations.

Small amplitude harmonic perturbations about the flat metric are given by $\delta h_{ij} \sim \hat{h}_{ij}e^{-i\omega t}e^{ik_i x^i}$. Conservation of momentum, $D^i K_{ij} = D_j K$, the lapse condition $-i\omega N = -\hat{K}$, $\delta\hat{h}_{ij} = -2i\omega^{-1}\hat{K}_{ij}$ and $k^i\hat{K}_{ij} = k_j\hat{K}$, give rise to

$$\partial_i\partial^e\bar{h}_{ej} \rightarrow k_i k^e \hat{h}_{ej} - \frac{1}{2}k_i k_j \delta\hat{h} = i\omega^{-1}(-2k_i k^e \hat{K}_{ej} + k_i k_j \hat{K}) = -i\omega^{-1}k_i k_j \hat{K}. \quad (17)$$

We then have

$$\hat{R}_{ij} - \partial_i\partial_j N = \frac{1}{2}k^2\hat{h}_{ij} - i\omega^{-1}k_i k_j \hat{K} + i\omega^{-1}k_i k_j \hat{K} = \frac{1}{2}k^2\hat{h}_{ij}, \quad (18)$$

whereby $\partial_t h_{ij} = -2R_{ij} + 2D_i D_j N$ gives rise to the dispersion relation

$$\omega^2 = k^2. \quad (19)$$

It follows that all small amplitude metric perturbations propagate along the light cone, which completes the proof. \square

Clearly, the system (15) is asymptotically stable, as metric perturbations become small at arbitrarily large distances. Asymptotic wave-motion is commonly studied in the so-called transverse traceless gauge, or harmonic coordinates—neither of these two coordinate conditions are used here.

Numerical integration of (13-14) on the donut $(\lambda, \theta, \varphi)$ can be pursued using a conformal decomposition to bring out invariance of the aforementioned C_{ij} with respect to scaling in ϕ . To this end, we may set $\phi = e^\eta$, whereby

$$C_{ij} = -2[D_i D_j \eta + g_{ij}\Delta\eta] + 4[D_i \eta D_j \eta - g_{ij}D^p \eta D_p \eta]. \quad (20)$$

It follows that $\partial_t K_{ij} = F_{ij}$ with $F_{ij} = -N(2K_i^m K_{jm} - K K_{ij}) + N(R_{ij} + C_{ij}) - D_i D_j N + \Omega_{ij}^k \partial_k N$, where $\Omega_{ij}^k = 2\phi^{-1} g^{ke} (g_{ie} \partial_j \phi + g_{je} \partial_i \phi - g_{ij} \partial_e \phi) = 2g^{ke} (g_{ie} \partial_j \eta + g_{je} \partial_i \eta - g_{ij} \partial_e \eta)$.

A closed system is obtained by choosing an equation for the conformal factor. A common choice is $\phi = h^{1/12}$ in terms of the determinant h of the three metric h_{ij} . For outgoing radiation, note that $\phi = 1$ up to including first order in the wave-amplitude, whereby $\phi = 1$ tracks the merger phase of black hole coalescence (here with overlapping horizon surfaces) and ringdown in collapse to a single black hole. A slight variation is to insist $g^{ij} \partial_t g_{ij} = 0$. The complete hyperbolic system for numerical integration using normal coordinates ($\beta_i = 0$) hereby becomes

$$\begin{cases} \partial_t N &= -K, \\ \partial_t \eta &= -\frac{1}{6} N K, \\ \partial_t g_{ij} &= -2N \left(K_{ij} - \frac{1}{3} g_{ij} \tilde{K}_{ij} \right), \\ \partial_t K_{ij} &= F_{ij}(\eta, g_{ij}, N), \end{cases} \quad (21)$$

where $\tilde{K}_{ij} = e^{-4\eta} K_{ij}$.

The cosmological event horizon $\lambda \pm \lambda_2$ represents a *turning point* in (4), representing an extremum of the radius $\rho\phi^2$ defined by the Neumann condition

$$\frac{d\eta}{d\lambda} = -\frac{1}{2}. \quad (22)$$

As such, (22) defines the cosmological event horizon $(\lambda_2(\theta, \varphi), \theta, \varphi)$ following superpositions (10). In dynamical evolutions, it can be used to define apparent horizon surfaces. A proper condition of lapse function N is that it preserves a simple zero across, i.e., the initial condition

$$N = \frac{d \log(\rho\phi^2)}{d\lambda}. \quad (23)$$

5. Conclusions

We have extended the superposition principle for the Hamiltonian energy constraint to (anti-)de Sitter cosmologies with cosmological constant Λ in terms of an eigenvalue problem for the Laplace operator on a metric with constant curvature.

A positive cosmological constant Λ is of increasing relevance to our study of cosmological spacetimes in view of its role in the early universe in view of observational evidence for a small positive value at the present epoch.

For one black hole, the conformal scale ϕ is periodic, representing the extension of the black hole-de Sitter space to an infinite tower of universes. Superposition of two eigenfunctions produces a binary black hole-de Sitter universe with one (adjacent) cosmological event horizon when the separation is small relative to the cosmological scale $1/\sqrt{\Lambda}$. However, it produces a binary of universes when their separation is large described by two adjacent cosmological event horizons.

The results demonstrate extended topologies of black hole-de Sitter cosmologies, wherein the metric is everywhere smooth, as the physical singularities inherent to black holes are moved away into the complex plane and space is given a finite physical extent. By virtue of a finite evaporation time of black holes by Hawking radiation, there is a strong cosmic censorship conjecture by which singularities are protected from direct observation, regardless whether they are located in real or complex coordinate space.

The dynamical interaction of a binary of two universes (4) is conceivably of interest to the early universe and the ensuing generation of primordial gravitational waves from an initial distribution of primordial black hole-de Sitter universes, when Λ was large at the onset of inflation. An extension of our approach to extra dimensions is readily given, which is conceivably of interest to dynamical evolution of our four-dimensional spacetime within a spacetime of extra dimensions. For computational purposes, we give a new hyperbolic formulation of the equations of motion based on a curvature-driven lapse function.

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