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Perturbative instabilities in Hořava gravity

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ABSTRACT: We investigate the scalar and tensor perturbations in Hořava gravity, with and without detailed balance, around a flat background. Once both types of perturbations are taken into account, it is revealed that the theory is plagued by ghost-like scalar instabilities in the range of parameters which would render it power-counting renormalizable, that cannot be overcome by simple tricks such as analytic continuation. Implementing a consistent flow between the UV and IR limits seems thus more challenging than initially presumed, regardless of whether the theory approaches General Relativity at low energies or not. Even in the phenomenologically viable parameter space, the tensor sector leads to additional potential problems, such as fine-tunings and super-luminal propagation.

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1. Introduction

Relativity is commonly assumed to be the foundation of every gravitational construction, especially in its cosmological application. However, since a quantum field theory of general relativity proves to be non-renormalizable, numerous attempts of its modification have appeared in the literature. Recently, a power-counting renormalizable, ultra-violet (UV) complete theory of gravity was proposed by Hořava in [1, 2, 3, 4]. Although presenting General Relativity as an infrared (IR) fixed point, in the UV the theory possesses a fixed point with an anisotropic, Lifshitz scaling between time and space.

Due to these novel features, there has been a large amount of effort in examining and extending the properties of the theory itself [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. Additionally, application of Hořava gravity as a cosmological framework gives rise to Hořava cosmology, which proves to lead to interesting behavior [20, 21]. In particular, one can examine specific solution subclasses [22, 23, 24, 25, 26, 27], the perturbation spectrum [28, 29, 30, 31, 32], the gravitational wave production

[33, 34, 35], the matter bounce [36, 37], the black hole properties [38, 39, 40, 62, 42, 43, 44, 45, 46], the dark energy phenomenology [47, 48, 49, 50] etc.

However, investigations of various aspects of Hořava gravity have started to reveal some potentially troublesome features. Although some of them can be alleviated or removed once the detailed-balance condition is relaxed [12, 51, 52], the most significant core remains unaffected. In [11] it was shown that the diffeomorphism-invariance breaking leads to an additional degree of freedom (known already in [2] as the longitudinal degree of freedom of metric perturbations), which is not needed to decouple in the IR, and thus General Relativity is not recovered at any scale. Similarly, in [53] it was shown that the explicit breaking of general covariance uncovers an extra scalar degree of freedom, with fast exponential instabilities at short distances and strong-coupling at extremely low cutoff scales. In addition, observational constraints may rule out the theory completely [54]. These features led many authors to examine various forms of “modified” Hořava gravity, such as versions with full diffeomorphism invariance [55, 56], with deformed action and zero cosmological constant (and thus with Minkowski and not AdS IR limit) [57, 58], with restored parity invariance beyond detailed-balance [13, 14, 15], with extra derivative-terms [59], and with “soft” detailed-balance breaking [60].

On the other hand, most works on the cosmological application of Hořava gravity focus straightaway on its IR limit, as it is realistically expected to happen, without examining the running of the theory towards this regime. Indeed, in the original papers of Hořava the running behavior was revealed, but not the precise flow. It is interesting to point out that, up to this point, there is an amount of ambiguity concerning the IR behavior of the theory. The presence of additional degrees of freedom appears to be in some sense background-dependent. Although strong coupling arises once the perturbations around a flat Minkowski spacetime are considered [11], the theory seems to be well behaved when perturbing a cosmological background [32]. It is therefore non-trivial to trace internal consistency problems of the theory directly at the IR limit, at least in the perturbative level, even though it is already known that non-perturbative solutions such as the Schwarzschild black hole are not recovered in the context of the conventional, detailed-balance-preserving Hořava gravity [22].

In the present work we are interested in performing a detailed investigation of the gravitational perturbations of Hořava gravity, using it as a tool to examine its consistency. We study both scalar and tensor sectors, around a Minkowski background. We find that instabilities do appear in the whole range of parameter values, between which the theory is supposed to flow in order to provide a consistent UV completion. Such instabilities seem to exclude any viable flow of the theory within the range needed to provide power-counting renormalizability. Breaking detailed balance also does not seem to improve the situation. Additionally, these results are not affected by the above mentioned ambiguity about the existence or not of additional degrees of freedom at the IR. Finally, we argue that the IR limit behavior may actually be

the least of the problems.

In summary, the gravitational perturbation investigation reveals that Hořava gravity, in its present form, suffers from instabilities and fine-tunings that seem to originate from its deep and implicit features and thus they cannot be overcome by simple tricks such as analytic continuation. The paper is organized as follows: In section 2 we extract the scalar and tensor perturbations under the ADM diffeomorphism group in Minkowski spacetime, setting the relevant degrees of freedom and fixing the gauge. In section 3 we discuss about the emerging instabilities, the fine-tunings and the possible causality problems. Finally, section 4 is devoted to a summary and discussion of the obtained results.

2. Gravitational perturbations in Minkowski background

One of the most decisive tests for the reliability of a gravitational theory is the examination of its perturbations. This investigation reveals some of the deep features of the theory, and additionally, one may hope to eventually extract possible observational signatures. Thus, the study of gravitational perturbations of Hořava gravity will be the basic tool of the present work.

2.1 Variables and gauge transformations

We consider coordinate transformations of the form $x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu$. Under this transformation the metric-perturbation around a given background changes as $\delta\tilde{g}_{\mu\nu} = \delta g_{\mu\nu} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu$.

As it is known, the perturbation analysis, especially its primordial phase, is sensitive to the background evolution. A usual choice for cosmological applications [20, 21], is to impose $N^{(0)} = 1$, $N_i^{(0)} = 0$, $g_{ij}^{(0)} = a^2(t)\delta_{ij}$ in the ADM (“foliation-preserving”) formalism. Although we will soon perform perturbations around a flat background, we keep for the moment the explicit dependence on the scale factor for the sake of generality. The background metric is given by

$$\begin{aligned} ds_{(0)}^2 &= (g_{ij}^{(0)} N^{i(0)} N^{j(0)} - N^2) dt^2 + 2g_{ij}^{(0)} N^{i(0)} dt dx^j + g_{ij}^{(0)} dx^i dx^j \\ &= a^2(n) (-dn^2 + \delta_{ij} dx^i dx^j). \end{aligned} \quad (2.1)$$

From now on the 0-coordinate denotes the conformal time n , defined through $dt^2/a^2 = dn^2$. Therefore, the general perturbations of the metric around this background read:

$$\delta g_{00} = -2a^2 \phi \quad (2.2)$$

$$\delta g_{0i} = a^2 \partial_i B + a^2 Q_i \quad (2.3)$$

$$\delta g_{ij} = a^2 h_{ij} - a^2 (\partial_i W_j + \partial_j W_i) - 2a^2 \psi \delta_{ij} + 2a^2 \partial_i \partial_j E. \quad (2.4)$$

The vector modes are assumed to be transverse, that is $\partial_i W^i = \partial_i Q^i = 0$, while the tensor mode is forced to be transverse and traceless, $\partial_i h^{ij} = \delta^{ij} h_{ij} = 0$. Finally, the

Christoffel symbols read

$$\Gamma_{00}^0 = H, \quad \Gamma_{0j}^i = H\delta_j^i, \quad \Gamma_{ij}^0 = H\delta_{ij}, \quad (2.5)$$

where H is the *conformal Hubble parameter*, $H = \frac{1}{a(n)} \frac{da(n)}{dn}$. In the following, we concentrate on the flat geometry, and since there is now no distinction between coordinate and conformal time we restore t as denoting the time coordinate.

The spatial part of the gauge transformation vector is defined as $\xi^i = \partial^i \xi + \zeta^i$ with $\zeta_i = \dot{\zeta}_i$, so that $\xi_i = a^2(\partial_i \xi + \zeta_i)$. Anticipating the application to the Hořava gravity framework, where only transformations of the time coordinate of this sort are allowed, we assume that $\xi^0 = \xi^0(t)$. Using these transformation rules, we obtain the following expressions for the gauge transformation of each mode (note that $\xi_0 = -a^2 \dot{\xi}^0$):

- Scalars

$$\tilde{\psi} = \psi + H\xi^0 \quad (2.6)$$

$$\tilde{\phi} = \phi - H\xi^0 - \dot{\xi}^0 \quad (2.7)$$

$$\tilde{B} = B - \dot{\xi} \quad (2.8)$$

$$\tilde{E} = E - \xi \quad (2.9)$$

- Vectors

$$\tilde{Q}_i = Q_i - \dot{\zeta}_i \quad (2.10)$$

$$\tilde{W}_i = W_i + \zeta_i \quad (2.11)$$

- Tensor

$$\tilde{h}_{ij} = h_{ij}, \quad (2.12)$$

where a dot denotes the coordinate time-derivative. Lastly, note that since we restrict ourselves to the flat case, in the following we set $H = 0$.

Let us now discuss about the gauge fixing, which is required for the action derivation and the determination of the physical degrees of freedom. The projectability condition of Hořava gravity [3] requires that the perturbation of the lapse-function N depends only on time, thus $\phi \equiv \phi(t)$. This allows us to “gauge away” the ϕ -perturbation, that is to set $\tilde{\phi} = 0$ imposing $\phi = \dot{\xi}^0$. Similarly, we fix $\tilde{B} = 0$ from setting $B = \dot{\xi}$. Finally, setting $Q_i = \dot{\zeta}_i$ we eliminate the Q_i degree of freedom. Therefore, the remaining degrees of freedom are ψ , E , W_i and h_{ij} .

In summary, in the aforementioned gauge we obtain

$$\delta N = \delta N_i = 0 \quad (2.13)$$

$$\delta_{ij} = h_{ij} - 2\psi\delta_{ij} + 2\partial_i\partial_j E - (\partial_i W_j + \partial_j W_i). \quad (2.14)$$

Note that since only perturbations imposed on the “same-time” spatial hypersurface are allowed, this is equivalent to a *synchronous gauge* choice.

2.2 Perturbed Action

In Hořava gravity the gravitational action can be decomposed into a kinetic and a potential part as $S_g = S_K + S_V$, where

$$S_K = \frac{2}{\kappa^2} \int dt d^3x \sqrt{g} N (K_{ij} K^{ij} - \lambda K^2), \quad (2.15)$$

with

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i), \quad (2.16)$$

the extrinsic curvature and κ a constant with mass dimension -1 .

One of the subjects that have led to discussion in the literature is the imposition of detailed balance [3], which apart from reducing the possible terms in the action, it allows for a quantum inheritance principle [1] (the $D+1$ dimensional theory acquires the renormalization properties of the D -dimensional one). In order to proceed and write explicitly the potential term for the moment we impose detailed balance too. However, in order to avoid possible accidental artifacts of this, possibly ambiguous, condition in the obtained results, later on we will extend our analysis beyond detailed balance.

Under detailed balance we can write

$$S_V = \int dt d^3x \sqrt{g} N \left[-\frac{\kappa^2}{2w^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2w^2} \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R_k^l - \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left(\frac{1-4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right) \right]. \quad (2.17)$$

In the above expression, the Cotton tensor of the spatial hypersurface is given by

$$C^{ij} = \frac{\epsilon^{ikl}}{\sqrt{g}} \nabla_k \left(R_l^j - \frac{1}{4} R \delta_l^j \right), \quad (2.18)$$

and the covariant derivatives are defined with respect to the spatial metric g_{ij} . ϵ^{ijk} is the totally antisymmetric unit tensor, Λ is a negative constant which is related to the cosmological constant in the IR limit, and the variables w and μ are constants with mass dimensions 0 and 1, respectively. Finally, λ is a dimensionless constant, which incorporates the running behavior of Hořava gravity. Although a priori arbitrary, it is now known that the important variation-regime is between $1/3$ and 1 , with the former corresponding to the UV limit and the later to the IR one.

We will now perturb the Hořava gravitational action up to second order. At this point, we should mention that varying the action once we have substituted our metric ansatz for the perturbations is equivalent to deriving the Hamilton and momentum constraints along with the remaining equations of motion for an arbitrary metric perturbation and then imposing the ansatz. We choose the former approach here

for reasons of simplicity. After non-trivial but straightforward calculations that are presented in Appendix A, for the perturbed kinetic part (2.15) we obtain

$$\delta S_K^{(2)} = \int dt d^3x \frac{2}{\kappa^2} \left[\frac{1}{4} \dot{h}_{ij} \dot{h}^{ij} + (1-3\lambda) \left(3\dot{\psi}^2 - 2\dot{\psi} \nabla^2 \dot{E} \right) + (1-\lambda) \dot{E} \nabla^4 \dot{E} \right], \quad (2.19)$$

while for the perturbed potential part (2.17) we acquire

$$\begin{aligned} \delta S_V^{(2)} = \int dt d^3x \left[\frac{\kappa^2}{8w^4} h_{ij} \nabla^6 h^{ij} + \frac{\kappa^2 \mu}{8w^2} \epsilon^{ijk} h_{il} \partial_j \nabla^4 h_k^l - \frac{\kappa^2 \mu^2}{32} h_{ij} \nabla^4 h^{ij} \right. \\ \left. + \frac{\kappa^2 \mu^2 \Lambda}{32(1-3\lambda)} h_{ij} \nabla^2 h^{ij} - \frac{\kappa^2 \mu^2 (1-\lambda)}{4(1-3\lambda)} \psi \nabla^4 \psi - \frac{\kappa^2 \mu^2 \Lambda}{4(1-3\lambda)} \psi \nabla^2 \psi \right. \\ \left. + \frac{27\kappa^2 \mu^2 \Lambda^2}{16(1-3\lambda)} \psi^2 - \frac{9\kappa^2 \mu^2 \Lambda^2}{8(1-3\lambda)} \psi \nabla^2 E + \frac{3\kappa^2 \mu^2 \Lambda^2}{16(1-3\lambda)} E \nabla^4 E \right]. \quad (2.20) \end{aligned}$$

2.3 Scalar perturbations

As can be observed from (2.19), (2.20) the action for scalar perturbations includes the two modes ψ and E and it is written as

$$\begin{aligned} \delta S_S^{(2)} = \int dt d^3x \left[\frac{2(1-3\lambda)}{\kappa^2} \left(3\dot{\psi}^2 - 2\dot{\psi} \nabla^2 \dot{E} \right) + \frac{2(1-\lambda)}{\kappa^2} \dot{E} \nabla^4 \dot{E} \right. \\ \left. - \frac{\kappa^2 \mu^2 (1-\lambda)}{4(1-3\lambda)} \psi \nabla^4 \psi - \frac{\kappa^2 \mu^2 \Lambda}{4(1-3\lambda)} \psi \nabla^2 \psi + \frac{27\kappa^2 \mu^2 \Lambda^2}{16(1-3\lambda)} \psi^2 \right. \\ \left. - \frac{9\kappa^2 \mu^2 \Lambda^2}{8(1-3\lambda)} \psi \nabla^2 E + \frac{3\kappa^2 \mu^2 \Lambda^2}{16(1-3\lambda)} E \nabla^4 E \right]. \quad (2.21) \end{aligned}$$

Varying it with respect to E and ψ we obtain the equations of motion:

$$\frac{8}{\kappa^2} \ddot{E} + \frac{\kappa^2 \mu^2 (1-\lambda)}{2(1-3\lambda)} \nabla^2 \psi + \frac{\kappa^2 \mu^2 \Lambda}{2(1-3\lambda)} \psi = 0 \quad (2.22)$$

$$\begin{aligned} \frac{8}{\kappa^2} \frac{1-3\lambda}{1-\lambda} \ddot{\psi} - \frac{9\kappa^2 \mu^2 \Lambda^2}{4(1-\lambda)(1-3\lambda)} \psi + \frac{3\kappa^2 \mu^2 \Lambda^2}{4(1-\lambda)(1-3\lambda)} \nabla^2 E \\ + \frac{\kappa^2 \mu^2 (1-\lambda)}{2(1-3\lambda)} \nabla^4 \psi + \frac{\kappa^2 \mu^2 \Lambda}{2(1-3\lambda)} \nabla^2 \psi = 0. \quad (2.23) \end{aligned}$$

As can be seen these two equations are coupled, not allowing for a straightforward stability investigation. However, we can still acquire information about the stability of the configuration by studying it at high and low momenta. Taking the IR limit of (2.22), (2.23), that is considering their low- k behavior, they reduce to

$$\frac{8}{\kappa^2} \ddot{E} + \frac{\kappa^2 \mu^2 \Lambda}{2(1-3\lambda)} \psi = 0 \quad (2.24)$$

$$\frac{8}{\kappa^2} \frac{1-3\lambda}{1-\lambda} \ddot{\psi} - \frac{9\kappa^2 \mu^2 \Lambda^2}{4(1-\lambda)(1-3\lambda)} \psi = 0. \quad (2.25)$$

Thus, the second equation is decoupled, acting as a low-momentum equation of motion for the scalar field ψ .

A straightforward observation from (2.25) is that it leads to a ghost-like behavior in the IR limit whenever $\frac{1}{3} < \lambda < 1$. Inverting the overall sign of the Lagrangian is not going to help, since as we will promptly see, the time derivative of the tensor perturbation has the opposite sign. In particular, (2.25) leads to

$$\ddot{\psi} - \frac{9\kappa^4\mu^2\Lambda^2}{32(1-3\lambda)^2}\tilde{\psi} = 0, \quad (2.26)$$

where

$$\psi(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \tilde{\psi}_k(t) e^{i\mathbf{k}\cdot\mathbf{x}}. \quad (2.27)$$

Therefore, we acquire the following dispersion relation

$$\omega^2 \equiv m^2 = -\frac{9\kappa^4\mu^2\Lambda^2}{32(1-3\lambda)^2} < 0, \quad (2.28)$$

which induces instabilities at the IR, regardless of the λ -value and of the sign of the cosmological constant. Finally, combination of both (2.24),(2.25) fixes E against ψ , and in particular it leads to

$$\psi = -\frac{9}{2}\frac{\Lambda}{1-3\lambda}E. \quad (2.29)$$

Thus, we are led to only one effective degree of freedom, or in other words the system has been diagonalized.

Now, for high k , (2.22), (2.23) reduce to

$$\frac{8}{\kappa^2}\ddot{E} + \frac{\kappa^2\mu^2(1-\lambda)}{2(1-3\lambda)}\nabla^2\psi = 0 \quad (2.30)$$

$$\frac{8}{\kappa^2}\frac{1-3\lambda}{1-\lambda}\ddot{\psi} + \frac{\kappa^2\mu^2(1-\lambda)}{2(1-3\lambda)}\nabla^4\psi = 0. \quad (2.31)$$

Therefore, the ghost-like coefficient in the time derivative part remains and (2.31) yields a high- k dispersion relation of the form

$$\omega^2 \equiv \frac{\kappa^4\mu^2}{16} \left(\frac{1-\lambda}{1-3\lambda} \right)^2 k^4. \quad (2.32)$$

In general the homogeneous system of equations (2.22),(2.23) leads to only one active degree of freedom (one of the fields should be defined in terms of the other, in order to have a non-trivial solution) and a dispersion relation of the form

$$\begin{aligned} \frac{64}{k^4} \left(\frac{1-3\lambda}{1-\lambda} \right) \omega^4 + \left[\frac{18\mu^2\Lambda^2}{(1-\lambda)(1-3\lambda)} + \frac{4\mu^2\Lambda}{1-3\lambda} k^2 - \frac{4\mu^2(1-\lambda)}{1-3\lambda} k^4 \right] \omega^2 \\ + \frac{3\kappa^4\mu^4\Lambda^2}{8(1-\lambda)(1-3\lambda)^2} [\Lambda + (\lambda-1)k^2] k^2 = 0. \end{aligned} \quad (2.33)$$

Therefore, the physical requirement of obtaining a positive solution for ω^2 will lead to restrictions on the various parameters of the theory.

2.4 Tensor perturbations

Let us now examine the tensor perturbations. Their action can be extracted from (2.19),(2.20) and it reads:

$$\delta S_T^{(2)} = \int dt d^3x \left[\frac{1}{2\kappa^2} \dot{h}_{ij} \dot{h}^{ij} + \frac{\kappa^2 \mu^2 \Lambda}{32(1-3\lambda)} h_{ij} \nabla^2 h^{ij} + \frac{\kappa^2}{8w^4} h_{ij} \nabla^6 h^{ij} + \frac{\kappa^2 \mu}{8w^2} \epsilon^{ijk} h_{il} \partial_j \nabla^4 h_k^l - \frac{\kappa^2 \mu^2}{32} h_{ij} \nabla^4 h^{ij} \right]. \quad (2.34)$$

Therefore, the graviton equation of motion writes as

$$\ddot{h}^{ij} - \frac{\kappa^4 \mu^2 \Lambda}{16(1-3\lambda)} \nabla^2 h^{ij} - \frac{\kappa^4}{4w^4} \nabla^6 h^{ij} - \frac{\kappa^4 \mu}{4w^2} \epsilon^{ilk} \partial_l \nabla^4 h_k^j + \frac{\kappa^4 \mu^2}{16} \nabla^4 h^{ij} = 0. \quad (2.35)$$

Assuming graviton propagation along the x^3 direction, that is $k_i = k^i = (0, 0, k)$, the h_{ij} can be written as usual in terms of polarization components as

$$h_{ij} = h^{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (2.36)$$

Using this parametrization we derive the two equations for the different polarizations

$$\ddot{h}_+ - \frac{\kappa^4}{4w^4} \nabla^6 h_+ + \frac{\kappa^4 \mu}{4w^2} \partial_3 \nabla^4 h_\times + \frac{\kappa^4 \mu^2}{16} \nabla^4 h_+ - \frac{\kappa^4 \mu^2 \Lambda}{16(1-3\lambda)} \nabla^2 h_+ = 0, \quad (2.37)$$

$$\ddot{h}_\times - \frac{\kappa^4}{4w^4} \nabla^6 h_\times - \frac{\kappa^4 \mu}{4w^2} \partial_3 \nabla^4 h_+ + \frac{\kappa^4 \mu^2}{16} \nabla^4 h_\times - \frac{\kappa^4 \mu^2 \Lambda}{16(1-3\lambda)} \nabla^2 h_\times = 0, \quad (2.38)$$

and thus we identify the light speed from the low k regime as

$$c^2 = \frac{\kappa^4 \mu^2 \Lambda}{16(1-3\lambda)}. \quad (2.39)$$

A significant observation is that the two polarization modes are coupled due to the ϵ^{ijk} term, which arises from the Cotton tensor. Thus, it is more convenient to shift to the Left-Right base defining

$$h_L \equiv \frac{1}{\sqrt{2}}(h_+ + ih_\times), \quad (2.40)$$

$$h_R \equiv \frac{1}{\sqrt{2}}(h_+ - ih_\times). \quad (2.41)$$

In this case, and after Fourier transforming we obtain the decoupled equations of motion (2.37),(2.38):

$$-\omega^2 \tilde{h}_L + c^2 k^2 \tilde{h}_L + \frac{\kappa^4 \mu^2}{16} k^4 \tilde{h}_L + \frac{\kappa^4 \mu}{4w^2} k^5 \tilde{h}_L + \frac{\kappa^4}{4w^4} k^6 \tilde{h}_L = 0, \quad (2.42)$$

$$-\omega^2 \tilde{h}_R + c^2 k^2 \tilde{h}_R + \frac{\kappa^4 \mu^2}{16} k^4 \tilde{h}_R - \frac{\kappa^4 \mu}{4w^2} k^5 \tilde{h}_R + \frac{\kappa^4}{4w^4} k^6 \tilde{h}_R = 0, \quad (2.43)$$

an equation system that accepts a non-trivial solution only if the corresponding determinant is zero, which leads to the dispersion relation

$$\omega^2 = c^2 k^2 + \frac{\kappa^4 \mu^2}{16} k^4 \pm \frac{\kappa^4 \mu}{4w^2} k^5 + \frac{\kappa^4}{4w^4} k^6. \quad (2.44)$$

Therefore, we can identify the effective light speed as

$$\begin{aligned} c_s^2(k) &= c^2 + \frac{\kappa^4 \mu^2}{16} k^2 \pm \frac{\kappa^4 \mu}{4w^2} k^3 + \frac{\kappa^4}{4w^4} k^4 \\ &= c^2 \left[1 + \frac{(1-3\lambda)}{\Lambda} k^2 \left(1 \pm \frac{2}{w^2 \mu} k \right)^2 \right]. \end{aligned} \quad (2.45)$$

In expressions (2.44),(2.45) the plus and minus branches correspond to Left-handed and Right-handed mode respectively.

Now, since the signs of the k^5 -term are different in the Left-handed and Right-handed modes, they propagate with a different speed, leading to a rotation of the polarization plane, and the rotation angle can be calculated by

$$\delta\chi = \int_i^f (\omega_R - \omega_L) dt, \quad (2.46)$$

where ω_R and ω_L represent the frequency of the Right-handed and Left-handed mode respectively and the subscripts "i" and "f" denotes the initial and final moment of the propagation. Note that due to this rotation, the spectrum of tensor perturbations would be expected to be suppressed at high energy scales, as was shown in a different context in [61].

2.5 Beyond Detailed Balance

In the above analysis we were restricted to the detailed-balance condition, which constraints the terms in the potential part of the action. Since we desire to examine the general features of Hořava gravity, and thus avoiding possible accidental artifacts of this condition, in this subsection we extend the investigation beyond detailed balance.

As a demonstration, and without loss of generality, we consider a detailed-balance-breaking term of the form $\nabla_i R_{jk} \nabla^i R^{jk}$. This term induces in the action a second order term which reads:

$$\nabla_i \delta R_{jk} \nabla^i \delta R^{jk} = \frac{1}{4} (\partial_i \nabla^2 h_{jk})^2 + (\partial_i \partial_j \partial_k \psi)^2 + 5 (\partial_i \nabla^2 \psi)^2. \quad (2.47)$$

Thus, the corresponding contribution to the action will be

$$\delta S_{new}^{(2)} = \eta \int dt d^3x \left(-\frac{1}{4} h_{ij} \nabla^6 h^{ij} - 6\psi \nabla^6 \psi \right), \quad (2.48)$$

where η is an additional parameter. It is straightforward to calculate the modifications that $S_{new}^{(2)}$ brings to the dispersion relations for scalar and tensor perturbations obtained above (expressions (2.32) and (2.44) respectively). The extended dispersion relations read:

- Scalar perturbations (UV-behavior):

$$\omega^2 \sim \frac{\kappa^2 (1-\lambda)^2}{16 (1-3\lambda)^2} k^4 - \frac{3\kappa^2 (1-\lambda)}{2 (1-3\lambda)} \eta k^6 \quad (2.49)$$

- Tensor perturbations:

$$\omega^2 = c^2 k^2 + \frac{\kappa^4 \mu^2}{16} k^4 \pm \frac{\kappa^4 \mu}{4w^2} k^5 + \left(\frac{\kappa^4}{4w^4} - \frac{\kappa^2 \eta}{2} \right) k^6. \quad (2.50)$$

As was expected, the new, detailed-balanced-breaking term, modifies mainly the UV regime of the theory.

3. Instabilities, fine-tunings and super-luminal propagation

In the previous section we investigated the gravitational perturbations of Hořava gravity. Such a study is crucial in order to examine the consistency of the theory itself. In the present section we analyze the obtained results, and focus on the problematic features.

Let us first discuss about the scalar perturbations. As was mentioned above, (2.25), (2.31) imply that the regime $\frac{1}{3} < \lambda < 1$ leads to instabilities. Unfortunately, this is exactly the flow-interval of λ -parameter between the UV and IR regimes. This unstable behavior of the scalar mode at $\frac{1}{3} < \lambda < 1$, has been already observed in the literature. In [22] it was argued that it could be amended by imposing an analytic continuation of the form

$$\mu \rightarrow i\mu, \quad w^2 \rightarrow -iw^2. \quad (3.1)$$

This could also allow for a positive cosmological constant solution. Unfortunately, performing the analytic continuation (3.1), we straightforwardly see that the UV behavior is spoiled (see (2.32)) and thus instabilities re-emerge at high energies. Finally, as we will see later on, such an analytic continuation would induce similar problems in the tensor sector, too.

Without analytic continuation, the only physically interesting case that remains, allowing for a possible flow towards General Relativity (at $\lambda = 1$) is the regime $\lambda \geq 1$ (since the region $\lambda \leq \frac{1}{3}$ is disconnected). Even in this case though, we cannot evade the instability coming from the negative mass term, whose sign is independent of

both λ and Λ . IR instabilities persist in this regime as long as we have a non-vanishing cosmological constant. Variations of the theory with $\Lambda = 0$ thus seem to be favored from such a viewpoint [62]. However, as we will see later on, the $\Lambda = 0$ subclass of the original version of Hořava gravity leads to phenomenological problems concerning the light-speed definition.

We now turn to the tensor sector. Here again, from (2.44) we see that if we desire a well-behaved UV regime we cannot impose the analytic continuation (3.1). Thus, consistency with the scalar-sector results means restriction to the $\lambda \geq 1$ regime.

Let us now proceed relaxing the detailed-balance condition, that is analyzing the results of subsection 2.5, according to which the behavior of detailed-balance Hořava gravity is modified mainly at the UV. A first and crucial observation is that the ghost instability of the scalar mode arises from the kinetic term of the action and thus the breaking of detailed balance, which affects the potential term, will not alter the aforementioned scalar-instabilities results concerning the exclusion of $\frac{1}{3} < \lambda < 1$ regime. In the physical $\lambda \geq 1$ region, the scalar dispersion relation (2.49) remains well-behaved in the UV, provided η is negative. Additionally, for negative η the UV behavior of the tensor perturbations is not affected. Finally, it is interesting to notice that if η is sufficiently negative, then analytic continuation would not bring problems to the tensor sector (as can be seen imposing (3.1) in (2.50)). Unfortunately, since analytic continuation cannot cure the instabilities of the scalar-sector, this possibility does not have any further physical utility. However, it can offer an additional indication that a consistent and well-behaved Hořava gravity should be sought beyond detailed balance.

In summary, we see that the combined scalar and tensor perturbation analysis excludes the regime $\frac{1}{3} < \lambda < 1$, since the involved ghost instabilities cannot be removed. Thus, we conclude that the allowed and physically interesting interval of Hořava gravity is that with $\lambda \geq 1$, and even in this case, the problem of IR instabilities persists. However, it seems unlikely that a RG flow in this regime will render the theory power-counting renormalizable, thus negating its initial motivation as a possible UV completion of gravity. We mention that this result is valid independently of the imposition of the detailed-balance condition, and this is the reason we did not perform an investigation beyond detailed balance in full generality, examining more such terms. It is also complementary to already known works, concerning the presence of the additional scalar modes and the way they may defer the theory from becoming equivalent to GR at the IR [11, 53, 63, 64]. This approach is however less ambiguous, since in the regime we are discussing the new degree of freedom is definitely present. There is also no need to invoke strong coupling arguments, i.e. bring matter into the picture [11].

Let us now discuss a different, but equally significant problem, that is related to some basic observable consequences of Hořava gravity. For simplicity we restrict ourselves to the detailed-balance, but this discussion is independent of that. In

expression (2.45) we provide the effective light speed, comparing to the standard light speed c^2 given by (2.39). If we desire the correction term to be of next-to-leading order, then Λ must be large in general. However, as it is known the effective cosmological constant in a universe governed by Hořava gravity is [20, 21]:

$$|\Lambda_{eff}| = \frac{\kappa^4 \mu^2}{16(1-3\lambda)^2} \Lambda^2 = \frac{c^2}{|1-3\lambda|} |\Lambda|, \quad (3.2)$$

where the second expression is obtained using (2.39). Thus, in general Λ should be very small. The aforementioned contradiction could be resolved by the addition of a suitable positive constant in an introduced matter sector. However, having in mind the physical values of the aforementioned quantities, this would lead to an incredible fine-tuning problem. In other words, the cosmological constant problem does remain in Hořava gravity, which is strange since this theory is constructed to incorporate the underlying and fundamental gravitational features. Furthermore, the fact that its solution would demand the contribution of the matter-sector is even more problematic. Finally, we mention that the aforementioned light-speed definition makes the $\Lambda = 0$ theory problematic and this was already mentioned in [20, 21]. Therefore, one cannot easily set $\Lambda = 0$ in order to cure the instabilities as described above, and thus the aforementioned instability results seem to be robust.

We close this section by referring to another potential problem. According to (2.45), in the physical case $\lambda \geq 1$ the effective light-speed is sub-luminal if $\Lambda > 0$, while it is super-luminal if $\Lambda < 0$. However, in the regime $\lambda \geq 1$, Λ must be indeed negative in order to assure for a well-defined light speed [20, 21]. Therefore, we observe a possible causality violation. At first it could be stated that such a violation is not a surprise, since Hořava gravity violates relativity and thus the light cone is not a bound anymore. However, it is the classical causality that is violated, and therefore, one could still construct extensions where both energy condition and causality would be restored. For such extensions one could use the formalism and ideas of reggeon field theory [65, 66] and of holographic correspondence used in studies of AdS/CFT [67, 68, 69, 70]. It seems that Hořava gravity could correspond to a subclass of models where non-relativistic (super)-conformality, required in AdS/CFT [71], can be acquired. Definitely, causality in Hořava gravity is a subject that requires further and thorough investigation.

4. Conclusions

In this work we have investigated the gravitational perturbations of Hořava gravity, with and without the detailed-balance condition. Setting the relevant degrees of freedom and fixing the gauge we have extracted the scalar and tensor dispersion relations and the corresponding effective speeds of propagation. As we have seen, the scalar sector is plagued by instabilities in the $\frac{1}{3} < \lambda < 1$ regime, which cannot

be cured by analytic continuation. Thus, the only physical regime, that additionally allows for an IR limit towards General Relativity, is the $\lambda \geq 1$ one. This result is in line with previous treatments discussing potential problems in recovering General Relativity as an IR limit of the theory. We stress though that in our case there is no ambiguity about the role of the additional scalar degree of freedom. Since we are dealing with λ away from unity, the scalar mode is certainly present and no ambiguity arises. Regardless of possible strong coupling issues occurring at $\lambda = 1$, we see that perturbative instabilities arise long before we reach the IR limit. The ability of Hořava gravity to fulfill its RG flow between the UV and IR, seems to be jeopardized.

This λ -restriction also leads to a significant fine-tuning problem, which is related to the value of the effective cosmological constant in the universe. This fact casts additional doubt on Hořava gravity, since any theory that desires to incorporate some of the fundamental gravitational features, should provide some route to alleviate the cosmological constant problem.

Additionally, in the present form of Hořava gravity, one could have causality violation, which would be either a signal of problematic behavior, or a novel property that originates from the relativity abandonment. Clearly, a detailed examination of this subject is crucial in revealing the underlying structure of the theory.

Although the treatment of the present work is robust at the usual perturbative framework, we would like to close by commenting on the possible limitations of our procedure. The presence of a ghost-like scalar mode may be rendered ambiguous, unless the entire set of perturbations of the metric is taken into account. In particular, if there is mixing between different types of perturbations (that is if the “orthogonality” assumption that forbids the mixing between scalar and tensor modes is not valid), it may lead to the appearance of “fictitious” scalar ghosts, such as the conformal ghost in General Relativity (see also [72] and [73]). Furthermore, it could be important to include backreactions of the higher spatial curvature terms to the geometry, since the presence of non-linear corrections may lead to non-trivial modifications, as was recently pointed out in [52]. Both these non-trivial extensions could provide additional information on Hořava gravity and deserve further inquiry.

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Note added

After the present work was pre-printed and after its submission to the journal, the subject of possible instabilities in Hořava gravity has gained a significant attention

in the literature. In [74] the authors claimed that a more general breaking of the detailed balance can cure the instabilities. However, as we showed in our analysis, the problematic behavior is not related to the strength of the detailed-balance breaking but it seems to be a deeper constituent of the theory, at least in its basic version (with or without detailed-balance) examined in the present work. These results were verified by many other authors, and led to generalizations of the basic Hořava gravity version in order to cure the instabilities. For instance, in [75, 76, 77] it was argued that the cause of the instabilities is the projectability condition itself, and thus a new non-projectable version of Hořava gravity was formulated. But as it was shown in [78, 79], even in such an extension it is ambiguous if the strong coupling problem can be avoided (see also [80]). In the same lines, some other authors claimed that in order to avoid the instabilities, one should study suitably modified versions of Hořava gravity [81] (see also [82] for a different approach). The evolution of the literature after the appearance of our work seems to verify our basic result about the problematic features of the basic version of Hořava gravity. Clearly, the subject is still open in extended Hořava-like gravitational theories and deserves further investigation, but this is the center of interest of separate works.

A. Perturbations of kinetic and potential parts of Hořava action

A.1 Perturbation of S_K

In order to obtain the perturbed kinetic and potential terms that constitute the Hořava gravitational action, we need to extract the perturbations of the basic geometrical quantities. Following the formalism of subsection 2.1, the perturbed spatial Christoffel symbols read

$$\begin{aligned} \delta\Gamma_{jk}^i &= \frac{1}{2} (\partial_j h_k^i + \partial_k h_j^i - \partial^i h_{jk}) - \partial_l \partial_k W^i \\ &\quad - (\delta_k^i \partial_j \psi + \delta_j^i \partial_k \psi - \delta_{jk} \partial^i \psi) + \partial_k \partial_j \partial^i E, \end{aligned} \quad (1.1)$$

and thus the perturbation of the Riemann tensor is $\delta R_{jkl}^i = \delta\Gamma_{jl,k}^i - \delta\Gamma_{jk,l}^i$, while that of the Ricci tensor is

$$\delta R_{ij} = -\frac{1}{2} \nabla^2 h_{ij} + \partial_i \partial_j \psi + \delta_{ij} \nabla^2 \psi, \quad (1.2)$$

with $\nabla^2 \equiv \partial_i \partial^i$. Therefore, the Ricci-scalar perturbation $\delta R = \delta(g^{ij} R_{ij}) = \delta g^{ij} R_{ij}^{(0)} + g^{ij(0)} \delta R_{ij}$ (since $R_{ij}^{(0)} = 0$ and $g^{ij(0)} = \delta^{ij}$) writes as

$$\delta R = 4 \nabla^2 \psi. \quad (1.3)$$

Since the extrinsic curvature of the background spacetime is zero, the only second-order contributions we may have will be of the form $\delta K_{ij} \delta K^{ij}$ and $(\delta K)^2$.

Higher order contributions arising from the $\sqrt{g}N$ will not enter here and this coefficient evaluated at zeroth-order is equal to one. We thus calculate

$$\begin{aligned}\delta K_{ij} &= \frac{1}{2} (\delta \dot{g}_{ij} - \partial_i \delta N_j - \partial_j \delta N_i) \\ &= \frac{1}{2} \left(\dot{h}_{ij} - 2\dot{\psi} \delta_{ij} + 2\partial_i \partial_j \dot{E} - \partial_i \dot{W}_j - \partial_j \dot{W}_i \right).\end{aligned}\quad (1.4)$$

Therefore, we also determine $\delta K = \delta(g^{ij} K_{ij}) = \delta g^{ij} K_{ij}^{(0)} + g^{ij(0)} \delta K_{ij} = \delta^{ij} \delta K_{ij}$, that is

$$\delta K = -3\dot{\psi} + \nabla^2 \dot{E}. \quad (1.5)$$

Assembling everything, the perturbed kinetic part of the action, up to second order, writes

$$\delta S_K^{(2)} = \int dt d^3x \frac{2}{\kappa^2} \left[\frac{1}{4} \dot{h}_{ij} \dot{h}^{ij} + \frac{1}{2} \partial_i \dot{W}_j \partial^i \dot{W}^j + (1 - 3\lambda) \left(3\dot{\psi}^2 - 2\dot{\psi} \nabla^2 \dot{E} \right) + (1 - \lambda) \dot{E} \nabla^4 \dot{E} \right], \quad (1.6)$$

where we have also used the transversality of fields and integration by parts to eliminate irrelevant terms and remove derivatives. Thus, switching off the W^i mode, and concentrating only on scalar and tensor perturbations, the perturbed kinetic part of the action (2.15), up to second order, yields the expression (2.19).

A.2 Perturbation of S_V

Let us now examine the perturbation of potential term (2.17). Note that again all quadratic terms must yield only contributions of the form $\delta R \delta R$ etc, since these tensors vanish for a flat background. The only terms that do not follow this rule are the fifth and sixth. The fifth is first order in R , so the second-order perturbation will be of the form $\delta(\sqrt{g})\delta R$, while the sixth only receives contributions at second order from the determinant \sqrt{g} . We will calculate each quadratic expression in turn.

For the first term we need to evaluate the perturbed Cotton tensor

$$\begin{aligned}\delta C^{ij} &= \frac{\epsilon^{ikl}}{\sqrt{g^{(0)}}} \partial_k \left(\delta R_l^j - \frac{1}{4} \delta R \delta_l^j \right) \\ &= -\frac{1}{2} \epsilon^{ikl} \partial_k \nabla^2 h_l^j,\end{aligned}\quad (1.7)$$

which allows to write for the first four-terms:

$$\delta C_{ij} \delta C^{ij} = -\frac{1}{4} h_{ij} \nabla^6 h^{ij} \quad (1.8)$$

$$\epsilon^{ijk} \delta R_{il} \partial_j \delta R_k^l = \frac{1}{4} \epsilon^{ijk} \nabla^2 h_{il} \partial_j \nabla^2 h_k^l \quad (1.9)$$

$$\delta R_{ij} \delta R^{ij} = \frac{1}{4} (\nabla^2 h_{ij})^2 + 6\psi \nabla^4 \psi, \quad (1.10)$$

together with the square of (1.3). Concerning the perturbation of the determinant part, we write:

$$\sqrt{g}NR = N^{(0)} \left(\sqrt{g^{(0)}} + \frac{1}{2\sqrt{g^{(0)}}}\delta g - \frac{1}{8}\frac{1}{\sqrt{g^{(0)}}^3}\delta g^2 + \dots \right) \quad (1.11)$$

$$\times \left(\delta^{ij}R_{ij}^{(1)} + \delta^{ij}R_{ij}^{(2)} - \delta g^{ij}R_{ij}^{(1)} \right) \quad (1.12)$$

$$= \delta^{ij}R_{ij}^{(1)} + \delta^{ij}R_{ij}^{(2)} - \delta g^{ij}R_{ij}^{(1)} + \frac{1}{2}\delta g\delta^{ij}R_{ij}^{(1)}, \quad (1.13)$$

since the perturbation of N is $\delta N = 0$ and $N^{(0)} = 1$. Note that the combination $\delta^{ij}R_{ij}^{(1)}$ is just δR , and thus the second-order perturbation becomes

$$(\sqrt{g}NR)^{(2)} = \delta^{ij}R_{ij}^{(2)} - \delta g^{ij}R_{ij}^{(1)} + \frac{1}{2}\delta g\delta R \quad (1.14)$$

where, to first order, $\delta g = g^{(0)}g^{ij(0)}\delta g_{ij} = \delta^{ij}\delta g_{ij} = -6\psi + 2\nabla^2 E$. Thus the third term in (1.14) becomes

$$\frac{1}{2}\delta g\delta R = -12\psi\nabla^2\psi + 4\nabla^2 E\nabla^2\psi. \quad (1.15)$$

Additionally, using integration by parts and imposing transversality, we obtain

$$\delta g^{ij}R_{ij}^{(1)} = -\frac{1}{2}h_{ij}\nabla^2 h^{ij} - 8\psi\nabla^2\psi + 4\psi\nabla^4 E. \quad (1.16)$$

Finally, writing the Ricci tensor as

$$R_{ij}^{(2)} = \Gamma_{ij,l}^{(2)l} - \Gamma_{il,l}^{(2)l} + \Gamma_{lk}^{(1)l}\Gamma_{ij}^{(1)k} - \Gamma_{jk}^{(1)l}\Gamma_{il}^{(1)k} \quad (1.17)$$

where

$$\Gamma_{jk}^{(1)i} = \frac{1}{2}(\partial_j\delta g_k^i + \partial_k\delta g_j^i - \partial^i\delta g_{jk}) \quad (1.18)$$

$$\Gamma_{jk}^{(2)i} = -\frac{1}{2}\delta g^{il}(\partial_j\delta g_{lk} + \partial_k\delta g_{jl} - \partial_l\delta g_{jk}) \quad (1.19)$$

with

$$\delta^{ij}\Gamma_{lk}^{(1)l}\Gamma_{ij}^{(1)k} = 3\psi\nabla^2\psi + 2\psi\nabla^4 E - E\nabla^6 E \quad (1.20)$$

$$\delta^{ij}\Gamma_{jk}^{(1)l}\Gamma_{il}^{(1)k} = \frac{1}{4}h_{ij}\nabla^2 h^{ij} + \psi\nabla^2\psi + 2\psi\nabla^4 E - E\nabla^6 E, \quad (1.21)$$

we conclude that the Ricci scalar can be expressed as

$$\begin{aligned} \delta^{ij}R_{ij}^{(2)} &= \delta^{ij}\Gamma_{ij,l}^{(2)l} - \delta^{ij}\Gamma_{il,l}^{(2)l} + \delta^{ij}\Gamma_{lk}^{(1)l}\Gamma_{ij}^{(1)k} - \delta^{ij}\Gamma_{jk}^{(1)l}\Gamma_{il}^{(1)k} \\ &= -\frac{1}{4}h_{ij}\nabla^2 h^{ij} + 2\psi\nabla^2\psi. \end{aligned} \quad (1.22)$$

Assembling (1.15),(1.16),(1.22) we find that the fifth term in the perturbed action reduces to

$$(\sqrt{g}NR)^{(2)} = \frac{1}{4}h_{ij}\nabla^2 h^{ij} - 2\psi\nabla^2\psi. \quad (1.23)$$

Lastly, using $\delta\sqrt{g} = \frac{1}{2\sqrt{g^{(0)}}}\delta g = -3\psi + \nabla^2 E$ and $(\delta\sqrt{g})^{(2)} = -\frac{1}{8}\delta g^2 = -\frac{9}{2}\psi^2 + 3\psi\nabla^2 E - \frac{1}{2}(\nabla^2 E)^2$, the sixth term in the perturbed action reads

$$(\delta\sqrt{g})^{(2)}N^{(0)} = -\frac{9}{2}\psi^2 + 3\psi\nabla^2 E - \frac{1}{2}(\nabla^2 E)^2. \quad (1.24)$$

Finally, assembling the aforementioned terms, we conclude that the perturbed potential part of the action is (2.20).

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