

Added mass for motion in density-stratified fluids

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In a density-stratified fluid, owing to the generation of internal gravity waves, the added mass of a moving body becomes anisotropic and frequency-dependent. Two definitions of added mass, equivalent in a homogeneous fluid¹, become different: one based on the impulse of the fluid, and the other on the pressure on the body.

We start by deriving a Kirchhoff–Helmholtz integral for internal waves, extending earlier attempts², and deduce from it the relation between the two definitions. We then calculate the motion of the fluid for oscillating spheres and cylinders, extending earlier results³, and deduce from it the coefficients of added mass, represented in figure 1(a) and consistent with experiment⁴. At frequencies large compared with the buoyancy frequency the coefficients take their values C_∞ in a homogeneous fluid (1/2 for the sphere and 1 for the cylinder); at frequencies smaller than the buoyancy frequency they become complex, of real part associated with added mass in the usual sense and imaginary part associated with wave drag. These variations are related by Fourier transformation to the free oscillations, represented in figure 1(b) and consistent with experiment⁵, of the bodies displaced from their neutrally buoyant levels then released.

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¹Batchelor, *An Introduction to Fluid Dynamics*, Cambridge University Press, §6.4 (1967).

²Miropol'skii, *Izv. Atmos. Ocean. Phys.* **14**, 673 (1978); Simakov, *J. Fluid Mech.* **248**, 55 (1993).

³Lai and Lee, *Int. J. Engng Sci.* **19**, 1411 (1981); Hurley, *J. Fluid Mech.* **351**, 105 (1997).

⁴Ermanyuk and Gavrilov, *J. Fluid Mech.* **451**, 421 (2002), **494**, 33 (2003).

⁵Larsen, *Deep-Sea Res.* **16**, 587 (1969); Winant, *Deep-Sea Res.* **21**, 445 (1974).

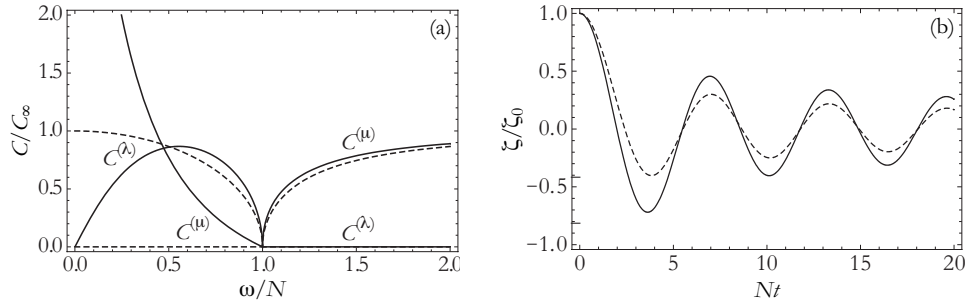


Figure 1: (a) Coefficients of added mass $C^{(\mu)} = \text{Re } C$ and damping $C^{(\lambda)} = (\omega/N) \text{Im } C$ of a sphere (solid lines) and horizontal circular cylinder (dashed lines) oscillating vertically at frequency ω in a fluid of buoyancy frequency N . (b) Vertical displacement ζ of a sphere (solid line) and horizontal circular cylinder (dashed line) displaced by ζ_0 from its neutrally buoyant level $\zeta = 0$ then released at time $t = 0$.