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The 1990s Rise in Swedish Earnings Inequality - Persistent or Transitory?

July 28, 2005

Abstract

This paper decomposes the rise in the cross-sectional variance of male annual earnings in Sweden between 1991 and 1999 into its persistent and transitory components. The results show that the persistent component accounts for basically all of the increase in earnings dispersion. This implies that the answer to the 1990s trend reversal in Swedish earnings inequality is to be found in explanations that focus on persistent changes in the labour market, such as changes in the price of skills

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1. Introduction

Sweden has an interesting history of marked changes in earnings and wage inequality. There was precipitous pay compression among all dimensions from the late 1960s through the 1970s which resulted in one of the most compressed wage structures visible among advanced countries.¹ Whilst several countries also experienced pay compression during the 1970s, few, if any, match the magnitude and rapidness of that in Sweden. In the 1990s, however, there was a noticeable trend reversal in Sweden as cross-sectional inequality began increase rapidly; see for instance Edin and Fredriksson (2000), Gustavsson (2005), and Lindquist (2005).

What are the causes and consequences of the 1990s move toward increased earnings dispersion in Sweden? Before one tries to answer questions along these lines it is crucial to recognize that the rise can be decomposed into a persistent and a transitory component, with very different ramifications for the duration of inequality. An increase in the persistent component increases both short- and long-term inequality. An increase in the transitory component, on the other hand, only increases short-term inequality. Consequently, if the cross-sectional increase stems from the persistent component, one should try to explain the trend reversal by focusing on factors that affect earnings differentials in a systematic and persistent way, such as increases in the price of skills. An increase in the transitory component, on the other hand, implies that one should focus on factors that can explain increased transitory earnings fluctuations, such as for instance larger job-instability or a closer connection between firm profits and earnings.

Using a large register-based longitudinal database, this paper offers a decomposition of the rise in cross-sectional inequality in Sweden between 1991 and 1999 into its persistent and transitory components. The paper starts with a description of the parametric model of earnings dynamics used in the analysis and section 3 describes the data. Section 4 presents the results, with the main conclusion being that the increased dispersion in persistent earnings accounts for basically all of the increase in cross-sectional inequality. The paper ends with concluding remarks.

¹ See Edin and Holmlund (1995) and Palme and Wright (1998) for an account of changes in Swedish pay differentials between 1968 and 1991. See for instance Katz and Autor (1999) for an international comparison of pay differentials.

2. Parametric models of earnings dynamics

This section describes the parametric model that I use to decompose changes in cross-sectional inequality into persistent and transitory components. To provide intuition behind this model, I first describe a more basic model of earnings dynamics.

Let Y_{ibt} denote the log of earnings in year t of the i^{th} individual born in year b . Then

$$(1) \quad Y_{ibt} = \mu_{bt} + y_{ibt}$$

expresses Y_{ibt} as the cohort specific mean μ_{bt} in year t plus an individual specific deviation y_{ibt} from that mean. A stripped down model for y_{ibt} is

$$(2) \quad y_{ibt} = p_t u_{ib} + \lambda_t v_{ibt}.$$

In equation (2), the variable u_{ib} and its year-specific factor loading, p_t , capture persistent, or permanent, earnings. The variable v_{ibt} and its factor loading, λ_t , capture stochastic transitory deviations from persistent earnings. The transitory component is serially uncorrelated with mean zero, and u_{ib} and v_{ibt} are uncorrelated with variances σ_u^2 and σ_v^2 respectively. With these assumptions, the variance of log earnings is

$$(3) \quad \text{Var}(y_{ibt}) = p_t^2 \sigma_u^2 + \lambda_t^2 \sigma_v^2,$$

and the auto-covariance between year t and $t-s$ is

$$(4) \quad \text{Cov}(y_{ibt}, y_{ib,t-s}) = p_t p_{t-s} \sigma_u^2.$$

Equation (3) shows that an increase in either factor loading generates increased cross-sectional earnings dispersion. The character of the change depends crucially, however, on which of the factor loadings that changes. A persistent rise in p_t increases long-run inequality as the relative labour market advantage of workers with chronically high earnings is enhanced. An increase in λ_t without any change in p_t generates increased cross-sectional earnings dispersion by raising year-to-year earnings instability but with no change in long-run inequality.

The model in equation (2), although intuitive, is likely to be too restrictive to adequately capture changes in persistent and transitory inequality. In particular, persistent and transitory inequality should be allowed to vary with age and transitory shocks should be allowed to last for several periods; see for instance the discussion in Baker & Solon (2003).

The model employed in this paper is as follows:

$$(5) \quad y_{ibt} = p_t u_{ia} + \varepsilon_{ibt},$$

where

$$(6) \quad u_{ia} = u_{i,a-1} + r_{ia},$$

$$(7) \quad \varepsilon_{ibt} = \rho \varepsilon_{ib,t-1} + \lambda_t v_{ibt},$$

and

$$(8) \quad \text{Var}(v_{ibt}) = \gamma_0 + \gamma_1 a + \gamma_2 a^2 + \gamma_3 a^3 + \gamma_4 a^4.$$

where $a = t - b - 27$, that is, years since age 27 (the lowest defined age in my sample).

Equations (5) and (6) model the persistent component as a random walk in age where the innovation at each age is $r_{i,a} \sim \text{iid}(0, \sigma_{r,a}^2)$.² The innovation variance $\sigma_{r,a}^2$ is allowed to differ between individuals aged 28-37 and 38-56 but is restricted to be the same within these two age intervals.³ I also estimate the variance of an initial persistent shock, as of age 27, denoted σ_u^2 . Equation (7) models the transitory component as a first order autoregressive process with year-specific factor loadings on the innovation v_{ibt} . Furthermore, equation (8) allows the variance of v_{ibt} to be a quartic function of age; a quartic function, rather than for example a quadratic function, is used since Wald tests indicate that this specification is required.

The parameters of the model in equations (5)-(8) are estimated by applying the minimum distance estimator of Chamberlain (1984). Basically, the implied variances and auto-covariances of the model are fitted to the corresponding empirical moments in the data by non-linear least squares; the Appendix contains a description of the estimation procedure.

Last, deriving the expressions for the variances and auto-covariances of the model in equations (5)-(8) shows that cohort b 's transitory variance in year t depends on its transitory variance in year $t-1$, which in turn depends on the transitory variance in $t-2$, and so forth (see the Appendix for details). This raises the question of what the cohort's transitory variance is in its first sample year. As pointed out by MaCurdy (1982), a time series approach to this problem is problematic since the assumption of infinite history is untenable. I therefore follow the approach of Baker and Solon (2003) and treat the initial transitory variance for each cohort as an additional parameter to be estimated. This approach recognizes that earnings volatility varies across cohorts because they are at different stages of the life cycle and have lived through different times.

² A random walk specification is also used, among others, in Dickens (2000) and Moffitt and Gottschalk (2002).

³ The age intervals are chosen based on graphs of how the longer-lag auto-covariances, which mainly reflect persistent inequality, vary over the life cycle; these graphs are available on request.

3. Data

The analysis is based on Statistics Sweden's register-based longitudinal database LINDA (SCB, 1999). Each year contains a representative sample of 3.35 percent of the Swedish population; see Edin and Fredriksson (2000) for details. An attractive feature of the database is that attrition from the sample can be due only to death or migration.

The definition of an individual's earnings used in the analysis is total earnings from all jobs during a year. Information about earnings comes from tax reports, so the earnings variable is free of the measurement errors that are common in survey data such as recall errors and rounding errors.

The target group in the sample selection is males born in Sweden aged 26 to 56. As in previous studies, the focus upon males is due to their relatively steady labour market participation. Comparatively irregular female labour market participation, e.g. due to parental leave, would complicate the interpretation of results regarding transitory and persistent earnings.⁴ The chosen age range is to ensure that included individuals are old enough to have completed their education and too young to be considered for early retirement.

Table 1: Cohorts included in the revolving balanced panel

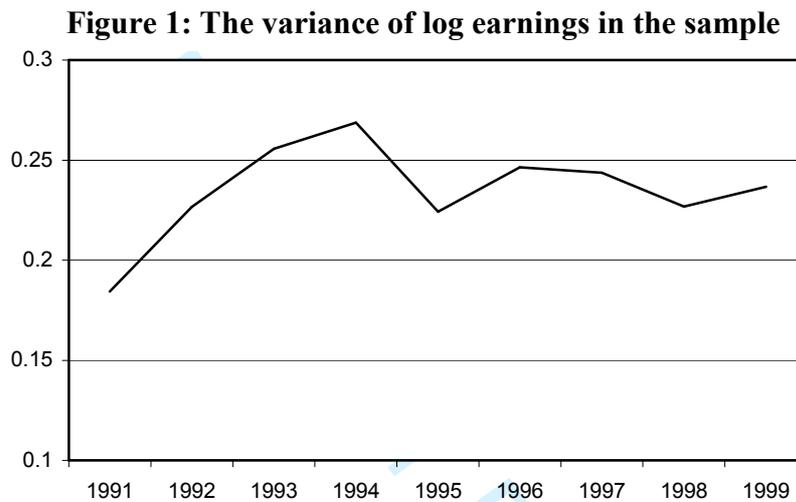
Birth year	Sample size	Years observed	Age in initial year
1939/40	1925	1991-95	52
1941/42	2173	1991-97	50
1943/44	2502	1991-99	48
1945/46	2617	1991-99	46
1947/48	2557	1991-99	44
1949/50	2425	1991-99	42
1951/52	2312	1991-99	40
1953/54	2268	1991-99	38
1955/56	2333	1991-99	36
1957/58	2161	1991-99	34
1959/60	2210	1991-99	32
1961/62	2139	1991-99	30
1963/64	2372	1991-99	28
1965/66	2604	1992-99	27
1967/68	2637	1994-99	27
1969/70	2601	1996-99	27

Note: Age is defined by the older of the birth cohorts in each two-year cohort.

In constructing the analysis sample the revolving balanced panel design recommended by Haider (2001) and Baker and Solon (2003) is applied. First, I identify the sixteen two-year birth cohorts who are between the ages 26 to 56 for at least four years between 1991 and

⁴ See for instance Haider (2001) and Baker and Solon (2003) for a further discussion on this subject.

1999. I then select all males who have positive earnings in each year that the age requirement is met and who did not received any form of pension. The end result is a balanced earnings panel for each cohort, with the panel length varying across cohorts. Table 1 describes the included cohorts. It is worth pointing out that the sample sizes *for each cohort* are similar to the total sample sizes used in the corresponding U.S. studies by Haider (2001) and Moffitt and Gottschalk (2002). The variance of log earnings for the pooled revolving balanced panel is depicted in Figure 1.



To estimate the parameters of the model in equations (5)-(8), I take each cohort separately and estimate the variances and auto-covariances of log annual earnings. Each auto-covariance matrix for the 11 cohorts observed for the whole 1991-99 period contains $45=(9 \times 10)/2$ distinct elements; the matrices for the other cohorts have fewer elements. Computing auto-covariance matrices for each cohort results in a total of 605 distinct variance and auto-covariance elements - these are the data to which the implied variances and auto-covariances of the model in equations (5)-(8) are fitted.

4. Results

Table 2 presents the parameter estimates and associated standard errors of the model in equations (5)-(8). Starting with the factor loadings for the persistent component, the value for 1991 is normalized to unity in order to obtain identification. The estimated factor loading for 1992 is significantly larger than unity and the estimates for the following years are all

significantly larger than that for 1992.⁵ The estimates thus indicate a rise in persistent earnings inequality in the beginning of the 1990s.

Moving next to the estimated year-specific factor loadings on the transitory innovation, the value for 1992 is normalized to unity because the innovation variance in 1991 must be left unrestricted to identify the initial variances of the cohorts, i.e. the transitory variance in a cohort's first sample year is estimated solely by their initial variance. Of the estimates, only those for 1995 and 1996 are significantly different (smaller) than unity. Hence, there is no rise in the dispersion of transitory shocks.

Table 2: Minimum distance estimates of the earnings dynamics model in equations (5)-(8)

Persistent component		Transitory component (cont.)	
<i>Factor loadings</i>		<i>AR-parameter</i>	
p_{91}	1.000	ρ	0.555 (0.011)
p_{92}	1.049 (0.013)	<i>Parameters in the variance of transitory shocks</i>	
p_{93}	1.128 (0.016)	γ_0	0.201 (0.017)
p_{94}	1.169 (0.019)	γ_1	-0.034 (0.006)
p_{95}	1.122 (0.019)	γ_2	0.003 (0.0008)
p_{96}	1.189 (0.021)	γ_3	-1.35E-4 (3.63E-5)
p_{97}	1.193 (0.021)	γ_4	2.11E-6 (5.86E-7)
p_{98}	1.153 (0.021)	<i>Initial cohort variances</i>	
p_{99}	1.151 (0.020)	$\sigma_{1939/40}^2$	0.069 (0.017)
<i>Variances of the random walk innovation</i>		$\sigma_{1941/42}^2$	0.071 (0.016)
σ_u^2	0.0469 (0.0035)	$\sigma_{1943/44}^2$	0.065 (0.014)
$\sigma_{r,28-37}^2$	0.0053 (0.0005)	$\sigma_{1945/46}^2$	0.038 (0.011)
$\sigma_{r,38-56}^2$	0.00089 (0.00032)	$\sigma_{1947/48}^2$	0.065 (0.014)
		$\sigma_{1949/50}^2$	0.031 (0.014)
		$\sigma_{1951/52}^2$	0.091 (0.016)
		$\sigma_{1953/54}^2$	0.071 (0.017)
Transitory component		$\sigma_{1955/56}^2$	0.077 (0.015)
<i>Factor loadings</i>		$\sigma_{1957/58}^2$	0.071 (0.015)
λ_{92}	1.000	$\sigma_{1959/60}^2$	0.101 (0.017)
λ_{93}	1.048 (0.038)	$\sigma_{1961/62}^2$	0.142 (0.019)
λ_{94}	0.937 (0.042)	$\sigma_{1963/64}^2$	0.180 (0.017)
λ_{95}	0.854 (0.039)	$\sigma_{1965/66}^2$	0.269 (0.018)
λ_{96}	0.803 (0.044)	$\sigma_{1967/68}^2$	0.325 (0.021)
λ_{97}	0.945 (0.041)	$\sigma_{1969/70}^2$	0.369 (0.023)
λ_{98}	0.943 (0.040)		
λ_{99}	1.030 (0.042)		

Note: Heteroskedasticity and auto-correlation robust standard errors are in parentheses.

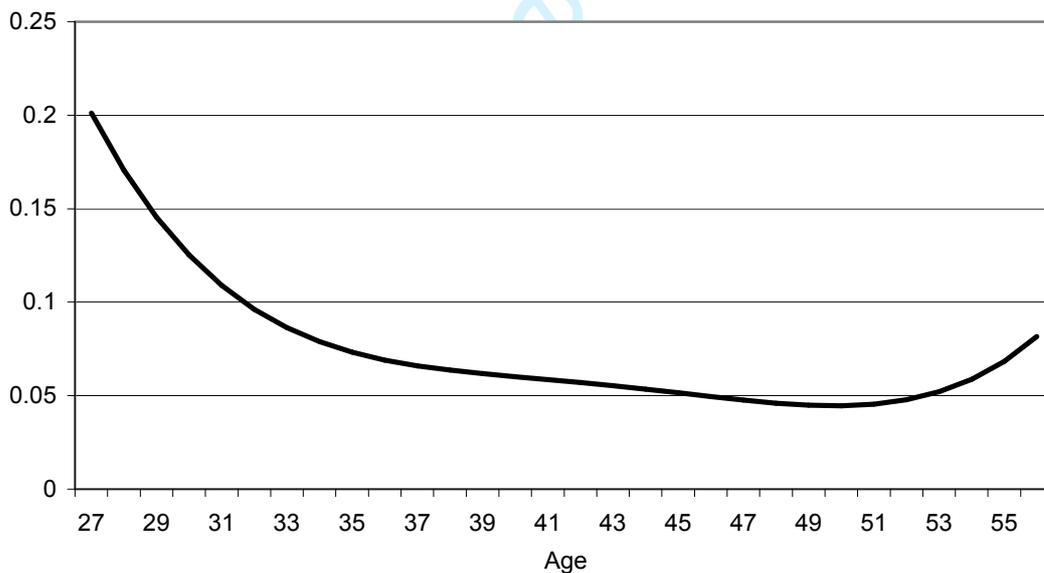
The other parameter estimates in Table 2 reveal some interesting information about individuals' earnings dynamics. For the persistent component, the variances of the innovation in the random walk are significantly larger than zero. As the variance of a variable that

⁵ Wald tests are used throughout the analysis.

follows a random walk is the sum of the variances of the innovation, this implies that persistent inequality increases over the whole studied age-range (with the time-varying factor loadings held constant). The estimate of $\sigma_{r,28-37}^2$ is significantly larger than the estimate of $\sigma_{r,38-56}^2$, indicating that persistent inequality increases faster at younger ages. The presence of larger persistent shocks at younger ages is consistent with matching models in which information about a worker's ability is revealed over time, as for instance in Jovanovic (1979).

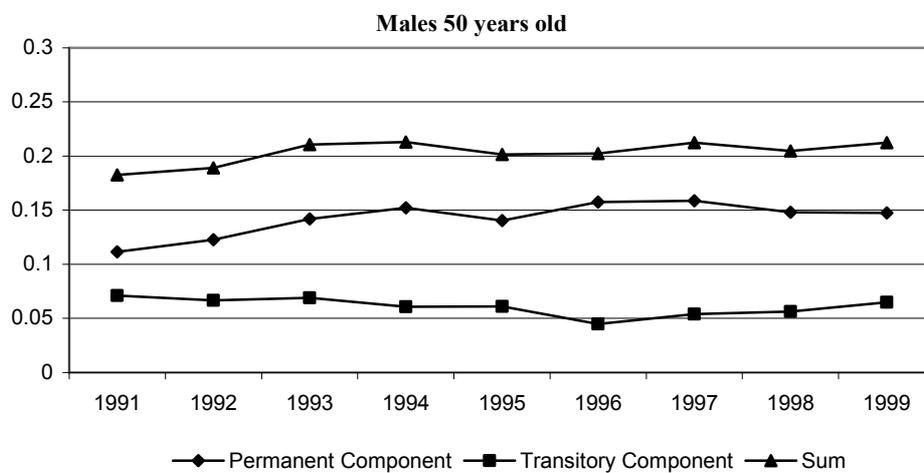
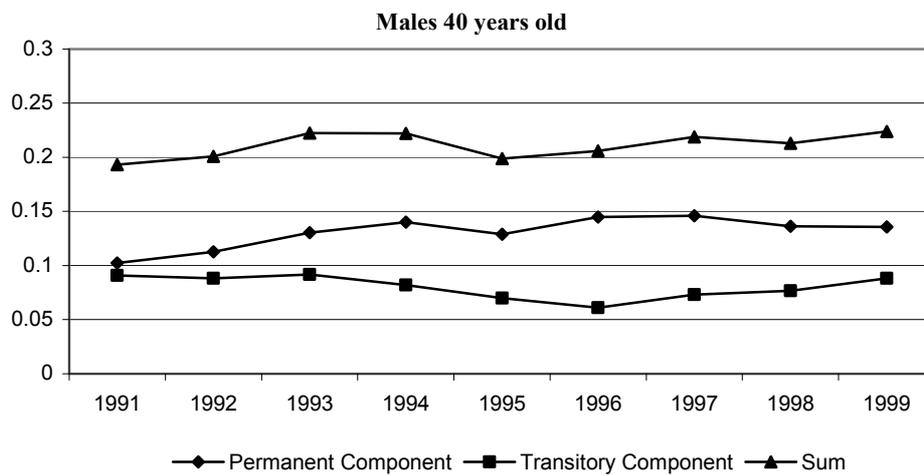
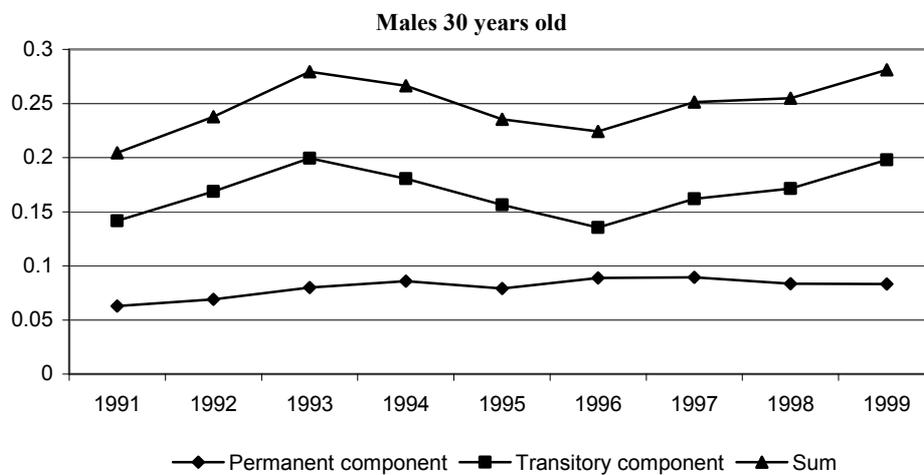
For the transitory component, the $\hat{\rho} = 0.555$ implies that more than 80 percent of a transitory shock has disappeared after 3 years. The estimated parameters in the variance of transitory shocks imply the U-shaped age profile displayed in Figure 2. As can be seen, the initial decline in the variance is quite sharp as it falls with over 60 percent to the mid 30's. Overall, the results of how transitory earnings fluctuations vary with age are similar to those obtained by Baker and Solon (2003) based on Canadian data.

Figure 2: Age profile of the variance of the transitory innovation



What do the estimates in Table 2 imply about changes in the persistent and transitory components of earnings inequality for different age groups? To investigate this I use the full model to predict the two components for each year, holding age constant. Figure 3 displays these predictions for males 30, 40, and 50 years old. In moving from year to year, not only do

Figure 3: Decompositions of the variance of log earnings



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3 factor loadings on the two components change, but also the cohort specific initial variances
4 that are used to generate the transitory variances up to these ages vary.⁶
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7 The persistent variances in Figure 3 display an upward trend during the 1990s. Another
8 interesting feature of Figure 3 is the different role played by the transitory variance in the
9 three age groups. Transitory earnings fluctuations dominate earnings dispersion among
10 individuals who are 30 years old, whereas it is much smaller among individuals aged 40 and
11 50. Figure 3 also shows that the evolution of the transitory variance differs between the age
12 groups. The transitory variance is roughly the same in 1991 and 1999 for 40- and 50-year olds
13 but higher in 1999 for 30-year olds.
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19 20 21 **5. Concluding remarks**

22 This paper decomposes the rise in the cross-sectional variance of male annual earnings in
23 Sweden between 1991 and 1999 into its persistent and transitory components. The results
24 show that the persistent component accounts for basically all of the increase in earnings
25 dispersion. This indicates that future research trying to explain the reversal in Swedish
26 inequality should focus on factors that can explain systematic and persistent changes in
27 earnings differentials, such as increases in the price of skills.
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30 A natural question, and where the answer may provide more clues to the Swedish
31 evolution, is how the changes in Sweden during the 1990s compare to changes in other
32 industrialized countries. This is however to a large extent a question for future research as
33 knowledge about the international evolution during the 1990s is still sparse, but the work of
34 Ramos (2003) permits comparison with the U.K. between 1991 and 1999. His results indicate
35 increased persistent earnings inequality in the U.K., whilst transitory inequality has been
36 constant. Thus, there appears to have been a similar evolution in Sweden and the U.K..
37 Results for two countries are of course not enough to be able to say that increased persistent
38 earnings inequality during the 1990s is an international trend. Studies that examine persistent
39 and transitory inequality for other countries during this period are therefore warranted.
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59 ⁶ In fact, the initial variances change every two years, corresponding to the cohort estimates in Table 2. For
60 example, the initial variance for cohort 1951/52 is a direct estimate of the variance of the transitory component
for individuals aged 40 in 1991. For 1993 I use the initial variance for cohort 1953/54, whose members are 40 in
this year.

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Appendix: The estimation procedure

The parameters of the model in equations (5)-(8) are estimated by applying the minimum distance estimator of Chamberlain (1984). Specifically, let \mathbf{C}_b contain the distinct elements of the population auto-covariance matrix of y_{ibt} for cohort b and let \mathbf{C} be an aggregate vector stacked with the \mathbf{C}_b vectors. Let the vector $\boldsymbol{\theta}$ contain all the parameters of the model and $\mathbf{C} = f(\boldsymbol{\theta})$ express the model's moment restrictions. The model in equations (5)-(8) then implies that the general variance element in \mathbf{C} is

$$(A1) \quad \text{Var}(y_{ibt}) = p_t^2 (\sigma_u^2 + \sum_a \sigma_{ra}^2) + \rho^2 \text{Var}(\varepsilon_{ib,t-1}) + \lambda_t^2 (\gamma_0 + \gamma_1 a + \gamma_2 a^2 + \gamma_3 a^3 + \gamma_4 a^4),$$

and that the general auto-covariance element for year t and $t-s$ is

$$(A2) \quad \text{Cov}(y_{ibt}, y_{ib,t-s}) = p_t p_{t-s} (\sigma_u^2 + \sum_{a-s} \sigma_{ra}^2) + \rho E[\varepsilon_{b,t-1} \varepsilon_{b,t-s}],$$

where $E[\cdot]$ is the expectation operator. The vector \mathbf{C} is estimated by the sample counterpart $\hat{\mathbf{C}}$, and $\hat{\boldsymbol{\theta}}$ is chosen to minimize a distance function

$$(A3) \quad \mathbf{D} = (\hat{\mathbf{C}} - f(\hat{\boldsymbol{\theta}}))' \mathbf{W} (\hat{\mathbf{C}} - f(\hat{\boldsymbol{\theta}})),$$

where \mathbf{W} is a positive definite weighting matrix.

The asymptotically optimal choice of \mathbf{W} is the inverse of a matrix that consistently estimates the covariance matrix of \mathbf{C} . However, Altonji and Segall (1996) and Clark (1996) provide Monte Carlo evidence of potentially serious finite sample bias in the estimate of $\boldsymbol{\theta}$ using this approach. I therefore follow the practice of the most recent literature and use the identity matrix as the weighting matrix. This "equally weighted minimum distance estimation" amounts to using non-linear least squares to fit $f(\hat{\boldsymbol{\theta}})$ to $\hat{\mathbf{C}}$.

As outlined in Chamberlain (1984), robust standard errors for $\hat{\boldsymbol{\theta}}$ are obtained from the formula

$$(A4) \quad (\mathbf{G}'\mathbf{G})^{-1} \mathbf{G}' \mathbf{V} \mathbf{G} (\mathbf{G}'\mathbf{G})^{-1},$$

where \mathbf{G} is the gradient matrix $\partial f(\boldsymbol{\theta})/\partial \boldsymbol{\theta}$ evaluated at $\hat{\boldsymbol{\theta}}$ and \mathbf{V} is a block diagonal matrix containing the estimated covariance matrices of each $\hat{\mathbf{C}}_b$ vectors.