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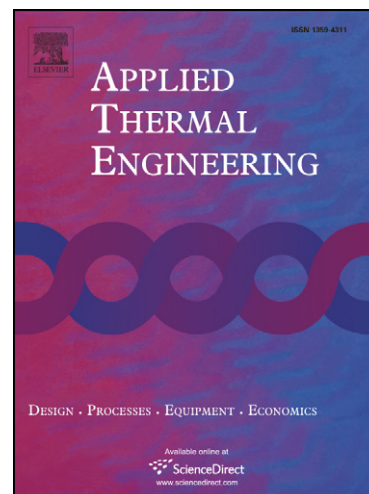
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Optimisation of tree path pipe network with nonlinear optimisation method

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Abstract

In this article the optimisation of pipe network with hot water is presented. The mathematical model, consisting of the non-linear objective function and system of non-linear equations for the hydraulics limitations is developed. On its basis the computer program for determination optimal tree path with the use of simplex method was solved. For economic estimation the capitalized value method, which consider all costs of investment and operation was used. The results are presented for real case study network with 24 nodes and 33 pipe sectors.

Keywords: District heating; Pipe network; Optimisation; Non-linear programming; Simplex method

1. Introduction

Nowadays efforts connected to energy savings demand the search for new technical scientific expertise in the field of heating techniques [1 – 4]. The focus of research is on better and more efficient use of primary energy [5].

If we limit ourselves to district heating energy systems, we may state that these systems ensure savings in the process of use of primary energy and are acceptable from the ecological point of view.

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Nomenclature

A, B, C, D, E, F	constants	L	pipe length
AC	annuity costs	L_0	salvage value
B_0	investment costs	n	device lifetime
CC	capitalized costs	N_v	number of nodes
C_0	operating and maintenance costs	N_c	number of pipes
C_1	pipe network expenditure	p	pressure
C_2	pump investment expenditure	P	power
C_3	pumping costs	Re	Reynolds number
C_4	construction expenditure	t	operating time of pipe network
C_e	electric price	v	velocity
$C(q_v)$	objective function	q_v	flow volume
C_p	pump price		
d	pipe diameter	<i>Greek symbols</i>	
i	interest rate	ρ	density
k	pipe roughness	λ	friction factor
K	coefficient of Darcy – Weisbach equation	ζ	coefficient of local losses
		η	pump efficiency

The whole system is represented as a non-linear target function by non-linear equations of hydraulic limitations and by minimizing the non-linear function the optimal design and dimensions of the pipe network are defined [6,7].

One method for modelling pipe networks is by linear programming, consisting of optimisation with limitations [8], meaning the search for the best possible, optimal solution of the stated problem within given conditions.

Nowadays two methods for solving linear programmes are used, both of them interactive, searching for a gradually better solution, until the optimum value is achieved. Mostly, simplex methods focused on searching for permissible solutions within the monotonous defined extreme point of the convex polyhedron of possible solutions and a defined base of the vector space, are used.

2. District heating systems

District heating systems are intended for the distribution of heat energy by a fluid from a heating source to different users [9, 10, 11]. The pressure losses of the system are defined by the non-linear Darcy-Weisbach equation [12]:

$$p_i - p_j = 0.81 \cdot \frac{\rho \cdot q_{vij}^2}{d^4} \cdot \left(\frac{\lambda \cdot L}{d} + \sum \zeta \right) = K_{ij} \cdot q_{vij}^2 \quad (1)$$

The Darcy friction coefficient (λ) is a function of the Reynolds number. In the range of the laminar flow the pressure losses depend only on the fluid viscosity, but in the range of turbulent flow the decisive factor is the relative pipe roughness coefficient, which depends upon age, corrosion and pipe material.

The relative pipe roughness coefficient (k/d) is a relation between absolute roughness of the inner pipe wall and the inner pipe diameter.

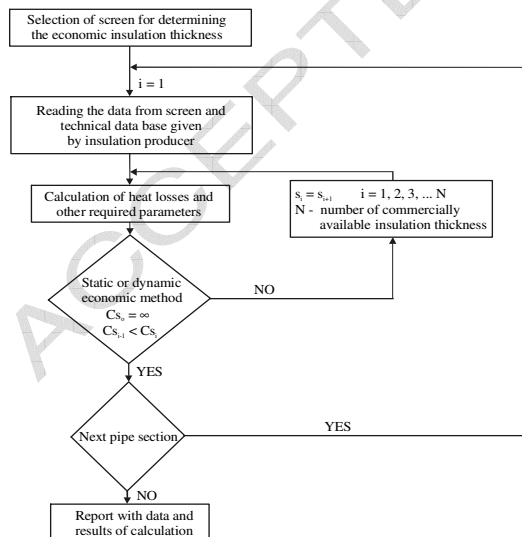


Figure 1: Simplified flow diagram for optimal thermal insulation thickness (c_s – insulation costs)

The heat losses through the pipes are negligible. Optimal thickness of insulation was calculated with the help of developed computer program [13]. The program flow diagram is presented on figure 1, and the difference between the static and dynamic economic methods on figure 2.

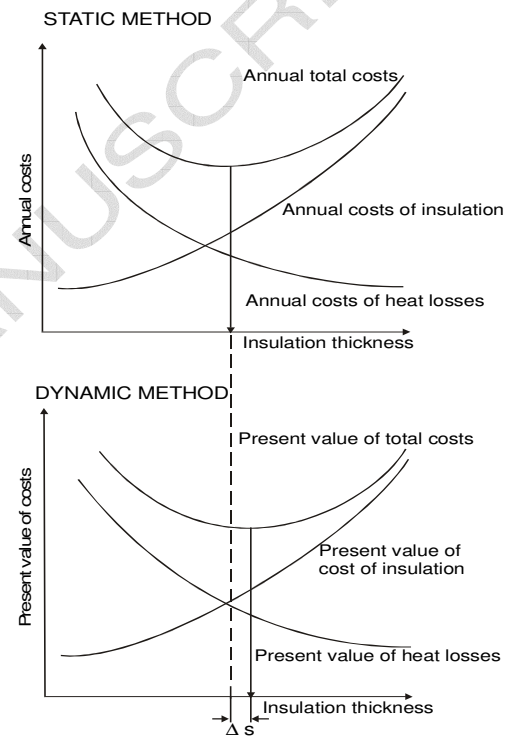


Figure 2: The difference between the static and dynamic economic methods by determining the economic thermal insulation thickness

3. The linear programming method

At the beginning the optimisation problem has to be designed. The common configuration of the mathematical model of the linear optimisation problem with m limitations and n variables is [14, 15]:

objective function $\min(c_1 \cdot x_1 + c_2 \cdot x_2 + \dots + c_n \cdot x_n)$

limitation

$$\begin{aligned} a_{11} \cdot x_1 + a_{12} \cdot x_2 + \dots + a_{1n} \cdot x_n & (\leq, \geq, =) b_1 \\ a_{21} \cdot x_1 + a_{22} \cdot x_2 + \dots + a_{2n} \cdot x_n & (\leq, \geq, =) b_2 \\ & \vdots \\ a_{m1} \cdot x_1 + a_{m2} \cdot x_2 + \dots + a_{mn} \cdot x_n & (\leq, \geq, =) b_m \\ x_1, x_2, \dots, x_n & \geq 0 \end{aligned} \quad (2)$$

The mathematical model (2) is soluble with a simplex algorithm [16].

The conditions in the form of non-equations are transferred into equations by introducing additional variables.

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + \dots + a_{1n} \cdot x_n \geq b_1 \quad (3)$$

The possible surplus is subtracted and the non-equation becomes an equation:

$$a_{11} \cdot x_1 + \dots + a_{1n} \cdot x_n - x_{n+1} = b_1 \quad (4)$$

The additional variable (x_{n+1}) was added and we now have to consider this with conditions, but it has no influence on the objective function due to the fact that its coefficient (c_{n+1}) is zero.

This modified linear programme is complemented with artificial variables (x_{n+m+1}) to find the primary interior point where the simplex calculations are to begin.

$$a_{11} \cdot x_1 + \dots + a_{1n} \cdot x_n - x_{n+1} + x_{n+m+1} = b_1 \quad (5)$$

We complement the objective function with a new variable and ascribe to it the coefficient (c_{n+m+1}):

$$\begin{aligned} c = c_1 \cdot x_1 + \dots + c_n \cdot x_n + c_{n+1} \cdot x_{n+1} + \\ + \dots + c_{n+m} \cdot x_{n+m} + c_{n+m+1} \cdot x_{n+m+1} \end{aligned} \quad (6)$$

The artificial variables serve for determination of the base allowed solution. In the last step the objective function is transferred from the minimum type to the maximum type:

$$-c = \sum_{j=1}^n -c_j \cdot x_j \quad (7)$$

Due to the fact that the first result is not necessarily optimal, we execute the optimality test. If the solution is not optimal a new base is required.

4. Optimisation of pipe network

For an effective and accurate description we handle the pipe network as a linear, directed graph, where the fluid entrance points are named as supplied nodes, fluid exit points are user nodes and the nodes where no external inflow or outflow is present as fictive nodes [17, 18].

If we define the number of pipes as N_c and N_v as the number of nodes for the tree pipe network then the number of nodes is always less than the number of pipes for the structure with loops.

The fluid flow in the net has the direction from the node with higher pressure to the node with lower pressure and is from i to j positive and in the other direction negative. With that we have to consider the first Kirchoff's law which says that the sum of all flows into a node equals the sum of all outflows.

The optimal path with minimal transport costs can be defined by two methods of linear programming, the transport method and the simplex method.

Garbai and Krope present a graph-theoretical method to set up a network with optimum operational costs. Their method is based on the Bellmann optimum principle and the application of discrete dynamic programming [19].

The mathematical model for defining the optimal tree path consists of the objective function of capitalised costs $C(q_v)$ that is minimised.

Capitalised costs of each pipe CC_j are the sums of discounted values of investment in pipes, pumps, building expenses and operational costs. As the capitalised costs depend on the pipe diameter, which is discrete variable, and the diameters depend on the flow, the object function is non-linear.

$$\min C(q_v) = \sum_{j=1}^{N_c} CC_j \cdot q_{vj} \quad (8)$$

Further elements of the mathematical model are non-linear equations of hydraulic limitations that represent the continuity of the pipe network and limitations of the simplex method representing the required non-negativity of the simplex variables.

$$\begin{aligned} q_{v1} - \sum_{i=2}^{N_v} q_{vi} &= 0 \\ \sum_{j=1}^{N_c} \pm q_{vij} \pm q_{vi} &= 0 \quad (i = 1, 2, 3, \dots, N_v) \\ q_{vij} &\geq 0 \quad (i = 1, 2, 3, \dots, N_v \quad j = 1, 2, 3, \dots, N_c) \end{aligned} \quad (9)$$

Figure 3 shows the nonlinear relationship between capitalised costs and volume flow.

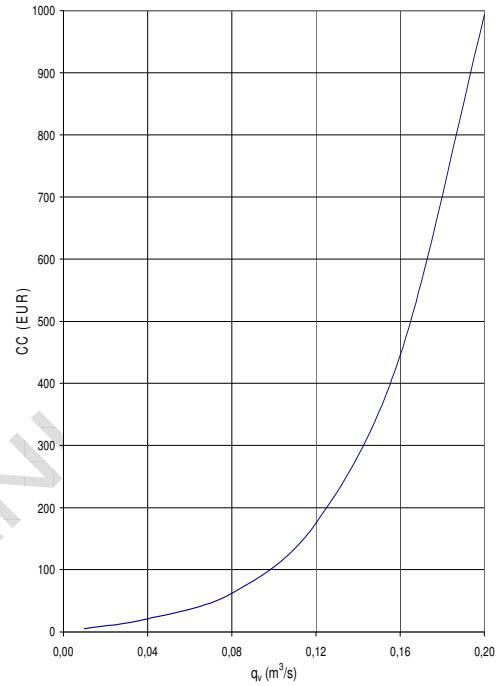


Figure 3: Relationship between CC and q_v

5. The financial analyse of district heating system

As the lifetime of warm water pipe networks is limited, let us suppose that after that lifetime the network is substituted by a new one. Pumps needed to transport fluids through the pipe network have an essential shorter lifetime than the pipe itself. If we are going to evaluate the project with the future or present value method the overall lifetime of all equipment parts has to be known. The capitalised costs method is used; including all costs of investment and operation to infinity [20]. The capitalised costs are the

present value of investment costs repeating to infinity and are presented as formula:

$$CC = B_0 + \frac{B_0 - L_0}{(1+i)^n - 1} + \frac{C_0}{i} \quad (10)$$

And the equation (10) became equation (11):

$$CC = L_0 + \frac{B_0 - L_0}{1 - (1+i)^{-n}} + \frac{C_0}{i} \quad (11)$$

Regarding capitalised costs the annuities are defined, based on a discount rate and composed of the sum of all costs and revenues transposed to equal yearly shares. Within defining the annual costs they start from the start of the pipe network operation and end with the closure of the pipe network operation and are presented at the end of each year in equal portions. Between the annuity costs and capitalised costs the following dependence is valid:

$$AC = CC \cdot i \quad (12)$$

Defining the fluid transport costs over the pipe network we have to consider that the initial investment costs are very high. In the first step we define the costs of insulated pipes that are a function of the diameter with the following equation:

$$C_1 = A + B \cdot d + C \cdot d^2 \quad (13)$$

The conversion on standard diameters is done with the use of economic method and is presented in [21].

The investment value for pumps depends on the utilisation rate and the pump price calculated per watt of power, the flow volume and the pressure drop that arises with the increase in pump capacity.

$$C_2 = C_p \cdot P = C_p \cdot \frac{q_v \cdot \Delta p}{\eta} \quad (14)$$

The pumping costs depend on the electrical energy price, the pump power and the operational lifetime of the pipeline.

$$C_3 = C_e \cdot P \cdot t \quad (15)$$

The construction costs depend on pipe diameter in defined pipe sectors and on the construction site environment of the pipe network.

$$C_4 = D + E \cdot d + F \cdot d^2 \quad (16)$$

$$\Sigma C = C_1 + C_2 + C_3 + C_4 \quad (17)$$

Figure 4 shows the relationship between the pipe costs, pump and pumping costs and costs for constructions.

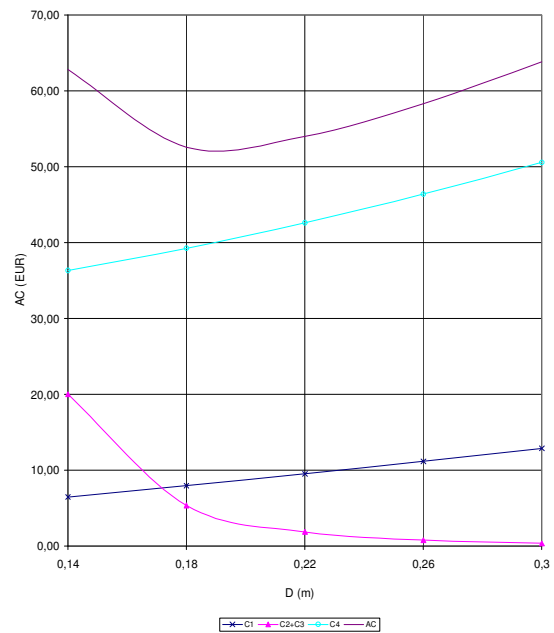


Figure 4: Relationship between all annuity costs vs. pipe diameter for the $q_v = 0.04 \text{ m}^3/\text{s}$ and $L = 1 \text{ m}$

Figure 5 presents the algorithm for determining the optimal branch tree path and pipe diameters with the use of developed computer program.

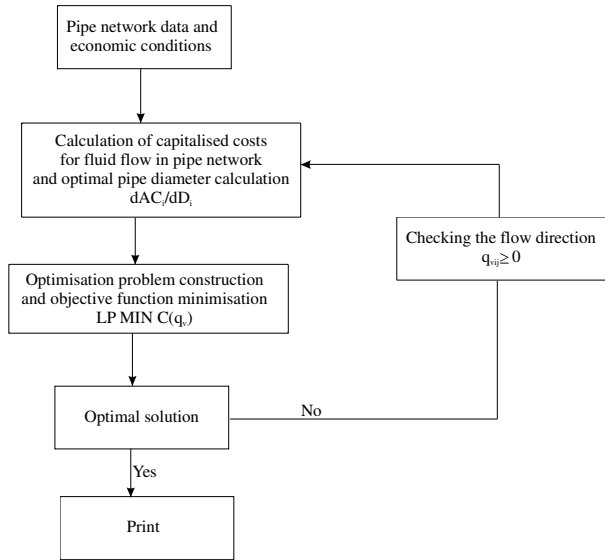


Figure 5: Algorithm for determining the optimal branch tree path and pipe diameters

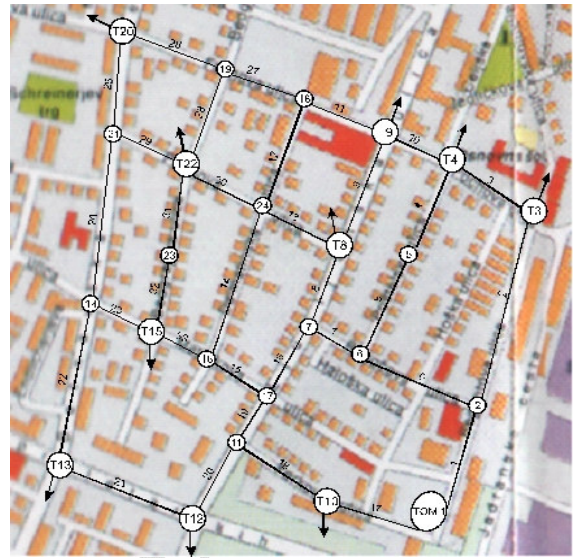


Figure 6: Loop pipe network of all possible routes of the district heating system

Table 1: Pipe data

Pipe	L [m]	ζ
1	210	10
2	400	5
3	150	5
4	200	15
5	200	5
6	220	5
7	100	5
8	160	5
9	210	15
10	90	5
11	140	5
12	210	5
13	150	15
14	320	5
15	110	10
16	130	10
17	160	5
18	180	15
19	90	5
20	140	15
21	230	5
22	310	5
23	90	10
24	340	5
25	180	5
26	180	10
27	150	5
28	170	15
29	120	10
30	160	5
31	150	15
32	120	10
33	80	5

6. The Optimisation of the District Heating System

The simplex method and the capitalised costs method were used for the optimisation of the district heating system (Figure 6).

The network consists of 33 pipeline sectors and 24 nodes. Hot water flowing at high pressure enters the system in the node TOM1 and exits in T3, T4, T8, T9, T10, T12, T13, T15, T20 and T22, other nodes on figure 3 are virtual, and represent the branching of pipes.

Data needed for the optimisation of the pipe network are presented in tables 1, 2 and 3.

Table 2: Input and output data in nodes

Node	$q_v \times 10^{-3} [\text{m}^3/\text{s}]$
1	47.50
2	0
3	-2.20
4	-7.60
5	0
6	0
7	0
8	-9.8
9	-7.3
10	-3.4
11	0
12	-4.8
13	-5.2
14	0
15	-2.5
16	0
17	0
18	0
19	0
20	-3.0
21	0
22	-1.7
23	0
24	0

Table 3: Physical and economical data

Input pressure	10^6 Pa
Input fluid temperature	110°C
Output fluid temperature	70°C
Density	934.8 kg/m^3
Kinematics viscosity	$0.226 \times 10^{-6} \text{ m}^2/\text{s}$
Operating time	8760 h/a
Interest rate	0.1
Price of electrical energy	$7.1 \times 10^{-5} \text{ EUR/Wh}$
Pump price	0.15 EUR/W
Pumps lifetime	10 year
Pumps efficiency	0.75
Pipe network lifetime	40 year
Pipe roughness	0.4 mm
Coefficient of pipe costs polynomial:	
A	18 EUR
B	291 EUR/m
C	229 EUR/m ²
Coefficient of construction costs polynomial:	
D	287 EUR
E	310 EUR/m
F	1275 EUR/m ²

7. Results and discussion

By using the developed computer programme, which enables optimization of the pipe network using the simplex method two optimal branch trees was defined.

The first optimal branch tree, Figure 7 and tables 4, 5, 6, was received by Version 1. This is for the case when at passage on standard diameters we consider only pipe costs while in objective function we consider all costs (C_1 , C_2 , C_3 and C_4).

Table 4: Output pressure in nodes

Node	p [Pa]	Node	p [Pa]
1	1000000	13	929678.
2	989448.	14	911975.
3	899868.	15	921361.
4	932760.	16	934913.
5	947095.	17	954457.
6	972226.	18	923717.
7	967123.	19	901952.
8	959943.	20	875753.
9	944151.	21	851575.
10	988733.	22	858976.
11	969521.	23	891215.
12	956542.	24	921451

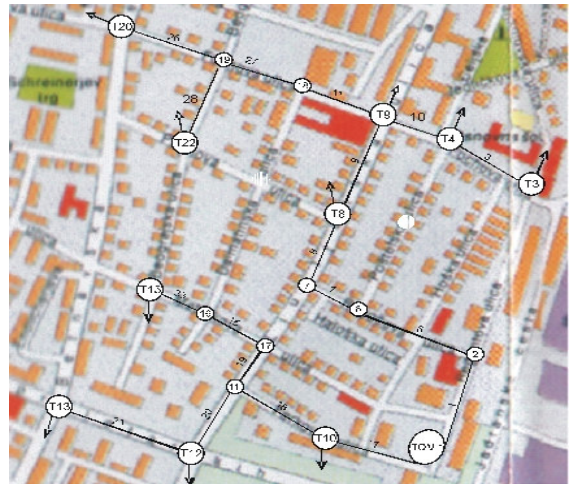


Figure 7: Optimised branch tree path of the district heating system

The subjects of table 5 are the pipe diameters, volume flows, flow velocities and pressure fall on particular pipe segments. The costs of pipes, pumps, pumping, construction work and common costs of first optimal branch tree path are defined by the cost capitalisation method and are presented for each pipe in table 6.

Table 5: Optimisation results by the first version

Pipe	D [m]	q _v [m ³ /s]	v [m/s]	Δp [Pa]
1	0.2191	0.03160	0.838	10552.
3	0.0570	0.00220	0.862	32892
6	0.1937	0.03160	1.072	17222
7	0.2191	0.03160	0.838	5103
8	0.2191	0.03160	0.838	7180
9	0.1778	0.02180	0.878	15792
10	0.11143	0.00980	0.955	11391
11	0.0825	0.00470	0.879	20434
15	0.0635	0.00250	0.789	19544
17	0.1524	0.01590	0.872	11267
18	0.1330	0.01250	0.900	19212
19	0.0635	0.00250	0.789	15064
20	0.1270	0.01000	0.789	12979
21	0.0889	0.00520	0.838	26864
26	0.0700.	0.00300	0.780	26199
27	0.0825	0.00470	0.879	21764
28	0.0510	0.00170	0.832	42976
33	0.0635	0.00250	0.789	13552

Table 6: Capitalized costs

Pi.	CC1 [EUR]	CC2 [EUR]	CC3 [EUR]	CC4 [EUR]	ΣCC [EUR]
1	19920	110	2760	89360	112150
3	5420	20	600	47370	53410
6	18660	180	4500	88840	112180
7	9480	50	1330	42550	53410
8	15180	70	1880	68080	85210
9	16530	110	2850	82120	101610
10	4990	40	920	31210	37160
11	6240	30	790	45990	53050
15	4210	20	400	35080	39710
17	11070	60	1480	59530	72140
18	11180	80	1990	64570	77820
19	3440	10	310	28700	32460
20	8400	40	1070	49670	59180
21	10740	50	1160	76350	88300
26	7270	30	650	57970	65920
27	6680	30	850	49280	56840
28	5810	20	600	53220	59650
33	3060	10	280	25510	28860
Σ	168280	960	24420	995400	1189060

The second optimal branch tree, Figure 8 and tables 7, 8, 9 was received by Version 2. This is for the case when at passage on standard diameters we consider pipe costs while in objective function we consider only C₁, C₂, C₃.

Table 7: Output pressure in nodes by second version

Node	p [Pa]	Node	p [Pa]
1	1000000	13	929678
2	989448.	14	911975
3	900614	15	921361
4	933506	16	934913
5	949518	17	954457
6	972226	18	914753
7	965478	19	893868
8	952674	20	967669
9	934341	21	869284
10	988733	22	876685
11	969521	23	891215
12	956542	24	914182

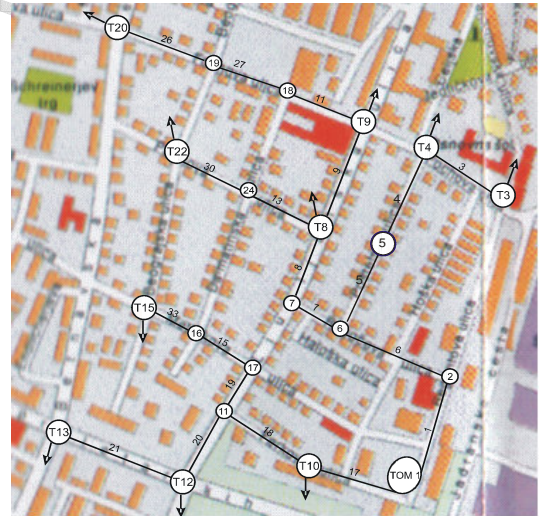


Figure 8: Optimised branch tree path of the district heating system by second version

Table 8: Optimisation results by the second version

Pipe	D [m]	q_v [m ³ /s]	v [m/s]	Δp [Pa]
1	0.2191	0.03160	0.838	10552
3	0.0570	0.00220	0.862	32892
4	0.1270	0.00980	0.774	16012
5	0.1143	0.00980	0.955	22708
6	0.1937	0.03160	1.072	17222
7	0.1778	0.02180	0.878	6748
8	0.1683	0.02180	0.9801	12804
9	0.1270	0.01030	0.813	18333
11	0.0700	0.00300	0.780	19588
13	0.0510	0.00170	0.832	38492
15	0.0635	0.00250	0.789	19544
17	0.1524	0.01590	0.872	11267
18	0.1330	0.01250	0.900	19212
19	0.0635	0.00250	0.789	15064
20	0.1270	0.0100	0.789	1279
21	0.0889	0.00520	0.838	26864
26	0.0700	0.00300	0.780	26199
27	0.0700	0.00300	0.780	20886
30	0.0510	0.00170	0.832	37497
33	0.0635	0.00250	0.789	13552

Table 9: Capitalized costs by the second version

Pi.	CC1 [EUR]	CC2 [EUR]	CC3 [EUR]	CC4 [EUR]	Σ CC [EUR]
1	19920	110	2760	0	22790
3	5420	20	60	0	5500
4	12000	50	1300	0	13350
5	11100	70	1840	0	13010
6	18660	180	4500	0	23340
7	7870	50	1220	0	9140
8	12020	90	2310	0	14420
9	12590	60	1560	0	14210
11	5650	20	490	0	6160
13	5130	20	540	0	5690
15	4210	20	400	0	4630
17	11070	60	480	0	11610
18	11180	80	990	0	12250
19	3440	10	310	0	3760
20	8400	40	70	0	8510
21	10740	50	160	0	10950
26	7270	30	650	0	7950
27	6060	20	520	0	6600
30	5470	20	530	0	6020
33	3060	10	280	0	3350
Σ	181260	1010	20970	0	203240

Of some interest are the influences of price changes of construction work, pumps and electrical energy regarding the course and characteristics of the branch tree pipe network.

With equal construction and pump costs, if the costs of electric energy increase by 50% we get by the first version the third path (Figure 9) and by the second version second path.

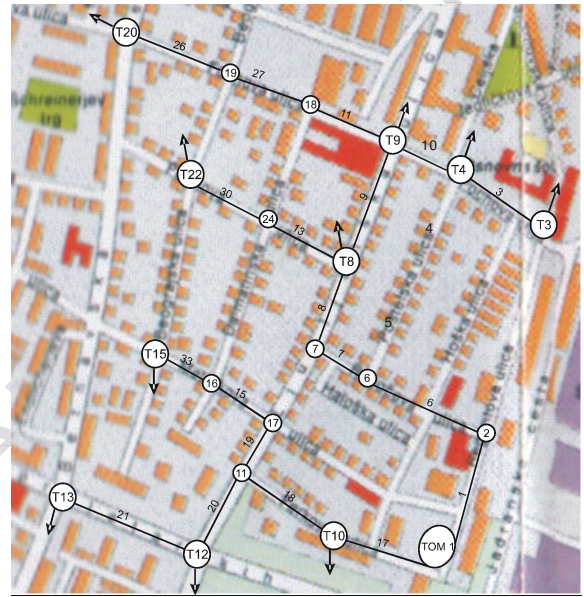


Figure 9: The third optimised branch tree path

Analysing the situation, when the prices for electrical energy and construction work remain the same but the pump price changes, we conclude that by both version the paths are unchanged (version 1 – 1st path, version 2 – 2nd).

8. Conclusion

Determination by the optimal tree branch path was executed using the simplex method. The basic factors for determination of the path are economy and functionality of the tree pipe network. Because the solution of nonlinear systems requires the use of computers, a computer

programme was developed, which considers all the limitations and requests of the given case.

The programme enables, after the determination of the optimal branch tree path and capitalised costs, also variations of different influential parameters and the evaluation of their influence on the district heating system as a whole. The optimal path, defined by the programme is not necessarily the shortest path, due to the fact that the branch tree pipe network is a function of the minimising both the investment and operational costs.

Within defining the optimal branch tree path the results show the importance of which costs do we consider into objective function and which costs do we use at passage on standard pipe diameters, also the construction and operating costs of the system have important role. The path itself varies with the changing of construction costs, pump and electric energy costs. With their increase the investment, operational and maintenance costs also go up.

Therefore using the simplex method for each sole case the solution, representing the most reasonable fluid transport path from entering nodes to exiting nodes, was selected.

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Figure captions

Figure 1: Relationship between CC and q_y

Figure 2: Relationship between all annuity costs vs. pipe diameter

Figure 3: Loop pipe network of all possible routes of the district heating system

Figure 4: Optimised branch tree path of the district heating system

Figure 5: Optimised branch tree path of the district heating system by second version

Figure 6: The third optimised branch tree path

Table captions

Table 1: Pipe data

Table 2: Input and output data nodes

Table 3: Physical and economical data

Table 4: Output pressure in nodes

Table 5: Optimisation results by the first version

Table 6: Capitalized costs

Table 7: Output pressure in nodes by second version

Table 8: Optimisation results by the second version

Table 9: Capitalized costs by the second version