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# THE EFFECTS OF WATER TEMPERATURE IN AQUACULTURE MANAGEMENT

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## THE EFFECTS OF WATER TEMPERATURE IN AQUACULTURE MANAGEMENT

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18 **THE EFFECTS OF WATER TEMPERATURE IN**  
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## THE EFFECTS OF WATER TEMPERATURE IN AQUACULTURE MANAGEMENT

### *Abstract*

This paper studies the impact of water temperature on the optimal management of the ration size and fish weight in off-shore farm aquaculture. A model for the expected returns of the farm is developed which includes a fish growth function influenced by fish weight, the ration size and water temperature. The output transportation cost has an ambiguous effect on the harvesting size, but the impact of water temperature is positive. These results explain empirical evidence in the Canary Islands that unfavorable economic conditions could be overcome by environmental advantageous conditions raising productivity.

*Keywords:* Expected returns, Farm aquaculture, Water temperature, Canary Islands, Seabream.

*JEL Classification:* C61, L11, Q12.

## 1. Introduction

The cultivation of marine species is expected to be an increasingly fast growing industry in the next decades as techniques for off-shore production develop. New species are being introduced and there is need for appropriate models that are useful for evaluating management decisions. In the last decades, several models have been developed for the optimal management of the most relevant control variables in the farm.

The early model by Bjorndal (1988) analyzed how the optimal harvesting time schedule is influenced by the price of fish and the most important costs involved in production, i.e. the feed and harvesting costs. Arnason (1992) incorporated the feeding schedule in the optimization problem, and considered various functional forms for the fish growth function. Heaps (1993, 1995) included the possibility of previous culling as dependent on density, while Mistiaen and Strand (1999) assumed a step-wise influence of fish's size on price. The general conclusion of these results is that an increase in the interest rate would result in shorter optimal harvesting time and weight. These models are based on fish growth functions which abstract from the influence of water temperature on fish growth.

In this paper we propose a model for aquaculture management that incorporates a fish growth function influenced by water temperature. This model is utilized to derive the optimal decisions on the feeding rate and the harvesting time, which have relevant economic implications in terms of expected returns for the farm.

In general, previous economic models in aquaculture have made basic assumptions about water temperature. The only work which has explicitly considered water temperature in the growth function is Cacho, Kinnucan and Hatch (1991). This model

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4 introduces water temperature in an optimization framework for catfish farming, but the  
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6 analysis is limited to the feeding rate and the costs of feeding.  
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10 Water temperature is a major determinant of fish growth and can not be controlled in  
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12 off-shore farming. Rather, it is determined by environmental and geographical  
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14 conditions. In addition, the variability of water temperature adds a risk component to  
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16 farming decisions, which makes theoretical predictions of standard models to depart  
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18 from empirical findings (Pascoe, Wattage and Naik, 2002).  
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23 Our results indicate that the input costs have a negative impact on the optimal  
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25 harvesting size if a particular relation between the economic and biological factors is  
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27 given. However, this is not the case for the output transportation costs. Since output  
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29 transportation costs are incurred at the end of the culture cycle they have an ambiguous  
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31 impact on the optimal harvesting size. That is, a high output transportation cost could be  
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33 overcome by higher revenues to be obtained with larger harvesting sizes.  
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38 The next sections present a model for a fish farm, with the fish weight as the state  
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40 variable and the ration size as control, and with harvesting time and harvesting weight  
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42 optimally determined. An empirical case study is presented for two scenarios of  
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44 seabream culture in off-shore cages. The first represents a farm in Mediterranean  
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46 waters, close to population demand centers in Europe, and the second is placed in the  
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48 Canary Islands, an archipelago in the Atlantic Ocean located 2.000 kms. from the  
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50 European coast but with favorable water temperature for seabream cultivation. The  
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52 simulation results indicate that product differentiation by the optimal size could be an  
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54 efficient strategy to overcome the economic disadvantages of the more distant farms.  
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## 2. The Model

Let us assume the producer has the objective to maximize profits, defined by the difference between revenues and costs. In aquaculture management, the production process is based on the growth process of the organisms that determines the amount of biomass generated in a period of time. Let us consider a growth model based on fish physiology proposed by Brett (1979) and applied by Muller-Feuga (1990), which has been shown to perform appropriately to represent the growth process of a large number of fish species. The model equations and their specifications are presented in Table 1. The growth function is

$$\dot{w}(t) = G(w(t), r(t), \theta(t)), \quad (1)$$

where  $w$  is the fish's weight,  $r$  is the ration size given to the fish,  $\theta$  is water temperature and  $t$  stands for time. The ration size  $r(t)$  is defined in normalized terms, i.e., in relation to the maximum ration which would lead to satiation.  $r$  is equal to 0 when there is no feeding and approaches 1 at satiation level.

As in Cacho, Kinnucan and Hatch (1991), water temperature is modeled by a cyclical function depending on the time of the year,

$$\theta(t) = \bar{\theta} + A \sin\left(\frac{2\pi(t + t_\theta)}{365}\right), \quad (2)$$

where  $\bar{\theta}$  is the average value of temperature during a year,  $A$  is the maximum level of temperature, and  $t_\theta = t_0 - t_{\bar{\theta}}$ , where  $t_0$  is the single day of the year when fingerlings are stocked and  $t_{\bar{\theta}}$  is the day of average temperature ( $1 \leq t_0, t_{\bar{\theta}} \leq 365$ ).

The influence of these factors on fish growth can assume a multiplicative form, i.e.

$$G(w(t), r(t), \theta(t)) = \Gamma G_1(w(t)) G_2(r(t)) G_3(\theta(t)), \quad (3)$$

where functions  $G_i(\cdot)$ ,  $i=1,2,3$ , represent the effect of each variable on fish weight and  $\Gamma$  is a correction parameter.

The firm grows a given number of species  $N_0=N(t=0)$  with a given initial weight  $w_0$  until they reach a commercial size at  $t=T$ . However, not all fingerlings reach maturity because of some mortality. Thus,  $N(t)=N_0e^{-Mt}$ , where  $M$  is the mortality rate.

On the other hand, the price of the product  $p$  is assumed to be dependent on the size of the fish harvested, i.e.  $p(w(t))$ . Without lack of generality,  $p(\cdot)$  is assumed increasing with weight ( $p_w > 0$ ) because larger sizes have larger market prices<sup>1</sup>.

The total cost function includes the costs of feed, labor, transportation, fingerlings and other costs. Because the efficiency of ration varies across different sizes of fish and is affected by water temperature (Brett, 1979), the costs of feed ( $C_f(\cdot)$ ) depend on the conversion ratio, i.e. the quantity of food which is necessary for the fish weight to grow by 1 gr. Therefore we assume the following expression:

$$C_f(N, w, \theta, r) = c_f N(t) f(w(t), r(t), \theta(t)) \dot{w}, \quad (4)$$

where  $c_f$  is the *per gram* price of feed and  $f(w, r, \theta)$  is the conversion rate for given levels of fish weight  $w(t)$ , ration size  $r(t)$  and temperature  $\theta(t)$ . The amount of feed in a day is given by multiplying the growth rate by the number of surviving individuals in the cages ( $N(t)$ ).

The labor cost function ( $C_L$ ) depends on the number of workers  $L$  and the daily cost of labor, which is assumed to be constant  $c_L$ . Thus,

$$C_L = c_L L. \quad (5)$$

The fingerling costs  $c_s$  are incurred at the beginning of the culture, while the output transportation costs  $C_\tau(\cdot)$  are seen at the end of the cycle and are given by

$$C_\tau(N(T), w(T)) = c_\tau N(T)w(T), \quad (6)$$

which  $c_\tau = \bar{c}_\tau - s_\tau$ , where  $\bar{c}_\tau$  is the output transportation cost per unit of weight and  $s_\tau$  is the subsidy to transportation cost per unit of weight.

The function of other costs ( $C_o(\cdot)$ ) includes sanitary expenditure, maintenance, etc. which depend on the cumulated biomass, i.e.

$$C_o(N, w) = c_o N(t)w(t), \quad (7)$$

where  $c_o$  represents the daily costs.

Let us consider that the farm is interested in optimally managing both the ration size and the final weight of the fish which is going to be sold in the market, i.e. the size of the product. The problem involves the maximization of discounted profits, and can be formulated as follows:

$$\begin{aligned}
& \underset{r(t), T, w(T)}{\text{Max}} \quad e^{-ht} p(w(T))w(T) - c_s - e^{-hT} w(T)c_r - \int_0^T e^{-ht} (c_f f(w, r, \theta) \dot{w}(t) + e^{Mt} K + c_o w) dt, \\
& \text{s.t.} \quad \dot{w} = G_1(w(t))G_2(r(t))G_3(\theta(t)), \\
& \quad w(0) = w_0, \\
& \quad T \text{ free, } w(T) \text{ free,} \\
& \quad \theta(t) = \bar{\theta} + A \sin\left(\frac{2\pi(t+t_0)}{365}\right), \\
& \quad 0 \leq r(t) \leq 1, \\
& \quad w(t) \geq 0,
\end{aligned} \tag{8}$$

where  $i$  is the interest rate,  $h=i+M$  is the sum of the interest and mortality rates, and  $K=c_L L/N_0$  is the daily cost of labor per individual. Thus, the state variable of this non autonomous problem is represented by the fish weight  $w(t)$ , which is controlled by the ration size  $r(t)$ . Both harvesting time and size are freely determined. The solution will lead us to the optimal ration size  $r^*(t)$  for each time period (i.e. per day) as well as to the optimal market size  $w^*(T)$  for the product that the firm should deliver to the market. This size is also grown in an optimal time span  $T^*$ .

The current value Hamiltonian is:

$$H = -c_f \Gamma G_1(w)G_2(r)G_3(\theta) f(w, r, \theta) - e^{Mt} K - c_o w + \psi(t) \Gamma G_1(w)G_2(r)G_3(\theta) + \mu_1 r + \mu_2 (1-r), \tag{9}$$

where  $\psi(t)$  is the costate variable and  $\mu_1(t)$ ,  $\mu_2(t)$  are non-negative multipliers with properties  $\mu_1(t)r(t)=0$  and  $\mu_2(t)(1-r(t))=0$  respectively. The specification of functions  $G_i$ ,  $i=1,2,3$  is included in Table 1. In particular,  $G_2(r)=Z(r)/Z(r_c)$ , where  $Z(r)$  is the normalized growth function and  $r_c$  is a constant ration of reference or culture ration. The conversion rate for different ration sizes  $f(w, r, \theta)$  is defined by means of the theoretical

normalized conversion rate  $Y(r)$  and function  $f_l(w, \theta)$ , which represents the conversion rate for a 100% ration size. Introducing these definitions and after some simplifications the Hamiltonian is transformed into

$$H = \Gamma G_1(w) G_3(\theta) \frac{1}{Z(r_c)} \left( -c_f r f_1(w, \theta) + \psi(t) Z(r) \right) - e^{Mt} K - c_o w + \mu_1 r + \mu_2 (1-r), \quad (10)$$

By applying the Pontryagin's maximum principle we have

$$H_r = \Gamma G_1(w) G_3(\theta) \frac{1}{Z(r_c)} \left( -c_f \cdot f_1(w, \theta) + \psi(t) Z'(r) \right) + \mu_1 - \mu_2 = 0. \quad (11)$$

The following results are obtained from the conditions on interior and corner solutions,

$$\begin{aligned} \text{if } 0 < r(t) < 1 &\Rightarrow -c_f \cdot f_1(w, \theta) + \psi Z'(r) = 0, \\ \text{if } r(t) = 0 &\Rightarrow -c_f \cdot f_1(w, \theta) + \psi Z'(0) \leq 0, \\ \text{if } r(t) = 1 &\Rightarrow -c_f \cdot f_1(w, \theta) + \psi Z'(1) \geq 0 \Leftrightarrow -c_f \cdot f_1(w, \theta) \geq 0. \end{aligned} \quad (12)$$

The last condition is not verified since both  $c_f$  and  $f_1(w, \theta)$  are positive functions. Thus, it is never optimal to feed at satiation<sup>2</sup>.

The first condition shows that the optimal ration size is determined such that the marginal benefits of weight ( $\psi Z'(r)$ ) have to be equal to the relative cost of the 100% ration size ( $c_f f_1(w, \theta)$ ). If the marginal benefits are less than the relative price for any ration size then the optimal ration is null ( $r=0$ ). The latter case is difficult to occur, since it would involve high feeding costs or high conversion rates for which commercial culture would not be viable.

Applying the first order optimal conditions we obtain the following differential equations which govern the state and co-state variables:

$$\begin{aligned}\dot{\psi} &= h\psi + \frac{c_f \Gamma}{Z(r_c)} G_3(\theta) \left( (f_1)_w(w, \theta) G_1(w) + f_1(w, \theta) G_1'(w) - \frac{\psi}{c_f Y(r)} G_1'(w) \right) \cdot r + c_o, \\ \dot{w} &= \Gamma G_1(w) G_2(r) G_3(\theta).\end{aligned}\quad (13)$$

On the other hand, if we impose the transversality conditions for  $w(T)$  and  $T$  (see Appendix 1 for details), we have

$$\begin{aligned}\psi(T) &= p_w w(T) + p - c_r, \\ p_w \dot{w}(T) w(T) + (p - c_r) \dot{w}(T) &= (i + M)(p - c_r) w(T) + c_f f(w(T), r, \theta) \dot{w}(T) + e^{MT} K + c_o w(T).\end{aligned}\quad (14)$$

From the first equation in (14), it is found that in the last period  $T$  the marginal benefits of allowing the individual to grow by one gram ( $\psi(T)$ ) are directly proportional to the increase in revenue to be obtained with this weight gain, i.e.  $(p + \dot{p} w)$ . The output transportation net costs reduce the potential benefits which would be obtained by an extra growth of the individual.

The second equation in (14) gives the harvesting weight  $w(T)$  as dependent on some economic and biological factors in the model. The influence of these factors on the optimal harvesting weight is determined by the following condition (see Appendix 1),

$$\frac{e^{MT} K}{w^2} + p_{ww} \dot{w} + 2p_w \frac{\dot{w}}{w} \leq (i + M) p_w. \quad (15)$$

In this condition were fulfilled, fish weight is inversely related to most of the input costs considered, as it is shown in Table 2. Therefore, the optimal weight decreases as the

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4 price of feed or other costs becomes larger. Similarly, if the wage rate increases then the  
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6 weight for the individual is lower. On the other hand, if the farm's size –as measured by  
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8 the number of fingerlings  $N_0$ - rises, then labor cost per individual would be lower,  
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10 which implies a larger optimal weight.  
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15 The main implication is that adverse cost conditions –i.e. higher wages or feed prices-  
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17 lead to earlier harvesting, with the result of lower optimal market sizes. The opposite  
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19 would be the case for those conditions which result in an increase in prospective  
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21 revenues for the farm. For the interest and mortality rate, the impact on harvesting  
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23 weight is also negative for those values which satisfy condition (15).  
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28 Condition (15) is more likely to be satisfied as the growth rate of fish price is positive,  
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30 i.e.  $p(w)$  is convex, or the labor cost per unity of biomass is very small. In this case, the  
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32 influence of output transportation costs on the optimal size is positive if  $\dot{w}/w < i + M$ .  
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34 For growth rates of fish at the harvesting time larger than the interest rate plus the  
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36 mortality index, a rise in transportation costs would reduce the optimal weight.  
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38 Consequently, transportation costs show an ambiguous effect on the optimal  
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40 management of fish weight. That is, the combination of the specific shape of the fish  
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42 price schedule and the growth rate makes the influence of these costs uncertain.  
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48 With respect to water temperature, Appendix 1 proves that its impact on the harvesting  
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50 weight at time  $T$  is always positive for values of the parameters in condition (15). This  
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52 means that higher temperature would induce larger market sizes because of the  
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54 advantage of a higher growth rate of fish weight. The implication is that favorable  
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56 environmental conditions could compensate for adverse cost conditions in production  
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58 and in the transportation of output.  
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### 3. Application

In this section we illustrate the model above using an empirical example in the Canary Islands, where water temperature has played an important role in the choice of the optimal harvesting weight. The particular expressions in Table 1 have been calibrated for seabream culture utilizing statistical techniques with empirical data (Appendix 2). More details of the calibration can be found in Hernández *et al.* (2003). For the price schedule, we assume a logistic-type function, which reflects the large degree of continuity and decreasing rate of the empirical prices published for the European market<sup>3</sup>, from €3.15 per kgm. for sizes around 200 g. to €6.00 per kgm. for 700 g. (see Figure 1). The empirical conversion rate was statistically fitted based on results from fish physiology and growth data.

We consider two alternative scenarios which vary in terms of the costs of feed and the average water temperature. Table 3 presents the assumptions made with respect to the parameters of the model for both scenarios. Scenario A represents average values for the Mediterranean waters while scenario B represents average values for the Canary Islands waters, off the coast of Northwestern Africa, in the Atlantic Sea. The latter incurs in somewhat higher costs of feed because the input has to be imported from the mainland. These input transportation costs can be incorporated in parameter  $c_f$ . It has also higher average water temperature and incurs in larger output transportation costs for access to the main consumer markets. However, local farms in the Canary Islands receive full compensatory subsidies for these costs (Gasca-Leyva, León and Hernández 2003), so we can assume identical parameters for the output transportation net costs for both

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4 scenarios. The rest of data have been obtained from previous market analysis and  
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6 observations of real culture in Canary Islands.  
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10 Table 4 presents the optimal results for the harvesting time ( $T$ ), final fish weight ( $w_T$ )  
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12 and the conversion rate ( $f$ ) for both scenarios respectively. The stocking is assumed in  
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14 May<sup>4</sup>. The solutions were obtained with the Runge-Kutta numerical method with order  
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16 five and one day step time. The algorithms were validated by finding very low local  
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18 errors. The results for the conversion rate respond to the accumulated rate over all the  
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20 production cycle<sup>5</sup>.  
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25 The optimal weight and harvesting time are larger for scenario B than for scenario A.  
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27 These results did not vary significantly with the capacity of the farm. The conversion  
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29 rate is also larger for the Canary Islands waters. Thus, producers in the Canary Islands  
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31 should optimally choose a product differentiation strategy based on larger sizes which  
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33 would compensate for its adverse cost conditions and would take advantage from higher  
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35 growth rates which are possible because the higher water temperature. The  
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37 Mediterranean scenario specializes in lower market sizes which are harvested at the  
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39 beginning of winter, when the conversion rate reaches its highest levels, and feeding is  
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41 more inefficient.  
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48 The optimal trajectories for the ration size under the assumptions of each scenario for  
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50 farm capacity of 200 tm. are presented in Figure 2. The optimal ration size is always  
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52 higher for the Canary Islands because of the advantageous water temperature. While  
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54 optimal ration size in the Mediterranean waters varies between a maximum of 84% and  
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56 a minimum of 64%, the range in scenario B is 92% when fish is stocked to 70% at the  
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58 end of the culture. The pattern is declining as the fish grows and enters the colder  
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4 seasons. Nevertheless, in both scenarios the ration size should rise as the cycle  
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6 approaches the summer peaks. This can be observed in scenario B after one year of  
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8 culture and the second summer of culture has begun. Thus the optimal feeding schedule  
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10 is critically influenced by the assumptions of the water temperature.  
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15 The differentiation strategy according to the optimal fish weight is not affected by  
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17 changes in the relevant parameters of the model. Figures 3 and 4 show respectively the  
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19 impacts of the output transportation costs and the interest rate on the optimal fish  
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21 weight. The theoretical model predicts that large labor cost per unity of biomass and a  
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23 large price response to weight could lead to an ambiguous effect of transportation costs  
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25 on market weight. For the numerical application, the output transportation costs have a  
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27 negative impact on the harvest size, but it is very small and does not produce significant  
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29 changes in the optimal choice across the relevant range. However, when the output  
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31 transportation costs are very high the optimal weight tends to zero because the negative  
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33 returns. These results are valid for both scenarios and match observed behavior, with  
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35 producers in the Canary Islands selecting a larger size than in the Mediterranean waters.  
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42 The impact of the interest rate on the optimal size is also negative but very small. The  
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44 lack of sensitivity found in the calibrated results is due to the particular assumptions  
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46 made with respect to the set of parameters. Nevertheless, the interest rate does not have  
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48 an impact on the differentiation strategy based on the choice of a larger fish size under  
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50 favorable conditions of water temperature.  
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#### 58 **4. Conclusions**

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4 This paper has focused the management of the key variables of the ration size and the  
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6 harvesting weight in off-shore aquaculture, taking into account the impact of water  
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8 temperature as a critical environmental condition. The model allows us to evaluate some  
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10 of the potential economic trade-offs farms can face between adverse economic  
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12 conditions and advantageous environmental factors.  
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17 The theoretical results show that most common input costs have a negative effect on the  
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19 optimal harvesting weight if a particular condition over these costs and the other  
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21 biological and economic factors is fulfilled. The particular condition that could lead to a  
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23 positive impact of these parameters on harvesting weight would be a large rate of  
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25 growth of the fish price with respect to weight and a large labor cost per unity of  
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27 biomass. Nevertheless, the effect of output transportation costs can be still ambiguous.  
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32 Water temperature has a positive effect on the optimal weight, indicating that those  
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34 farms with favorable environmental conditions could improve their performance by  
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36 producing larger sizes. That is, the results suggest that product differentiation on the  
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38 basis of fish's weight could be an optimal strategy for those distant scenarios that enjoy  
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40 advantageous conditions of water temperature. Past research has shown that the location  
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42 of firms could be a major determinant of technical efficiency (see for instance, Gumbau-  
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44 Albert and Maudos (2002) for the Spanish industry). Thus, further research is needed on  
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46 the impact of advantageous environmental conditions on the location of firms'  
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48 investment.  
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55 The application of the model to two alternative scenarios in the Mediterranean and the  
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57 Canary Islands indicate that water temperature is the main variable influencing the  
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59 choice of the optimal size. The optimal size for each scenario was not significantly  
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4 sensitive to changes in the output transportation costs and the interest rate. The  
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6 differentiation according to the market size was neither affected by changes in other  
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8 parameters of the model.  
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## 11 12 13 14 15 16 Notes

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19 1. As will be shown below this assumption is appropriate for the case of some  
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21 aquaculture products, such as gilthead seabream. The assumption of a stepwise  
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23 function for the price schedule, as in Mistiaen and Strand (1999) can be  
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25 considered a degenerated case which would limit the range of quantitative  
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27 results obtained with our model. On the other hand, the stepwise assumption  
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29 becomes somewhat intractable with more than two steps, and complicates  
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31 unnecessarily the analysis. Nevertheless, the general assumption of a non-  
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33 linear continuous function reflects observed prices for some aquaculture  
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35 species. From a market perspective, Bjorndal et al. (1993) find out that the  
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37 oligopoly pricing models perform well in estimating the price of farmed  
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39 salmon in the US market, suggesting that price discrimination and product  
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41 differentiation are viable strategies among firms.  
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- 49 2. However, feeding at satiation is a common practice (Azevedo *et al.* 1998,  
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51 Glasser and Oswald 2001).  
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- 54 3. <http://www.globefish.org/marketreports/Groundfish/seabass-seabream.htm>  
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- 58 4. The assumption about the stocking date affected the optimal harvesting weight  
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60 in our model, but did not change the conclusions obtained.

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5. The algorithm used in the numerical solution was implemented with MATLAB® and consisted on determining the trajectory  $r(t)$  for the state variables in (13) to satisfy the transversality conditions (14) with a tolerance of one thousandth. This is available from authors upon request.

## 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60

### References

- Arnason, R. (1992). Optimal Feeding Schedules and Harvesting Time in Aquaculture. *Marine Resource Economics* 7(1):15-35.
- Azevedo, P.A., Cho, C.Y., Leeson, S. and Bureau, D.P. (1998). Effects of feeding level and water temperature on growth, nutrient and energy utilization and waste outputs of rainbow trout (*Oncorhynchus mykiss*). *Aquatic Living Resources* 11:227-38.
- Bjorndal, T. (1988). Optimal Harvesting of farmed Fish. *Marine Resource Economics* 5: 139-59.
- Bjorndal, T., Gordon, D.V. and Singh, B. (1993). A Dominant Firm Model of Price Determination in the US Fresh Salmon Market: 1985-88, *Applied Economics*, 25: 743-750.
- Brett, J.R. (1979). Environmental Factors and Growth. *Fish Physiology*, vol. 8, W.S. Hoar, D.J. Randall and J.R. Brett, eds, pp. 599-667. London: Academic Press.
- Cacho, O.J., Kinnucan, H. and Hatch, U. (1991). Optimal control of fish growth. *American Journal of Agricultural Economics* 73:174-83.
- Gasca-Leyva, E., León, C.J., Hernández, J.M. (2003). Management strategies for Gilthead Seabream (*Sparus Aurata*) cultivation in floating cages in the Mediterranean sea and Atlantic Ocean. *Journal of the World Aquaculture Society* 34:29-39.

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Glasser, F. and Oswald, M. (2001). High stocking densities reduce *Oreochromis niloticus* yield: model building to aid the optimisation of production. *Aquatic Living Resources* 14:319-26.

Gumbau-Albert, M. and Maudos, J. (2002). The Determinants of Efficiency: the Case of the Spanish Industry, *Applied Economics*, 34 (15): 1941-1948.

Heaps, T. (1993). The Optimal Feeding of a Farmed Fish. *Marine Resource Economics* 8(1):89-99.

Heaps, T. (1995). Density Dependent Growth and the Culling of Farmed Fish. *Marine Resource Economics* 10(3):285-98.

Hernández, J.M., Gasca-Leyva, E., León, C.J. and Vergara, J.M. (2003). A growth model for gilthead seabream (*Sparus aurata*). *Ecological Modelling* 165: 265-83.

Mistiaen, J.A. and Strand, I. (1999). Optimal Feeding and Harvest Time for Fish Weight-Dependent Prices. *Marine Resource Economics* 13:231-46.

Muller-Feuga, A. (1990). Modélisation de la croissance des poissons en élevage. *Rapports scientifiques et techniques de l'IFREMER*. Institut Français de Recherche pour l'Exploitation de la Mer.

Pascoe, S, Wattage, P. and Naik, D. (2002). Optimal harvesting strategies: practice versus theory. *Aquaculture Economics and Management*, 6(5-6):295-308.

Ravagnan, G. (1984). L'élevage du loup et de la daurade en valliculture. *L'Aquaculture Du Bar Et Des Sparidés*, G. Barnab, et R. Billard, eds., pp. 435-46. Paris: INRA Publ.

## Appendix 1

### A1.1 Transversality conditions

Let  $\psi(t)$  be the costate variable in problem (8). The transversality conditions are given by

$$\psi(T) = e^{hT} \frac{\partial \phi}{\partial w(T)} \quad (\text{A.1})$$

with  $\phi = e^{-hT} w(T) p(w(T)) - c_s - e^{-hT} c_\tau w(T)$ . We deduce from here that

$$\psi(T) = p_w(w(T)) w(T) + p(w(T)) - c_\tau. \quad (\text{A.2})$$

The second transversality condition is obtained by solving expression

$$e^{-hT} H_1(T) + \frac{\partial \phi}{\partial T} = 0, \quad (\text{A.3})$$

where  $H_1$  is the Hamiltonian introduced in (9) and removing the last two terms. The calculations are

$$\begin{aligned} \frac{\partial \phi}{\partial T} &= -h e^{-hT} (p(w(T)) w(T) - c_\tau w(T)) \\ H_1(T) &= -c_f f(w(T), r(T), \theta(T)) \dot{w}(T) - c_o w(T) - e^{MT} K + \psi(T) \dot{w}(T) \\ &= -c_f f(w(T), r(T), \theta(T)) \dot{w}(T) - c_o w(T) - e^{MT} K + (p_w(w(T)) w(T) + p(w(T)) - c_\tau) \dot{w}(T) \end{aligned} \quad (\text{A.4})$$

Thus, applying (A.3) and simplifying we have

$$p_w \dot{w}(T) w(T) + (p - c_\tau) \dot{w}(T) = (i + M)(p - c_\tau) w(T) + c_f f(w(T), r, \theta) \dot{w}(T) + e^{MT} K + c_o w(T). \quad (\text{A.5})$$

## A1.2 Sensitivity analysis

We note

$$L(w, c_\tau) = p_w \dot{w} w + (p - c_\tau) \dot{w},$$

$$R(w, c_\tau, i, M, c_f, \bar{\theta}, K, c_o) = (i + M)(p - c_\tau)w + c_f f(w, r, \theta) \dot{w} + e^{MT} K + c_o w,$$

the left hand side and the right hand side in equation (A.5) respectively. For the sake of simplicity, we omit the argument of variable  $w$ . The influence of any factor  $\xi \in \{c_\tau, i, M, \bar{\theta}, K, c_o\}$  in the optimal weight is obtained through the sign of the implicit

derivative  $\frac{\partial w}{\partial \xi} = \frac{R_\xi - L_\xi}{L_w - R_w}$ . Using equation (A.5), the condition for the denominator to be

negative is

$$L_w - R_w < 0 \Leftrightarrow \frac{p_w \dot{w} w}{2(p - c_\tau)} + \frac{m+1}{2}(i + M) + \frac{2 - \gamma(\theta)}{2} \frac{c_f f}{p - c_\tau} \frac{\dot{w}}{w} + \frac{m+2}{2} \frac{e^{MT} K}{(p - c_\tau)w} + \frac{m+1}{2} \frac{c_o}{p - c_\tau} < \frac{\dot{w}}{w} + \frac{i + M}{2} \frac{p_w w}{p - c_\tau}, \quad (\text{A.6})$$

where  $m$  is the growth function parameter and  $\gamma(\theta)$  the empirical conversion rate exponent (see Table 1). As  $m < 1$  and  $\gamma(\theta) > 0$ , a sufficient condition for the inequality (A.6) to be satisfied is

$$\frac{e^{MT} K}{w^2} + p_{ww} \dot{w} + 2p_w \frac{\dot{w}}{w} \leq (i + M) p_w. \quad (\text{A.7})$$

Assuming (A.7) is fulfilled, we obtain the sign of the derivative of weight with respect to the parameters:

- *Output transportation net costs ( $c_\tau$ ):*

$$\frac{\partial w}{\partial c_\tau} = \frac{-\dot{w} + (i + M)w}{w(2(\dot{p}w + p - e^{iT} c_\tau) - c_f f)^2} > 0 \Leftrightarrow \frac{\dot{w}}{w} < i + M.$$

- *Interest rate ( $i$ ):*  $\frac{\partial w}{\partial i} = \frac{(p - c_\tau)w}{L_w - R_w} < 0.$

$$- \text{Mortality rate } (M): \frac{\partial w}{\partial M} = \frac{(p - c_\tau)w + Me^{M\tau} K}{L_w - R_w} < 0.$$

$$- \text{Feed cost } (c_f): \frac{\partial w}{\partial c_f} = \frac{(m + \gamma(\theta))f(w, r, \theta)\dot{w}}{L_w - R_w} < 0.$$

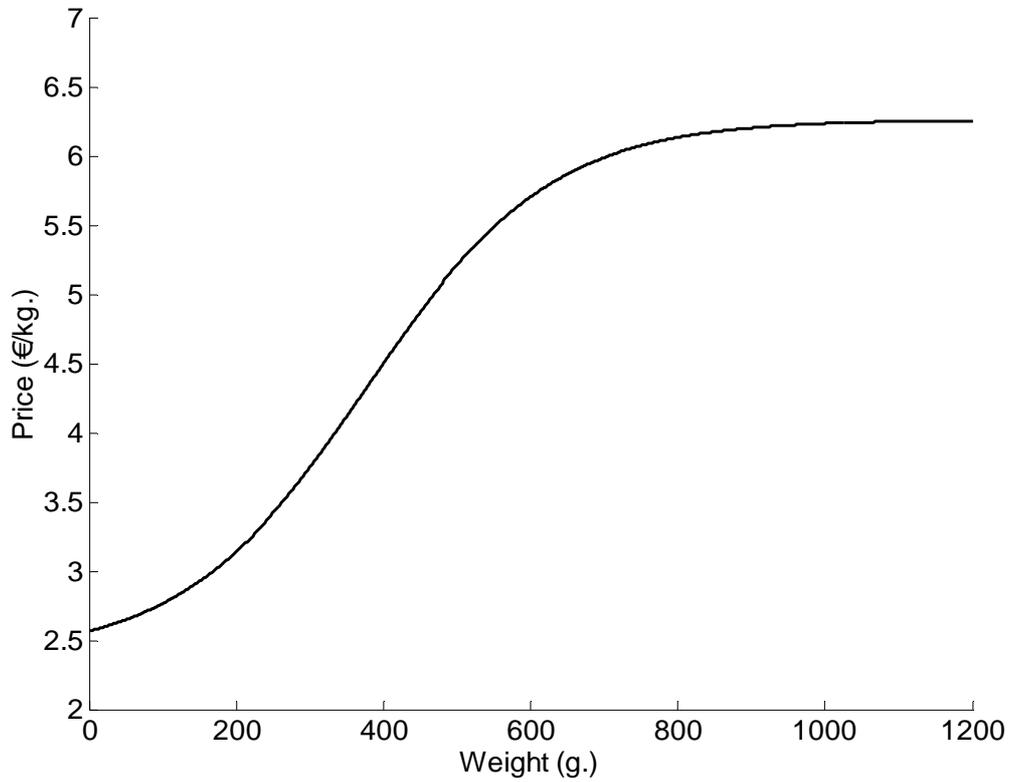
$$- \text{Average water temperature } (\bar{\theta}): \frac{\partial w}{\partial \bar{\theta}} = \frac{(m + \gamma(\theta))c_f Y(r)(f_1)_{\bar{\theta}} + \gamma_{\theta}(\theta)c_f f}{L_w - R_w} > 0.$$

$$- \text{Labor cost per individual } (K): \frac{\partial w}{\partial K} = \frac{e^{M\tau}}{L_w - R_w} < 0.$$

$$- \text{Other costs } (c_o): \frac{\partial w}{\partial c_o} = \frac{w}{L_w - R_w} < 0.$$

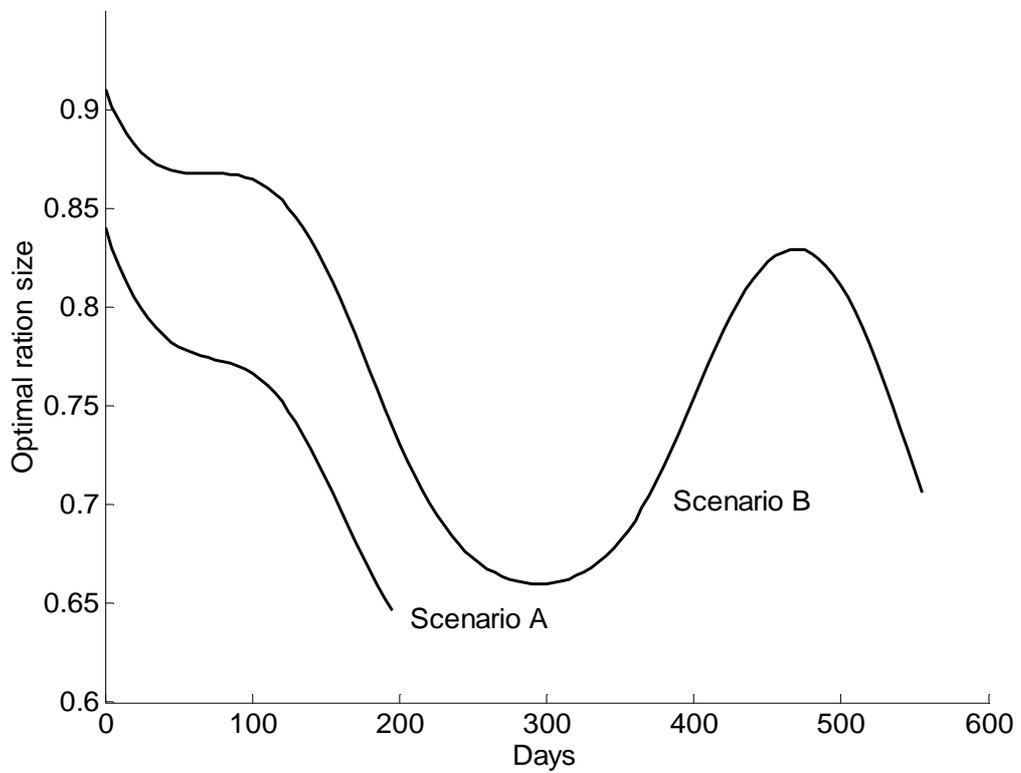
## Appendix 2

(Please, insert Table A2 here)



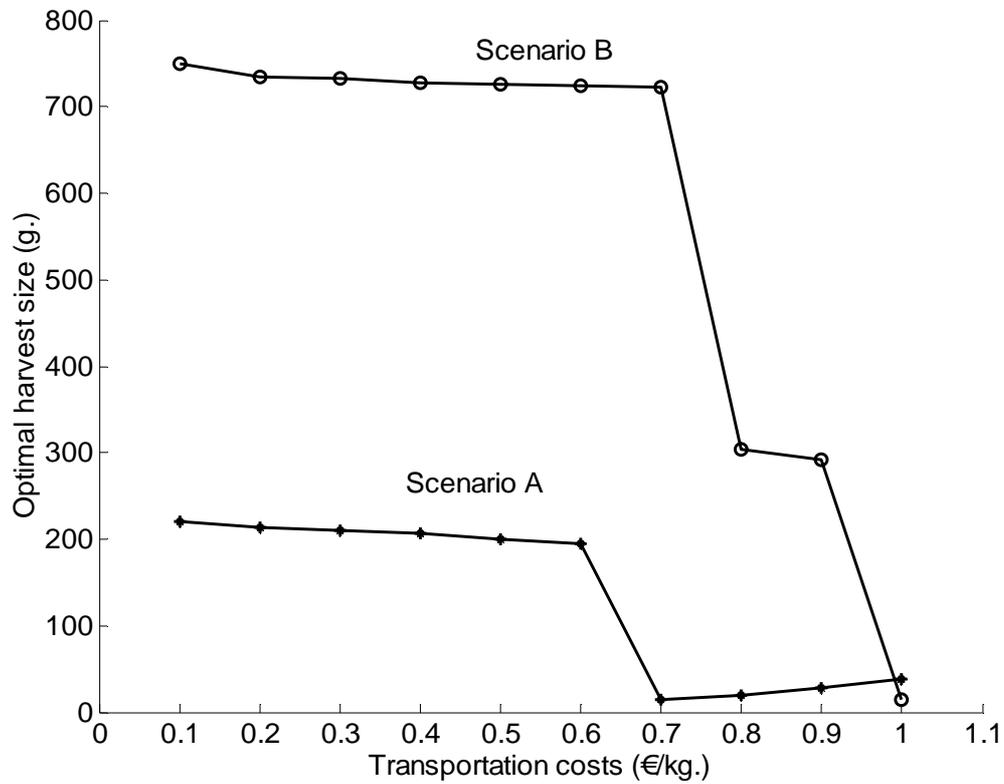
**Figure 1.** Fish price function:  $p(w) = \frac{18.78e^{0.008w} + 145.32}{3e^{0.008w} + 61}$

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**Figure 2.** Optimal trajectory of the ration size for both scenarios.

Farm size: 200 Tons. Initial month: May



**Figure 3.** Influence of the output transportation costs on optimal harvesting size

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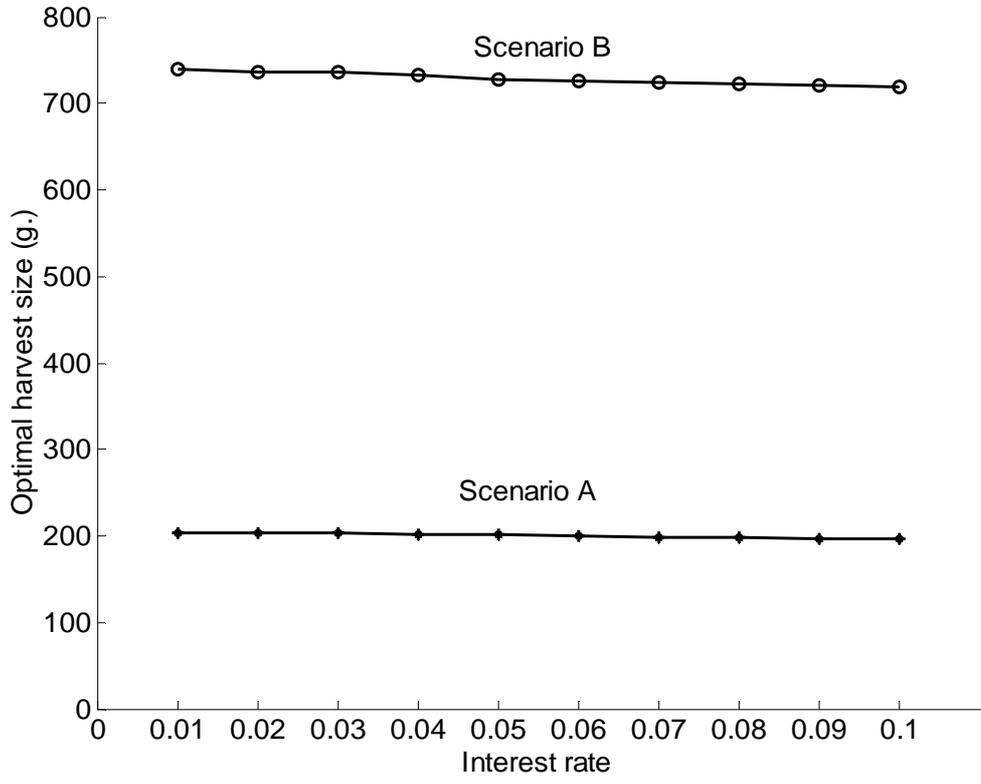


Figure 4. Influence of the interest rate on optimal harvesting size

**Table 1**

Growth model equations

Description	Expression
Fish growth function	$G(w(t), r(t), \theta(t)) = \Gamma G_1(w(t))G_2(r(t))G_3(\theta(t))$
Weight function	$G_1(w) = w^m$
Ration function	$G_2(r) = \frac{Z(r)}{Z(r_c)}$
Thermal function	$G_3(\theta) = D(e^{\alpha(\theta_M - \theta)} - e^{\beta(\theta_M - \theta)})$
Normalized conversion rate	$Y(r) = r \left( 1 + \frac{(r_m - r_o)^2(1-r)^2}{(r_o - 1)(3r_m r_o - r_m - 2r_o^2)(r - r_m)} \right)$
Normalized growth rate	$Z(r) = \frac{r}{Y(r)}, \quad Z' > 0, Z'' < 0^{(*)}$
Conversion rate	$f(w, \theta, r) = Y(r)f_1(w, \theta)$
Empirical conversion rate	$f_1(w, \theta) = H(\theta)w^{\gamma(\theta)},$ $H, \gamma > 0, \gamma_\theta < 0, (f_1)_\theta < 0, (f_1)_w > 0$

Parameters description:  $m$ , growth function parameter, specific for each species,  $0 < m < 1$ ;  $\theta_M$ , maximum lethal temperature;  $\alpha$  and  $\beta$ , thermal function parameters, specific for each species;  $r_c$ , culture ration, recommended levels of feed suppliers;  $r_m$ , maintenance ration;  $r_o$ , optimal ration, where a minimum conversion rate is reached.

(\*) While  $r > r_m$ , and  $r_m < 0.5$ , generally accepted.

**Table 2**

Effect of changes in unitary costs over harvesting weight if condition (15) is fulfilled.

Sign (+) indicates positive influence; sign (-) indicates negative influence; 0 indicates no influence.

<b>Unitary cost</b>	<b>Effect over harvesting weight</b>
$c_f$ (feed price)	-
$c_L$ (wage rate)	-
$c_S$ (fingerling cost)	0
$c_o$ (other costs)	-
$L$ (number of employees)	-
$N_0$ (number of fingerlings)	+
$i$ (interest rate)	-
$M$ (mortality index)	-
$c_\tau$ (output transportation net costs)	+/-
$\bar{\theta}$ (Average water temperature)	+

**Table 3**

## Model parameters

Parameters	Description	Scenario A (Mediterranean)	Scenario B (The Canary Islands)
$c_f$	Feed price	0,55 €/kg.	0,60 €/kg.
$c_L$	Wage rate	12621,25 €/year	12621,25 €/year
$c_S$	Fingerling cost	0,30 €	0,30 €
$c_o$	Other costs	0,0025 €/kg.	0,0025 €/kg.
$L$	Number of employees	*	*
$N_0$	Number of fingerlings	*	*
$w_0$	Fingerling weight	14 g.	14 g.
$i$	Interest rate	6 % annual	6 % annual
$h$	Mortality plus interest rate	11 % annual	11 % annual
$p$	Price	**	**
$\bar{c}_\tau$	Output transportation cost	0,5 €/Kg.	1,5 € kg.
$\bar{s}_\tau$	Subsidy to transportation cost	0 €/Kg.	1 €/Kg.
$\bar{\theta}$	Average water temperature	17.5° C.	21° C.

\* Depends on the size of the plant.

\*\* Depends on fish weight.

\*\*\*Subsidy included.

**Table 4**

Results of the optimization model for different farm sizes

Production <sup>a</sup> (Tm/year)	<i>T</i> (days)		$w_T$ (grams)		$f_A^b$	
	A	B	A	B	A	B
200 (10)	197	556	200	726	1,93	2,86
400 (15)	210	558	211	729	2,01	2,87
600 (19)	216	559	217	731	2,07	2,87
800 (25)	213	559	214	731	2,03	2,87

<sup>a</sup> Number of workers in brackets.<sup>b</sup> Conversion rate (accumulated).

**Table A2**

Description, values and sources of the growth model parameters

Parameter	Description	Value	Source
$m$	Growth function parameter	0.23	Hernández <i>et al.</i> (2003)
$\theta_M$	Maximum lethal temperature	32.9°C	Ravagnan (1984)
$\alpha$	Temperature function parameter	-0.12	Muller-Feuga (1990)
$\beta$	Temperature function parameter	-0.15	Muller-Feuga (1990)
$D$	Temperature calibration parameter	4.93	Calibration
$r_o$	Optimal ration rate	0.50	Brett (1979)
$r_m$	Maintenance ration rate	0.12	Muller-Feuga (1990)
$r_c$	Culture ration rate	0.80	Muller-Feuga (1990)
$\Gamma$	Other factors parameters	1	Calibration
$H(\theta)$	Conversion rate empirical parameter	0.40	Calibration
$\gamma(\theta)$	Conversion rate empirical parameter	*	Calibration

\* Depends on water temperature. Concretely,  $\gamma(\theta) = \frac{0.109}{0.2175 + 0.0325e^{0.25 \cdot (\theta - 20)}}$ .