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Ample thoughts

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AMPLE THOUGHTS

FRANK O. WAGNER

ABSTRACT. Non- n -ampleness as defined by Pillay [10] and Evans [4] is preserved under analysability.

1. INTRODUCTION

Recall that a type p over a set A in a simple theory is *one-based* if for any tuple \bar{a} of realizations of p and any $B \supseteq A$ the canonical base $\text{Cb}(\bar{a}/B)$ is contained in $\text{bdd}(\bar{a}A)$. One-basedness implies that the forking geometry is particularly well-behaved; for instance one-based groups are bounded-by-abelian-by-bounded. The principal result in [13] is that one-basedness is preserved under analyses: a type analysable in one-based types is itself one-based. This generalized earlier results of Hrushovski [6] and Chatzidakis [3]. One-basedness is the first level in a hierarchy of possible geometric behaviour of forking independence first defined by Pillay [10] and slightly modified by Evans [4], n -ampleness. Not 1-ample means one-based; not 2-ample is equivalent to a notion previously introduced by Hrushovski [7], CM-triviality. In [10] Pillay defines n -ampleness locally for a single type and shows that a superstable theory of finite Lascar rank is n -ample if and only if all its types of rank 1 are; his proof implies that in such a theory, a type analysable in non- n -ample types is again non- n -ample.

We shall give a definition of n -ampleness for invariant families of partial types, and generalize Pillay's result to arbitrary simple theories. Note that for $n = 1$ this gives an alternative proof of the main result in [13]. Since for types of infinite rank the algebraic (bounded) closure used in the definition is not necessarily appropriate (for a regular type p one might, for instance, replace it by p -closure), we also generalize the notion to Σ -closure for some \emptyset -invariant collection of partial types

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(thought of as small), giving rise to the notion of n - Σ -ample. This may for instance be applied to consider ampleness modulo types of finite SU -rank, or modulo superstable types. Readers not interested in this additional generality are invited to simply replace Σ -closure by bounded closure. However, this will only marginally shorten the proofs.

Our notation is standard and follows [12]. Throughout the paper, the ambient theory will be simple, and we shall be working in \mathfrak{M}^{heq} , where \mathfrak{M} is a sufficiently saturated model of the ambient theory. Thus tuples are tuples of hyperimaginaries, and $\text{dcl} = \text{dcl}^{heq}$.

2. Σ -CLOSURE

In this section we shall recall the definitions and properties of Σ -closure from [11, Section 4.0] in the stable and [13, Section 3.5] in the simple case (where it is called P -closure: Our Σ corresponds to the collection of all P -analysable types which are co-foreign to P). Buechler and Hoover [1, Definition 1.2] redefine such a closure operator in the context of superstable theories and reprove some of the properties [1, Lemma 2.5].

Let Σ be an \emptyset -invariant family of partial types.

Definition 1. Let π be a partial type over A . Then π is

- (almost) Σ -internal if for every realization a of π there is $B \downarrow_A a$ and \bar{b} realizing types in Σ based on B , such that $a \in \text{dcl}(B\bar{b})$ (or $a \in \text{bdd}(B\bar{b})$, respectively).
- Σ -analysable if for any $a \models \pi$ there are $(a_i : i < \alpha) \in \text{dcl}(A, a)$ such that $\text{tp}(a_i/A, a_j : j < i)$ is Σ -internal for all $i < \alpha$, and $a \in \text{bdd}(A, a_i : i < \alpha)$.

A type $\text{tp}(a/A)$ is *foreign* to Σ if $a \not\downarrow_{AB} \bar{b}$ for all $B \downarrow_A a$ and \bar{b} realizing types in Σ over B .

Definition 2. The Σ -closure $\Sigma\text{cl}(A)$ of a set A is the collection of all hyperimaginaries a such that $\text{tp}(a/A)$ is Σ -analysable.

Remark 3. We think of Σ as small. We always have $\text{bdd}(A) \subseteq \Sigma\text{cl}(A)$; equality holds if Σ is the family of all bounded types. One could also take Σ to be the family of all types of SU -rank $< \omega^\alpha$ for some ordinal α , the family of all supersimple types in a properly simple theory, or the family of p -simple types of p -weight 0 for some regular type p , giving rise to Hrushovski's p -closure [5].

Fact 4. *The following are equivalent:*

- (1) $\text{tp}(a/A)$ is foreign to Σ .
- (2) $a \downarrow_A \Sigma\text{cl}(A)$.
- (3) $a \downarrow_A \text{dcl}(aA) \cap \Sigma\text{cl}(A)$.
- (4) $\text{dcl}(aA) \cap \Sigma\text{cl}(A) \subseteq \text{bdd}(A)$.

Unless it equals bounded closure, Σ -closure has the size of the monster model and thus violates the usual conventions. The equivalence (2) \Leftrightarrow (3) can be used to cut it down to some small part.

Fact 5. *Suppose $A \downarrow_B C$. Then $\Sigma\text{cl}(A) \downarrow_{\Sigma\text{cl}(B)} \Sigma\text{cl}(C)$. More precisely, for any $A_0 \subseteq \Sigma\text{cl}(A)$ we have $A_0 \downarrow_{B_0} \Sigma\text{cl}(C)$, where $B_0 = \text{dcl}(A_0B) \cap \Sigma\text{cl}(B)$. In particular, $\Sigma\text{cl}(AB) \cap \Sigma\text{cl}(BC) = \Sigma\text{cl}(B)$.*

3. Σ -AMPLENESS

Let Φ and Σ be \emptyset -invariant families of partial types.

Definition 6. Φ is n - Σ -ample if there are tuples a_0, \dots, a_n , with a_n a tuple of realizations of partial types in Φ over some parameters A , such that

- (1) $a_n \not\downarrow_{\Sigma\text{cl}(A)} a_0$;
- (2) $a_{i+1} \downarrow_{\Sigma\text{cl}(Aa_i)} a_0 \dots a_{i-1}$ for $1 \leq i < n$;
- (3) $\Sigma\text{cl}(Aa_0 \dots a_{i-1}a_i) \cap \Sigma\text{cl}(Aa_0 \dots a_{i-1}a_{i+1}) = \Sigma\text{cl}(Aa_0 \dots a_{i-1})$ for all $0 \leq i < n$.

Remark 7. Pillay [10] actually requires $a_n \downarrow_{Aa_i} a_0 \dots a_{i-1}$ for $1 \leq i < n$ in item (2). We follow Evans variant [4] which seems more natural and which implies $a_n \dots a_{i+1} \downarrow_{\Sigma\text{cl}(Aa_i)} a_0 \dots a_{i-1}$.

Remark 8. [10, Remark 3.7] In definition 6 one may require a_0, \dots, a_{n-1} to lie in Φ^{heq} , replacing a_{n-1} by $a'_{n-1} = \text{Cb}(a_n/\Sigma\text{cl}(a_{n-1}A))$ and a_{i-1} by $a'_{i-1} = \text{Cb}(a'_i/\Sigma\text{cl}(a_{i-1}A))$.

Remark 9. [10, Lemma 3.2 and Corollary 3.3] If a_0, \dots, a_n witness n - Σ -ampleness over A , then $a_n \not\downarrow_{\Sigma\text{cl}(Aa_0 \dots a_{i-1})} a_i$ for all $i < n$. Hence a_i, \dots, a_n witness $(n - i)$ - Σ -ampleness over $Aa_0 \dots a_{i-1}$. Thus n - Σ -ample implies i - Σ -ample for all $i \leq n$.

For $n = 1$ and $n = 2$ there are alternative definitions of non- n - Σ -ampleness:

- Definition 10.** (1) Φ is Σ -based if $\text{Cb}(a/\Sigma\text{cl}(B)) \subseteq \Sigma\text{cl}(aA)$ for any tuple a of realizations of partial types in Φ over some parameters A and any $B \supseteq A$.
- (2) Φ is Σ -CM-trivial if $\text{Cb}(a/\Sigma\text{cl}(AB)) \subseteq \Sigma\text{cl}(A, \text{Cb}(a/\Sigma\text{cl}(AC)))$ for any tuple a of realizations of partial types in Φ over some parameters A and any $B \subseteq C$ such that $\Sigma\text{cl}(ABa) \cap \Sigma\text{cl}(AC) = \Sigma\text{cl}(AB)$.

Remark 11. It is clear from the latter definitions that even though Φ might be a complete type p , if p is one-based (or CM-trivial, or n -ample), so is any extension of p , not only the non-forking ones.

- Fact 12.** (1) Φ is Σ -based if and only if Φ is not 1- Σ -ample.
 (2) Φ is Σ -CM-trivial if and only if Φ is not 2- Σ -ample.

Proof: This is standard. For (1), take $a = a_1$ and $B = Aa_0$ and note that $a_1 \downarrow_{\Sigma\text{cl}(A)} a_0$ iff $a_1 \downarrow_{\Sigma\text{cl}(A)} \Sigma\text{cl}(Aa_0)$ by Fact 5. For (2), take $a = a_2$, $B = a_0$ and $C = a_0a_1$ and note that

$$\begin{aligned} a_2 \downarrow_{\Sigma\text{cl}(A)} a_0 &\Leftrightarrow a_2 \downarrow_{\Sigma\text{cl}(A)} \Sigma\text{cl}(Aa_0), \quad \text{and} \\ a_2 \downarrow_{\Sigma\text{cl}(Aa_1)} a_0 &\Leftrightarrow a_2 \downarrow_{\Sigma\text{cl}(Aa_1)} \Sigma\text{cl}(Aa_0a_1). \end{aligned}$$

For the converse, $a_0 = \text{Cb}(a/\Sigma\text{cl}(AB))$ and $a_1 = \text{Cb}(a/\Sigma\text{cl}(AC))$. \square

Lemma 13. If Φ is not n - Σ -ample, neither is $\text{tp}(a/A)$ for any $a \in \Sigma\text{cl}(\bar{a}A)$, where \bar{a} is a tuple of realizations of partial types in Φ over A .

Proof: Suppose $\text{tp}(a/A)$ is n - Σ -ample, as witnessed by a_0, \dots, a_n over some parameters $B \supseteq A$. There is a tuple \bar{a} of realizations of partial types in Φ over A such that $a_n \in \Sigma\text{cl}(\bar{a}A)$; we may choose it such that

$$\bar{a} \downarrow_{a_n A} a_0 \dots a_{n-1} B.$$

Then

$$\bar{a} \downarrow_{a_{n-1} a_n B} a_0 \dots a_{n-2},$$

and hence

$$\bar{a} \downarrow_{\Sigma\text{cl}(a_{n-1} a_n B)} a_0 \dots a_{n-2}.$$

As $a_n \downarrow_{\Sigma\text{cl}(a_{n-1} B)} a_0 \dots a_{n-2}$ implies

$$\Sigma\text{cl}(a_{n-1} a_n B) \downarrow_{\Sigma\text{cl}(a_{n-1} B)} a_0 \dots a_{n-2}$$

by Fact 5, we get

$$\bar{a} \downarrow_{\Sigma\text{cl}(a_{n-1}B)} a_0 \dots a_{n-2}.$$

We also have $\bar{a} \downarrow_{a_0 \dots a_{n-2} a_n B} a_{n-1}$, whence

$$\Sigma\text{cl}(a_0 \dots a_{n-2} \bar{a} B) \downarrow_{\Sigma\text{cl}(a_0 \dots a_{n-2} a_n B)} \Sigma\text{cl}(a_0 \dots a_{n-2} a_{n-1} B);$$

since Σcl is boundedly closed,

$$\begin{aligned} \Sigma\text{cl}(a_0 \dots a_{n-2} \bar{a} B) \cap \Sigma\text{cl}(a_0 \dots a_{n-2} a_{n-1} B) \\ \subseteq \Sigma\text{cl}(a_0 \dots a_{n-2} a_n B) \cap \Sigma\text{cl}(a_0 \dots a_{n-2} a_{n-1} B) \\ = \Sigma\text{cl}(a_0 \dots a_{n-2} B). \end{aligned}$$

Finally, $\bar{a} \downarrow_{\Sigma\text{cl}(B)} a_0$ would imply $\Sigma\text{cl}(\bar{a}A) \downarrow_{\Sigma\text{cl}(B)} a_0$ by Fact 5, and hence $a_n \downarrow_{\Sigma\text{cl}(B)} a_0$, a contradiction. Thus $\bar{a} \not\downarrow_{\Sigma\text{cl}(B)} a_0$, contradicting non- n - Σ -ampleness of Φ . \square

Lemma 14. *If $\Sigma\text{cl}(C) = \Sigma\text{cl}(A) \cap \Sigma\text{cl}(B)$ and $D \downarrow_C AB$, then $\Sigma\text{cl}(AD) \cap \Sigma\text{cl}(BD) = \Sigma\text{cl}(CD)$.*

Proof: This is just an adaptation of [8, Fact 2.4]. By Fact 5

$$\Sigma\text{cl}(AD) \downarrow_{\Sigma\text{cl}(A)} \Sigma\text{cl}(AB) \quad \text{and} \quad \Sigma\text{cl}(BD) \downarrow_{\Sigma\text{cl}(B)} \Sigma\text{cl}(AB),$$

so

$$\text{Cb}(\Sigma\text{cl}(AD) \cap \Sigma\text{cl}(BD) / \Sigma\text{cl}(AB)) \subseteq \Sigma\text{cl}(A) \cap \Sigma\text{cl}(B) = \Sigma\text{cl}(C).$$

Hence

$$\Sigma\text{cl}(AD) \cap \Sigma\text{cl}(BD) \downarrow_{\Sigma\text{cl}(CD)} \Sigma\text{cl}(AB)$$

and again by Lemma 5

$$\Sigma\text{cl}(AD) \cap \Sigma\text{cl}(BD) \downarrow_{\Sigma\text{cl}(CD)} \Sigma\text{cl}(ABD).$$

This yields the result. \square

Lemma 15. *If a_0, \dots, a_n witness n - Σ -ampleness over A and*

$$B \downarrow_A a_0 \dots a_n,$$

then a_0, \dots, a_n witness n - Σ -ampleness over B .

Proof: Clearly $B \downarrow_{a_0 \dots a_{i-1} A} a_0 \dots a_{i+1} A$, so

$$\Sigma \text{cl}(Aa_0 \dots a_{i-1} a_i) \cap \Sigma \text{cl}(Aa_0 \dots a_{i-1} a_{i+1}) = \Sigma \text{cl}(Aa_0 \dots a_{i-1})$$

implies by Lemma 14

$$\Sigma \text{cl}(Ba_0 \dots a_{i-1} a_i) \cap \Sigma \text{cl}(Ba_0 \dots a_{i-1} a_{i+1}) = \Sigma \text{cl}(Ba_0 \dots a_{i-1}).$$

Moreover, $a_n \downarrow_{\Sigma \text{cl}(A)} \Sigma \text{cl}(B)$ by Lemma 5, so $a_n \downarrow_{\Sigma \text{cl}(B)} a_0$ would imply $a_n \downarrow_{\Sigma \text{cl}(A)} a_0$, a contradiction. Hence $a_n \not\downarrow_{\Sigma \text{cl}(B)} a_0$.

Finally, $a_{i+1} \downarrow_{Aa_0 \dots a_i} B$, whence $a_{i+1} \downarrow_{\Sigma \text{cl}(Aa_0 \dots a_i)} \Sigma \text{cl}(Ba_i)$ by Lemma 5; as $a_{i+1} \downarrow_{\Sigma \text{cl}(Aa_i)} a_0 \dots a_{i-1}$ implies $a_{i+1} \downarrow_{\Sigma \text{cl}(Aa_i)} \Sigma \text{cl}(Aa_0 \dots a_i)$ this yields

$$a_{i+1} \downarrow_{\Sigma \text{cl}(Ba_i)} a_0 \dots a_{i-1}$$

for $1 \leq i < n$. □

Lemma 16. *Let Ψ be an \emptyset -invariant family of types. If Φ and Ψ are not n - Σ -ample, neither is $\Phi \cup \Psi$.*

Proof: Suppose $\Phi \cup \Psi$ is n - Σ -ample, as witnessed by $a_0, \dots, a_n = bc$ over some parameters A , where b and c are tuples of realizations of partial types in Φ and Ψ , respectively. As Ψ is not n - Σ -ample, we must have $c \not\downarrow_{\Sigma \text{cl}(A)} a_0$. Put $a'_0 = \text{Cb}(bc/a_0 \Sigma \text{cl}(A))$. Then a'_0 is definable in a Morley sequence $(b_i c_i : i < \omega)$ of $\text{tp}(bc/a_0 \Sigma \text{cl}(A))$; since $c \not\downarrow_{\Sigma \text{cl}(A)} a_0$ we get $(c_i : i < \omega) \not\downarrow_{\Sigma \text{cl}(A)} a'_0$, so a'_0 is internal in $\text{tp}(b/A)$. Put

$$a'_n = \text{Cb}(a'_0/a_n \Sigma \text{cl}(A)).$$

Then $\text{tp}(a'_n/\Sigma \text{cl}(A))$ is $\text{tp}(a'_0/\Sigma \text{cl}(A))$ -internal and hence $\text{tp}(b/A)$ -internal. Note that $a_n \not\downarrow_{\Sigma \text{cl}(A)} a_0$ implies $a_n \not\downarrow_{\Sigma \text{cl}(A)} a'_0$, whence

$$a'_n \not\downarrow_{\Sigma \text{cl}(A)} a'_0 \quad \text{and} \quad a'_n \not\downarrow_{\Sigma \text{cl}(A)} a_0.$$

Moreover $a'_n \in \Sigma \text{cl}(Aa_n)$, so $a_0, \dots, a_{n-1}, a'_n$ witness n - Σ -ampleness over A .

As $\text{tp}(a'_n/\Sigma \text{cl}(A))$ is $\text{tp}(b/A)$ -internal, there is $B \downarrow_{\Sigma \text{cl}(A)} a'_n$ and a tuple \bar{b} of realizations of $\text{tp}(b/A)$ with $a'_n \in \text{dcl}(B\bar{b})$. We may assume $B \downarrow_{a'_n} \Sigma \text{cl}(Aa_0 \dots a_{n-1})$, whence $B \downarrow_{\Sigma \text{cl}(A)} \Sigma \text{cl}(Aa_0 \dots a_{n-1} a'_n)$. So $a_0, \dots, a_{n-1}, a'_n$ witness n - Σ -ampleness over B by Lemma 15. As $a'_n \in \text{dcl}(B\bar{b})$, this contradicts non- n - Σ -ampleness of Φ by Lemma 13. □

Corollary 17. *For $i < \alpha$ let Φ_i be an \emptyset -invariant family of partial types. If Φ_i is not n - Σ -ample for all $i < \alpha$, neither is $\bigcup_{i < \alpha} \Phi_i$.*

Proof: This just follows from the local character of forking and Lemma 16. \square

Lemma 18. *If $\text{tp}(a/A)$ is not n - Σ -ample and $a \perp A$, then $\text{tp}(a)$ is not n - Σ -ample.*

Proof: Suppose $\text{tp}(a)$ is n - Σ -ample, as witnessed by a_0, \dots, a_n over some parameters B , where $a_n = (b_i : i < k)$ is a tuple of realizations of $\text{tp}(a)$. For each $i < k$ choose $B_i \perp_{b_i} (B, a_0 \dots a_n, B_j : j < i)$ with $B_i b_i \equiv Aa$. Then $B_i \perp b_i$, whence $(B_i : i < k) \perp Ba_0 \dots a_n$. Now $\text{tp}(b_i/B_i)$ is not n - Σ -ample, and neither is $\{\text{tp}(b_i/B_i) : i < k\}$ by Lemma 16. But a_0, \dots, a_n witness n - Σ -ampleness over $(B, B_i : i < k)$ by Lemma 15, a contradiction. \square

Corollary 19. *Let Ψ be an \emptyset -invariant family of types. If Ψ is Φ -internal and Φ is not n - Σ -ample, neither is Ψ .*

Proof: Immediate. \square

The following theorem is of independent interest, and has been shown by Buechler [2, Proposition 3.1] for superstable theories of finite Lascar rank and Φ the family of types of Lascar rank 1 (his first level $\ell_1(a)$ corresponds to our a_0 below).

Theorem 20. *Suppose $\text{tp}(a/A)$ is Φ -analysable, and $a_0 \subseteq \text{bdd}(Aa)$ is maximal such that $\text{tp}(a_0/A)$ is Φ -internal. Then a and a_0 are domination-equivalent over A .*

Proof: Since $a_0 \in \text{bdd}(Aa)$, clearly a dominates a_0 over A . For the converse, suppose $b \not\perp_A a$; we have to show $b \not\perp_A a_0$.

Let $b' = \text{Cb}(a/Ab)$. Then $\text{tp}(b'/A)$ is $\text{tp}(a/A)$ -internal, and hence Φ -analysable. So there is a sequence $(b_i : i \leq \alpha)$ in $\text{bdd}(Ab')$ such that $\text{tp}(b_i/A, b_j : j < i)$ is Φ -internal for all $i \leq \alpha$ and $b' \in \text{bdd}(A, b_i : i \leq \alpha)$. Since $a \not\perp_A b'$ there is minimal $i \leq \alpha$ such that $a \not\perp_{A, b_j : j < i} b_i$. Put $a' = \text{Cb}(b_j : j \leq i/Aa)$, and let $(b_j^\ell : j \leq i, \ell < \omega)$ be a Morley sequence in $\text{tp}(b_j : j \leq i/Aa)$. Then $a' \in \text{dcl}(b_j^\ell : j \leq i, \ell < \omega)$; as $a' \perp_A (b_j : j < i)$ by minimality of i we have $a' \perp_A (b_j^\ell : j < i, \ell < \omega)$. Now $\text{tp}(b_i^\ell/A, b_j^\ell : j < i)$ is Φ -internal by \emptyset -invariance of Φ , so $\text{tp}(a'/A)$ is Φ -internal, and $a' \subseteq a_0$. Clearly $a' \not\perp_A (b_j : j \leq i)$, whence $a' \not\perp_A b$ and finally $a_0 \not\perp_A b$. \square

Theorem 21. *Let Ψ be an \emptyset -invariant family of types. If Ψ is Φ -analysable and Φ is not n - Σ -ample, neither is Ψ .*

Proof: Suppose Ψ is n - Σ -ample, as witnessed by a_0, \dots, a_n over some parameters A , where a_n is a tuple of realizations of Ψ . Let $a'_n \subseteq \text{bdd}(Aa_n)$ be maximal such that $\text{tp}(a'_n/\Sigma\text{cl}(A) \cap \text{bdd}(Aa_n))$ is Φ -internal. Then a_n and a'_n are domination-equivalent over $\Sigma\text{cl}(A) \cap \text{bdd}(Aa_n)$ by Theorem 20; moreover a_n and hence a'_n are independent of $\Sigma\text{cl}(A)$ over $\Sigma\text{cl}(A) \cap \text{bdd}(Aa_n)$ by Fact 13, so a_n and a'_n are domination-equivalent over $\Sigma\text{cl}(A)$. Thus a_0, \dots, a'_n witnesses non- Σ -ampleness over A , contradicting Corollary 19. \square

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