Influence of the lateral friction on the surface flow: a 3D numerical approach

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To understand the complexity of dense granular flows, we used three-dimensional (3D) numerical simulations which appears as complementary approaches to the experimental ones. Numerical simulations allow (i) to investigate the influence of parameters difficult to control experimentally and (ii) to measure quantities inacessible to experiments. In this paper we focus on numerical 3D rotating drums and shed light on the role of lateral friction on the rheology of dense granular systems. Our framework is the Non Smooth Contact Dynamics approach, powerful to perform accurate simulations of large systems of particles, essential to deal with simultaneous contacts. We point out the influence of lateral friction coefficient on the granular flow and compare the results to previous 2D simulations.

1 INTRODUCTION

The transition between *liquid* and *solid* regimes in dense granular flows is one among the several interesting and unusual behaviors of this media. Rheology of both regimes has been widely studied experimentally, numerically and theoretically (see (G.D.R. Midi 2004) for a review), but some features still remain non understood. Some models describe dense granular flows by taking into account non-local effects (Mills, Loggia, and Texier 1999; Jenkins and Hanes 2002; Bonamy and Mills 2003; Rajchenbach 2003), by extending kinetic theory (Savage 1998) or by considering dense flows as quasi-static flows (Pouliquen and Gutfraind 1996), but to our knowledge, none of them succeeds to account for all the features experimentally observed. *Superimposed on the complexity of the internal rheology of the granular flow*, role of parameters such as the friction of beads on the lateral boundaries of the drum remains mysterious and may thus confuse the interpretation of experiments.

For these reasons, numerical simulations are essential to complete and assist experimental knowledges. Those allow to track the evolution of local quantities like stress or coordinence that are not accessible in experiments. Moreover, they allow to tune parameters such as friction on the boundary and in the systems. In a previous study (Renouf, Bonamy, Dubois, and Alart) we have investigated surface flows rheology through 2D numerical simulations. In this paper we present the first investigations on 3D systems. We will focus here on the influence of the lateral friction on the velocity profiles. The numerical simulations were performed using the contact dynamic method (Jean and Moreau 1992) based on a fully implicit resolution of the contact forces, with no resort to regularization schemes as Molecular Dynamics methods. We present in Sec. 2 our numerical framework. Section 3 is dedicated to the simulation methodology. Analysis of simulations results are gathered in Sec. 4, and Sec. 5 concludes the paper.

2 NUMERICAL BACKGROUND

Numerical simulations dedicated to the dynamic evolution of granular media can be based either on explicit (Kishino, Akaisawa and Kaneko 2001; Drake 1991) or implicit (Moreau 1993; Radjai, Wolf, Roux, Jean, and Moreau 1997) methods. The *Non Smooth Contact Dynamics* approach used here is implicit. Unilateral contact and dry Coulomb's friction are described by a nonsmooth formulation instead of compliant laws. For each step of the dynamical simulation, all kinematic and force constraints are simultaneously taken into account to determine all the contact forces in the packing. Contrary to Molecular Dynamics schemes traditionally used that reduces the system evolution to a succession of binary collisions, this approach allows to account properly for the non-local momentum transfers implied in multiple collisions generated by the rotation of the drum. The method is extensively described in (Moreau 1994; Jean 1999). we will give only its outlines concerning contact law.

The unilateral feature of contact evoked previously means that the gap between contactor bodies is non negative. Since we consider only dry systems, the reaction force is positive and when the contact vanishes, it is equal to zero. This is summarized by the so-called *velocity Signorini condition*

$$u_n \geq 0 \quad r_n \geq 0 \quad u_n . r_n = 0, \tag{1}$$

where the index n denotes the normal component of the various quantities: **u** the relative velocity between bodies in contact and **r** the local contact force. For explicit method the normal force is usually proportional to the penetration between two particles. Here there is no bilateral conditions. The dry frictional law is the Coulomb's one for which the basic features are: The friction force lies in the Coulomb's cone ($||\mathbf{r}_t|| \leq \mu r_n$, μ friction coefficient), and if the sliding relative velocity is not equal to zero, its direction is opposed to the friction force ($||\mathbf{r}_t|| = \mu r_n$). We can summarize previous explanations by the following relation:

$$||\mathbf{r}_t|| \leq \mu r_n \quad ||\mathbf{u}_t|| \neq 0 \rightarrow \mathbf{r}_t = -\mu r_n \frac{\mathbf{u}_t}{||\mathbf{u}_t||} \tag{2}$$

The velocity Signorini condition does not provide enough information when collisions occur. For rigid bodies we also need to adopt a collision law based on the Newton restitution law, $u_n^+ = -e_n u_n^-$, realistic for collection of spheres and extended to multicontact systems (Moreau 1994). To solve our contact problem we can use a classical non linear Gauss-Seidel algorithm (Jean 1999) or a Conjugate Projected Gradient one (Renouf and Alart 2004a). To face long CPU-time of 2D and 3D simulations, previous algorithms benefit of parallel versions (Renouf, Dubois and Alart 2004; Renouf and Alart 2004b).

3 SIMULATION METHODOLOGY

Our simulated granular systems are similar to those investigated experimentally in (Rajchenbach 2000; Bonamy, Daviaud, and Laurent 2002). We model a 3D rotating drum of diameter D equal to 250 mm half-filled with 13 000 rigid spheres of density $\rho_0 = 2.7$ g.cm^{-3} and diameter uniformly distributed between 3 mm and 3.6 mm (e.g. Fig.1).

Normal restitution coefficient between two particles is set to 0.46 and the friction coefficient to 0.4. The normal restitution is equal to 0.46 and the friction one to 0.95 between particles and cylindrical boundary of the drum. The drum/spheres friction coefficient

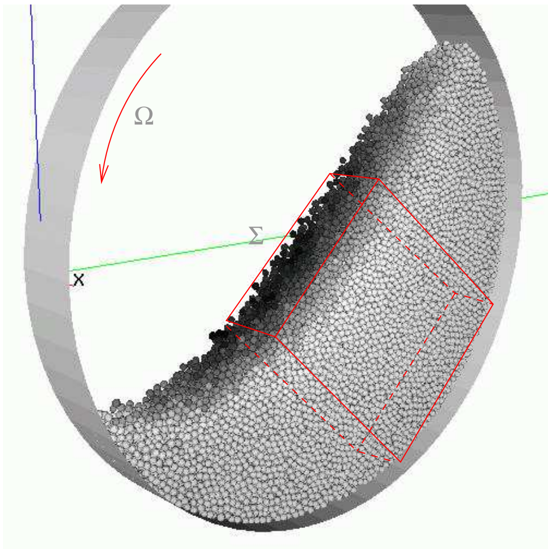

Figure 1. Typical snapshot of a 3D rotating drum. Its radius and its rotation speed are respectively $D = 250$ mm and $\Omega = 18$ rpm. It is filled with 13000 spheres of density $\rho^0 = 2.7$ g.cm^{-3} and diameter uniformly distributed in the interval [3 mm; 3.6 mm].

was set close to 1 to prevent sliding. Normal restitution coefficients and sphere/sphere friction coefficient were chosen to mimic the experimental flows of aluminum beads investigated in (Bonamy, Daviaud, and Laurent 2002). The normal restitution is equal to 0.46 between particles and the lateral wall and the friction coefficient, noted μ_l, takes three values : 0, 0.1 and 0.2.

For each simulation we place the particles in a drum with a constant rotation speed Ω (= 2 rpm, 6 rpm and 18 rpm). After a transient regime, a steady continuous surface flow is reached. We start then to capture snapshots equally distributed during the rotation of the drum. The time step is set to 2.10^{-3} s. The number of time steps necessary to achieve a complete simulation ranges from 4.10^3 to 1.10^4 depending on the rotating speed. All simulations have been performed with LMGC90 software (Dubois and Jean 2003). On a SGI Origin 3800 with 24 processors, about 96 h are required to achieve one of these simulations.

For each bead of each frames we record its position and its velocity. We note g the gravity constant and d the mean disk diameter. Distances, time and velocities are respectively given in units of d, $\sqrt{d/g}$ and $\sqrt{gd}$. In the elementary slice Σ (see Fig. 1) the flow can be considered as quasi-homogeneous. Placed at the center of the drum, Σ is 20 particles diameter wide and 4 particles diameter thick. This slice is divided into layers of one mean particle diameter wide parallel to the flow. The given value of the velocity at depth z is then defined as the average of the corresponding quantity defined at the grain scale over all the particles in all the frames that composes the sequence whose center of mass is inside the layer.

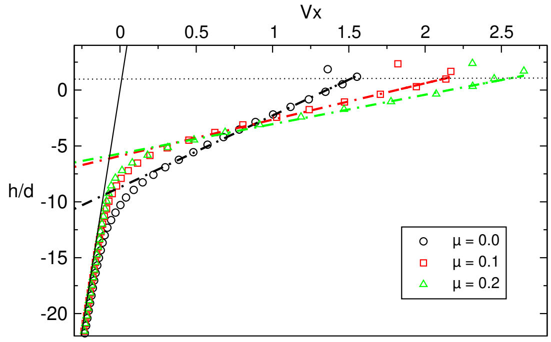

Figure 2. Velocity profiles for a constant value of $\Omega = 6$ rpm and three values of μ_l : 0, 0.1 and 0.2.

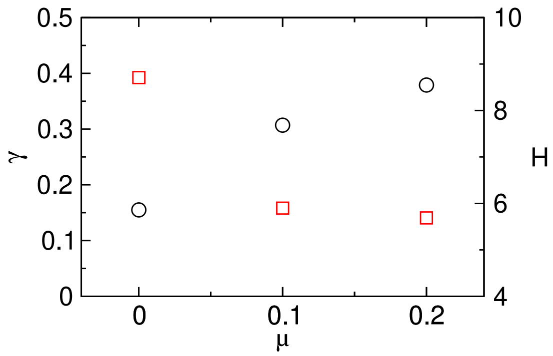

Figure 3. Velocity gradient (circles) and flowing thickness (squares) as function of μ_l for $\Omega = 6$ rpm.

4 ANALYSIS

To compare heap and rotating drum flows, (G.D.R. Midi 2004) suggest that the main control parameter for the surface flow is the non-dimensionalized flow rate $Q^{\star}$, defined as

$$Q^{\star} = \frac{\Omega D^2}{8d\sqrt{gd}}. \quad (3)$$

With this definition, in our simulation $Q^{\star}$ ranges from 2.77 to 24.95. We check first for the evolution of the velocity profiles. For a constant value of Ω (=6 rpm), corresponding to a value of $Q^{\star}$ equal to 8.318, we compare the profiles for different values of μ_l (see Fig. 2).

For each value two phases can clearly be observed: a flowing layer exhibiting a linear velocity profile (fitted by dashed lines) and a static phase corresponding to a driven motion related to the drum. Such shapes are very similar to the ones observed experimentally (Rajchenbach 2000; Bonamy, Daviaud, and Laurent 2002). However the three profiles do not exhibit the same gradient γ and the the same thickness H where H is defined as the intersection between the linear velocity profile and the continue line associate to the motion of the drum. We report on Fig. 3 the different values of this two quantities.

The frictionless case gives values very close to what we observed in our 2D rotating drym (Renouf, Bonamy, Dubois, and Alart), *i.e.* $\gamma = 0.15$ and $H = 10$ for $Q^{\star} = 8.318$. Without lateral friction, no confinement occurs. Thus this case is similar to the 2D simulations that do not take into account the friction of particles on the lateral boundaries. This computations may be considered as a reference for comparisons between 2D and 3D simulations.

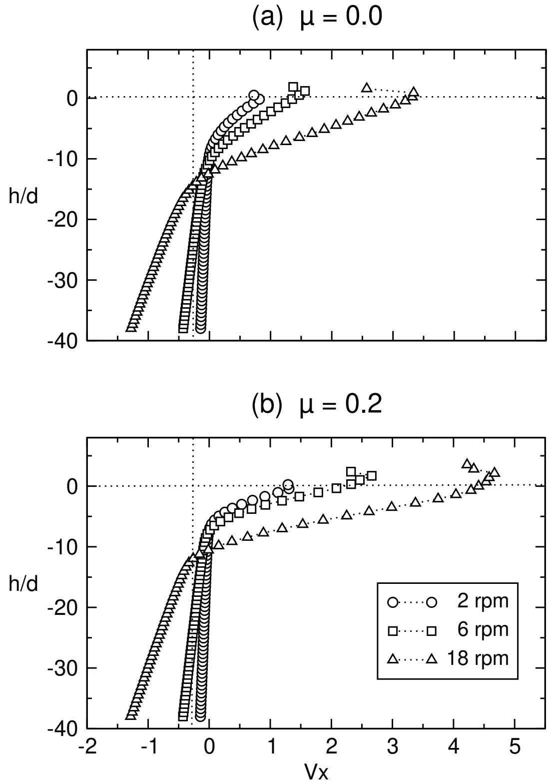

Figure 4. Velocity profiles for different values of Ω (= 2 rpm, 6 rpm and 18 rpm) and two values of μ_l: 0 (a) and 0.2 (b).

When friction is activated, the velocity gradient increases to reach values range from 0.3 to 0.4. The flowing layer thickness decreases to value equal to 4. The friction between particles and lateral walls restricts the volume of flowing particles but these last ones have a larger velocity on average on the flowing layer.This last phenomenon is essentially due to increasing of the mean angle of the particles' flow.

We represent on Fig.4 the velocity profiles obtained for different rotation speeds and with different values of μ_l. In our previous work (Renouf, Bonamy, Dubois, and Alart) we pointed out the fact that in 2D simulations, velocity gradient was depending on the drum's rotation speed. This previous observations is confirmed by the 3D lateral frictionless simulation results (e.g. Fig.4a).

When lateral friction becomes superior to zero, the velocity gradient stops to depend on the rotation speed. If we translate each velocity profile (e.g. 5) of the corresponding value of H, all linear velocity profile are superimposed. In other words, our numerical simulations show that the origin of the selection of a constant velocity gradient independent of the rotation speed Ω as commonly observed experimentally (Bonamy, Daviaud, and Laurent 2002; G.D.R. Midi 2004) should be found in effects of the friction

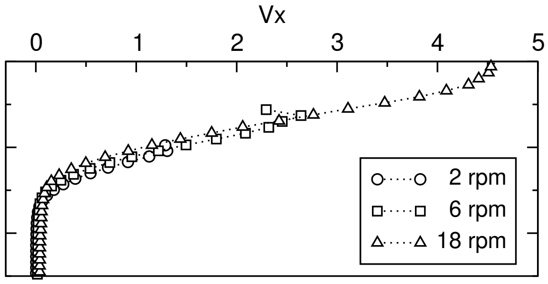

Figure 5. Translation of velocity profiles for different values of Ω. The lateral friction coefficient was μ_l =???.

of beads on the lateral drum boundaries rather than in the internal rheology of the granular flow.

5 CONCLUSIONS

Our numerical simulations exhibit clearly the influence of the lateral friction on the velocity gradient. In our previous study, we observed that the gradient was depending on the rotation speed in a 2D rotating drum. This coincides with what is observed in the 3D rotating drum with no friction on the lateral boundaries. But even for small value of μ_l this dependency disappears and numerical profiles match with experimental ones. Analysis of global and local quantities as stress or compacity profiles represents a great challenge for future investigations.

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