

# Shear strength properties of a thin granular interface

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**ABSTRACT:** We study shear strength properties of a thin two-dimensional layer of polygonal particles confined between two horizontal flat walls and confining lateral stresses. Both vertical compression tests at constant velocity and relaxation tests under the action of an overload are performed. It is shown that the evolution of the system is governed by the formation and collapse of “rigid” paths of contiguous particles joining the top and bottom walls. The chains of side-to-side contacts play a crucial role in the stability of these major force-bearing structures. The vertical strength is a decreasing function of thickness. The latter decreases under the action of an increasing vertical load, and tends to a critical value which is a function of wall friction and the aptitude of the particles to form side-to-side contacts. At this point, the force bearing structures become infinitely rigid in the vertical direction.

## 1 INTRODUCTION

Ballast is widely employed for the construction of railway tracks because of its mechanical properties and its flexibility from the point of view of construction and maintenance. This granular material, produced by crushing hard rocks, is confined between concrete sleepers supporting the rails and the platform. Its thickness is only about 10 grain diameters. Ballast has three main functions: 1) Evacuating rain water, 2) Transmitting static and dynamic stresses induced by passing trains to the platform and 3) Damping efficiently vibrations. The ballast layer needs to be relevelled from time to time because of its cumulative settlement under repeated loading. The increasing cost of such operations with the advent of high-speed trains has recently motivated a number of studies on the physical origins of settlement (Alias 1984; Oviedo 2001; Saussine 2004).

Obviously, a confined ballast layer is a *thin* granular interface since its thickness is below the correlation length of contact forces and particle displacements during a quasi-static flow (Radjaï et al. 1999). The space correlations are expected to be enhanced for two more reasons. First, the ballast grains are faceted and angular. Face-to-face contacts can form stable structures that will influence force transmission across the layer. Secondly, the confinement of ballast between the sleepers and the sublayer induces strong wall effects.

In this paper, we present a numerical study of the mechanical behavior of a thin granular interface in two dimensions by means of the contact dynamics method

(Moreau 1994; Jean 1999). We focus on the influence of thickness on shear strength and its relation with the microstructure. Our results suggest that friction with the walls plays a major role in resistance to settlement. In the presence of wall friction, highly localized rigid structures appear frequently and they support a large of fraction the vertical load. We show that the vertical strength is a decreasing function of thickness, and the force bearing structures become infinitely rigid in the vertical direction at a critical value of the thickness.

## 2 NUMERICAL PROCEDURES

In order to account for the faceted shapes of ballast grains, we use pentagonal grains in a two-dimensional space. The grains are confined inside a rectangular box with thickness  $H$  and length  $L$ . The two side walls are free to move horizontally, the base is fixed, and the top wall is subjected to a compressive load or displacement. The numerical sample is composed of approximately 1600 grains: 50% of diameter equal to 2.5 cm, 34% of diameter equal to 3.75 cm, 16% of diameter equal to 5 cm. The coefficient of friction between the grains is  $\mu_g = 0.6$ . We used a coefficient of friction  $\mu_w$  equal to 0 or 0.6 between the grains and the top and bottom walls.

Since we have a thin layer, a meaningful statistical information about the microstructure requires a long box, such that  $L > 4H$ . This ensures longitudinal homogeneity of the system. Nevertheless, as we shall see below, the stress transmission is highly

concentrated within a small portion of the layer. It is also important to note that in the case of railway ballast, the lateral stresses are induced by particle weights whereas the vertical stresses (below the sleepers) are due to the weight of the sleepers and dynamic loads by passing trains. These loads are far larger than grain weights. For this reason, we set the gravity to zero and the lateral confining stresses are imposed directly on the side walls.

For the simulations, we used the contact dynamics method on the LMGC90 platform developed in LMGC. This software allows for discrete-element and finite-element simulations of multibody materials with deformable or undeformable particles or contacts. Several results, presented here for a two-dimensional system, have been also established for a three-dimensional system by means of the same software using polyhedral particles (Saussine 2004).

### 3 VERTICAL STRENGTH

We first consider a compression test by imposing a downward velocity of  $-1$  cm/s on the top wall. The initial thickness of the layer is about 20 times the mean particle diameter  $D$ . In the following, all lengths will be normalized by  $D$ . We evaluate the strength of the system in terms of two quantities: 1) The normalized stress deviator  $q/p$  with  $q = (\sigma_1 - \sigma_2)/2$  and  $p = (\sigma_1 + \sigma_2)/2$ , where  $\sigma_1$  and  $\sigma_2$  are the principal values of the stress tensor over the whole volume of the layer including the contacts with the top and bottom walls. 2) The normalized difference of normal pressures  $R = (p_y - p_x)/(p_y + p_x)$ , where  $p_x$  is the average horizontal pressure and  $p_y$  is the average vertical pressure.

Figure 1 displays the evolution of  $q/p$  and  $R$  as a function of the thickness  $H$  for  $\mu_w = 0.6$ . We see that both quantities increase initially on average as  $H$  is decreased, and they tend to a constant value below  $H \simeq 14D$  up to fluctuations. For a representative elementary volume (far thicker than our system) the asymptotic value  $q/p \simeq 0.45$  would characterize the shear strength of the material. However, our system at  $H < 14D$  is not in a usual stress state since  $R \simeq 1$ ! In other words, the vertical load is supported with a nearly zero lateral pressure! This in turn implies that horizontal spreading (or extrusion) of the grains is hindered mainly by friction forces at the top and bottom walls. Indeed, under reasonable assumptions, it can be shown that  $R$  is equal to  $q/p$  if  $\mu_w = 0$ . The simulations with  $\mu_w = 0$  confirm this although  $R$  remains in all cases slightly above  $q/p$ ; see Figure 2. Hence, for the whole layer and in the presence of wall friction,  $R$  is a better descriptor of the vertical strength as compared to  $q/p$ .

The strong fluctuations of  $R$  and  $q/p$  reflect numerous instants of loss of stability that lead to the

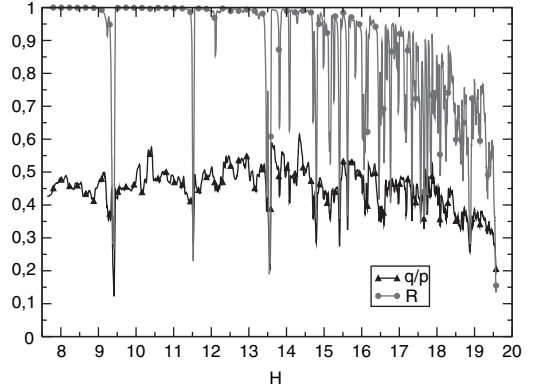


Figure 1. Evolution of the stress ratio  $q/p$  and the strength parameter  $R$  (see text) as a function of thickness  $H$  normalized by the mean particle diameter in a vertical compression test for  $\mu_w = 0.6$ .

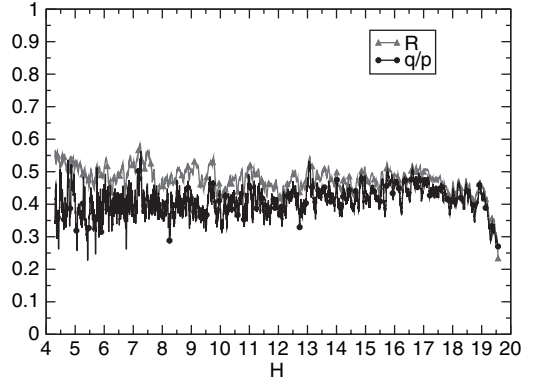


Figure 2. Evolution of stress ratio  $q/p$  and strength parameter  $R$  (see text) as a function of thickness in a vertical compression test for  $\mu_w = 0$ .

mobilization and demobilization of friction at the walls. These events correspond also to rapid rotations of principal stress directions while the major principal direction remains vertical on average.

### 4 FORCE TRANSMISSION

How does friction at the top and bottom walls influence the microstructure and stress transmission across the granular layer? The distribution of forces is known to be strongly inhomogeneous in granular media (Radjaï et al. 1999). In our thin granular layer, we observe still larger inhomogeneities. Most of time, nearly the whole vertical load is supported only by a few columnar structures. Figure 3 shows three snapshots of the central part of the layer. Both contact forces and highly

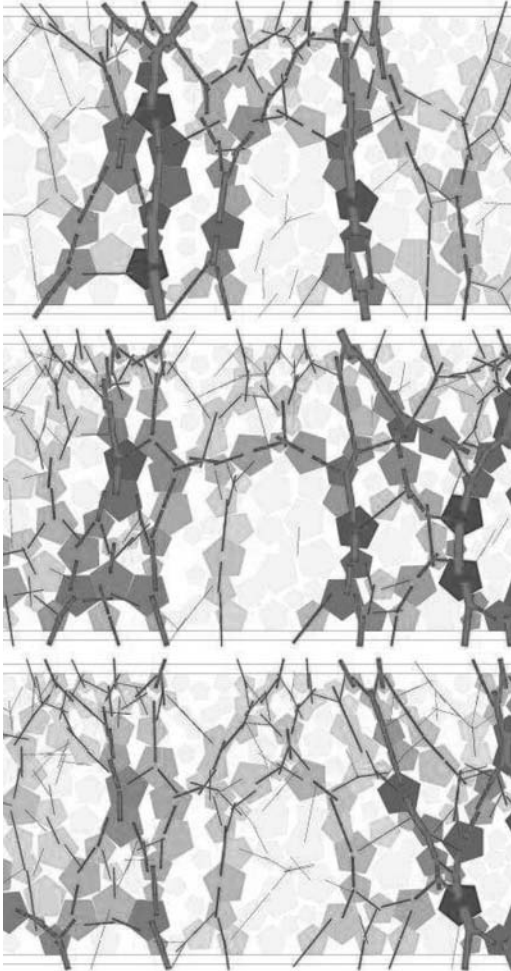


Figure 3. Snapshots of the main force-bearing structure at the center of the granular layer. Only highly loaded particles are shown in grey level. The line thickness is proportional to the force.

loaded grains have been displayed. The forces are much weaker in the rest of the layer. The forces are represented by segments centered on their application point, of width equal to the magnitude of the force and oriented along the force vector. We observe the strongest columnar force line to the left of the first snapshot. In the second and third snapshots, we can see that this column together with its “counterforts” disappears and a new structure is born to the right of the third snapshot. In the following, we will refer to these structures as “rigid localized structures” (RLS). The first snapshot of Figure 3 corresponds to a local peak of  $q/p$  as a function of  $H$  marked by a cross in Figure 4. This shows that the low-frequency oscillations of  $q/p$  reflect the dynamics of the RLS.

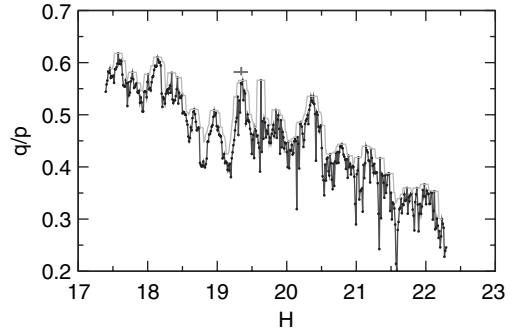


Figure 4. Evolution of  $q/p$  as a function of thickness  $H$ . The envelop of the maxima of  $q/p$  over small intervals of  $H$  is shown. The first snapshot of Fig. 3 corresponds to the peak marked by a cross.

The RLS are mostly composed of consecutive line (side-to-side) contacts. Nearly 20% of particles in the whole sample have two line contacts (involved mostly in the RLS) whereas 40% of particles have no line contact at all. The coordination number is about 2.2 and the fraction of line contacts is 42%. Although the line contacts are less frequent, they support on average a normal force that is 20% larger than the mean normal force. Figure 5 shows the probability density function  $P$  of normal forces for line contacts and point contacts. We see that  $P$  is larger for line contacts than for point contacts at large forces, and this trend is inverted below the mean force. The largest forces have a nearly exponential distribution. The forces below the mean force seem to follow a power law distribution. This power-law behavior of weak forces can be observed in a representative elementary volume but with a less pronounced divergence as the force goes to zero (Radjai et al. 1999).

The same structures can be observed also for  $\mu_w = 0$  but at very low thickness (around 5 particle diameters) and with a slightly different morphology. The stability of the RLS is thus ensured, for the most part, by wall friction. That is how friction with the walls works in a thin granular interface to sustain vertical loads.

## 5 INFLUENCE OF THICKNESS

In axial compression by downward displacement, the parameter  $R$  remains in the vicinity of 1 for  $H < 14D$  but the deformation of the sample continues in spite of wall friction and rigid structures. This is because the displacement is imposed numerically and the particle rearrangements can still take place due to (weak) particle interpenetrations which often lead to the destabilization of the LRS. However, the LRS seem so rigid that we expect them to support much higher loads.

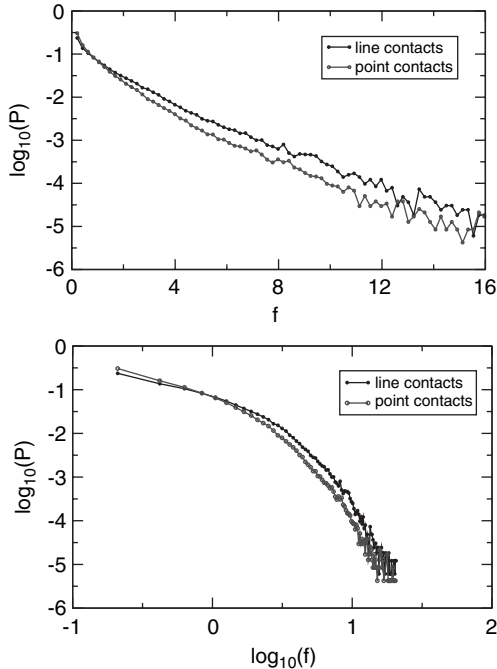


Figure 5. Probability density function  $P$  of normal forces in log-linear and log-log scales.

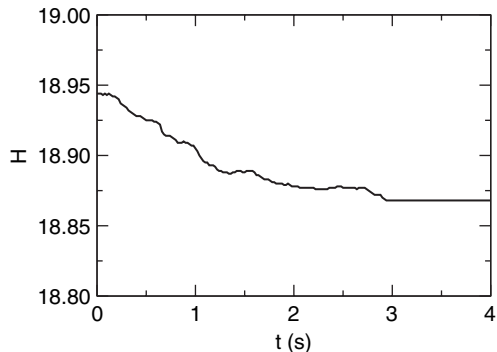


Figure 6. Evolution of thickness  $H$  as a function of time  $t$  in a relaxation test.

In order to evaluate the strength of the LRS, we performed four successive relaxation tests. We start with an initial thickness  $H_0 = 19D$  and we add a high vertical overload  $p_y = 10p_x$  on the top wall. The granular layer flows under the action of the overload and relaxes in a few seconds to a new equilibrium state. The thickness decreases slightly as shown in Figure 6. Then, we start a new relaxation test from the equilibrium state reached at the end of the first test by multiplying  $p_y$

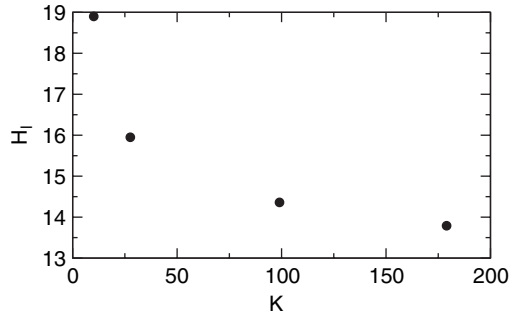


Figure 7. The locking thickness  $H_l$  (see text) normalized by the mean particle diameter as a function of the overload  $K = p_y/p_x$  in four relaxation tests.

by 4. Again, the material flows under the action of the new load and a new equilibrium state is reached at  $H \simeq 16D$ . We repeat the same tests two more times by multiplying the overload by 4 and then, for the last test, by 2. To each value  $K \equiv p_y/p_x$  of the overload corresponds a “locking thickness”  $H_l$ , the thickness reached after relaxation. Figure 7 shows  $H_l$  as a function of  $K$ .  $H_l$  decreases with  $K$  and tends to  $H_c = 14D$ . The latter corresponds thus to transition to a “locked” state. This transition was also observed in Figure 1.

These results suggest that the vertical strength  $R$  of a thin granular layer is a decreasing function of thickness. The highest strength is practically equal to 1 for a critical thickness  $H_c \simeq 14D$  in our system. This thickness is a function of wall friction and the aptitude of the particles (their shapes and size distribution) to set up stable force-bearing structures.

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## REFERENCES

- Alias, J. 1984. *La voie ballastée – Techniques de construction et d’entretien*. Paris: Eyrolles.
- Jean, M. 1999. *The Non Smooth Contact Dynamics method*. Computational Methods in Applied Mechanics Engineering.
- Moreau, J.J. 1994. Some numerical methods in multibody dynamics: application to granular materials. *Eur. J. Mech. A* 13: 93.
- Oviedo, X. 2001. *Thèse de doctorat, “Etude du comportement du ballast par un modèle micromécanique – Application aux opérations de maintenance de la voie ferrée ballastée”*. Marne-La-Vallée, France: ENPC.
- Radjaï, F., Roux, S. & Moreau, J.J. 1999. Contact forces in a granular packing. *Chaos* 9(3): 544–550.
- Saussine, G. 2004. *Thèse de doctorat: “Contribution à la modélisation de granulats tridimensionnels: application au ballast”*. Montpellier, France: Université Montpellier II.