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# Vibrational motion of the radical cation $C_3^+$ in its degenerate electronic ground state

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The cation  $C_3^+$  plays an important role in astrophysical reactions but so far it has not been characterized spectroscopically. Theory has established a complicated degenerate electronic ground state with a conical intersection of  $D_{3h}$  structure and three local minima of  $D_{\infty h}$  structure. The small isomerization energy is not well determined with estimates ranging from 3 to 7 kcal/mol.

In this paper the first attempt is made to generate a reliable global potential energy surface. The MR-CI method is used to cope with a very strong multi-configurational electronic structure. Including Davidson corrections and complete basis extrapolation, we predict a Jahn-Teller stabilization energy of 1334  $\text{cm}^{-1}$ , a pseudorotation barrier of 376  $\text{cm}^{-1}$ , a well depth of the linear minima of 289  $\text{cm}^{-1}$  and an electronic isomerization energy of 2238  $\text{cm}^{-1}$  (6.4 kcal/mol).

An analytical form of the potential surface is used in variational calculations of low-energy vibrational motions which are based on hyperspherical coordinates. Vibronic coupling is treated in a diabatic framework for the J-T region as well as the near-linear region. We find a unique mixture of vibrational states which are either located around the three cyclic minima or the three linear minima, or they are global in the sense that they fill the energy valleys between these minima. Assignments are proposed in terms of local vibrator quantum numbers. Zero point energies in the cyclic global and linear local minima are 1626  $\text{cm}^{-1}$  and 1195  $\text{cm}^{-1}$ , respectively, i.e. the isomerization energy is reduced by vibrational effects to 1668  $\text{cm}^{-1}$  (4.75 kcal/mol).

## I. INTRODUCTION

The cation  $C_3^+$  has been intensively investigated since it was suggested that it may play a crucial role in the process of forming larger homonuclear carbon clusters in interstellar space via hydrogenation steps [1]. The room temperature reaction rate of the hydrogen abstraction reaction  $C_3^+ + H_2 \rightarrow C_3H^+ + H$  was found abnormally low as compared to the rates of other  $C_n^+$  ion clusters [2, 3] which has been tentatively linked to a cyclic  $C_3^+$  structure. Recent ion trap studies revealed a surprisingly steep increase of the reaction rate down to about 50 K where it suddenly levels off [4]. At this low temperatures even the radiativ association reaction  $C_3^+ + H_2 \rightarrow C_3H_2^+ + h\nu$  becomes competitive. So far there is no well-based explanation of this inverse temperature dependence, speculation envisions either a long-living precursor complex or a structural peculiarity of  $C_3^+$  such as very facile bent-to-linear isomerization.

Apart from these reaction rates, there are indeed only very few experimental data available. Coulomb explosion experiments [5] have been interpreted as indicating a cyclic structure in contrast to older theoretical predictions of a linear structure. While early measurements of the first IP of  $C_3$  left a range from 11 to 13 eV [6, 7], the recently measured photoionization efficiency curve of  $C_3$  [8] shows a clear step for vertical ionization at 11.70  $\pm 0.05$  eV and in particular a rather long tail to lower energies which is believed to indicate unresolved vibrational states of a bent  $C_3^+$ . However, a firm experimental determination of its structure and a detailed spectroscopic characterization of  $C_3^+$  are still missing.

Sunil et al [9] reported the first careful theoretical investigation of ionization potentials of  $C_3$  and noted a

particularly strong role of electron correlation for the properties of (linear)  $C_3^+$ . It can be traced to rather low-lying nonbonding  $1\pi_g$  orbitals which are not occupied in the Hartree-Fock (HF) configurations of the  $^2\Sigma_u^+$  and  $^2\Sigma_g^+$  states,  $4\sigma_g^2 3\sigma_u 1\pi_u^4$  and  $4\sigma_g 3\sigma_u^2 1\pi_u^4$ , respectively. They give rise to large left-right correlation via strong admixture of pair excitations into  $1\pi_g^2$  or  $\sigma_{u,g} 1\pi_g$ . This results in a reordering of the electronic states by moving the  $^2\Sigma_u^+$  and  $^2\Sigma_g^+$  states below the  $^2\Pi_u$  state, the lowest at the HF level. It turned out that SD-CI still gave a wrong ordering of states and the Moller-Plesset perturbation series  $MP_n$  showed large oscillations of the correlation energies in particular from triple excitations. Only multi reference configuration treatments - MC-SCF as well as MR-CI - produced the correct ordering and a first IP of 11.5 eV in good agreement with the recent experimental value quoted above.

The observation of a complex electronic structure of  $C_3^+$  and the suggestion of a cyclic structure [5] triggered a rush of high-level *ab initio* calculations in which all possible methods and a large number of basis sets were explored, with the main purpose to ascertain the bent-to-linear isomerization energy (which we henceforth abbreviate by  $\Delta E^e$ ). Raghavachari [10] used the quadratic configuration interaction method (QCISD(T)) with a relatively small basis set and was the first to predict a bent  $^2B_2$  global minimum with apex angle of 67°. This minimum is only 3 kcal/mol lower than the  $^2\Sigma_u^+$  minimum of linear  $C_3^+$  but separated from it by a barrier which rises about 4 kcal/mol above the linear minimum. An imaginary force constant for the asymmetric stretch mode indicated symmetry breaking even at the level of a restricted QCISD(T). Applying a similar basis and the same method, but based on a UHF reference function, Martin et al [11] proposed 1.9 kcal/mol as best estimate

1 for  $\Delta E^e$  and stressed that this value would be further  
2 reduced by a zero point energy (ZPE) difference of about  
3 0.7 kcal/mol. Grev et al. [12] tried the single reference  
4 CI method, exploring substitution levels up to quadrup-  
5 les and various active orbital spaces to conclude that  
6 the  $\Delta E^e$  should be about 4 kcal/mol, however with an  
7 uncertainty of  $\pm 4$  kcal/mol. They found a small imag-  
8 inary bending frequency for the  ${}^2\Sigma_u^+$  state indicating a  
9 direct decent to the  ${}^2B_2$  minimum and also report sym-  
10 metry breaking for the HF wavefunctions of the  ${}^2\Sigma_u^+$  and  
11  ${}^2B_2$  states which prevails for SD-CI.

12 The newly established singles and doubles coupled  
13 cluster method with perturbative triples (CCSD(T)) was  
14 applied by Scuseria [13] with a focus on basis set sat-  
15 uration. Using atomic natural orbitals in symmetry-  
16 restricted calculations,  $\Delta E^e$  was found to increase mono-  
17 tonically with basis set size and reached 6.8 kcal/mol  
18 for the largest set. The performance of the CCSD(T)  
19 was investigated by Watts et al. [14] trying out various  
20 schemes for handling triple excitations, including the full  
21 CCSDT. They encountered variations up to 5 kcal/mol  
22 and concluded that the generally accepted CCSD(T) ap-  
23 proximation artificially favors the linear structure by 2.0  
24 kcal/mol. The results for the antisymmetric stretch  
25 frequency were indicative for the problems of single-  
26 reference methods with  $C_3^+$ : It changed from 1431i for  
27 CCSD(T) to 451i for CCSDT but was real for other vari-  
28 ants.

29 Taylor et al. [15] stressed the strong multi-reference  
30 character of correlation in the lower electronic states of  
31  $C_3^+$  as the primary cause of the problems. They per-  
32 formed a series of CASSCF/MRCI calculations with a  
33 full-valence active orbital space, state-averaged orbital  
34 optimization, reference configuration selection with vary-  
35 ing thresholds for their coefficients and optionally ap-  
36 plying a multireference analogue of the Davidson correc-  
37 tion. Their best calculation yielded a  $\Delta E^e$  value of 5.18  
38 kcal/mol. A critical evaluation of various energy incre-  
39 ments and an estimate for the deficiency of their basis  
40 set data led them to propose an energy difference of 5.2  
41 kcal/mol as best estimate, with a confidence interval from  
42 4.2 to 6.7 kcal/mol. This nicely includes the estimate of  
43 ref. [12] at the lower end and the best calculated value  
44 of ref. [13] at the upper end and it has not been chal-  
45 lenged since. It should not be overlooked, however, that  
46 they encountered imaginary frequencies for antisymmet-  
47 ric stretch of the linear species even at their full-valence  
48 CASSCF level.

49 The problems encountered with all the methods tried  
50 seem to have so far prevented any investigation of larger  
51 parts of the ground state energy surface. Only har-  
52 monic frequencies have been reported in some of the  
53 work discussed above, as they come along with geometry  
54 optimization. However, for bending and antisymmetric  
55 stretch vibrational modes they are of very limited use in  
56 view of the obvious strong anharmonicities. Symmetric  
57 stretch frequencies for the linear form vary between 1154  
58  $\text{cm}^{-1}$  from CASSCF [15] and 1261  $\text{cm}^{-1}$  from SDCI [12],

not very different from the experimental  $C_3$  frequency of  
1225  $\text{cm}^{-1}$ . For the bent form, symmetric stretch fre-  
quencies reported from well correlated treatments vary  
between 1590 and 1724  $\text{cm}^{-1}$ . The only attempt to pro-  
vide a global potential energy surface has recently been  
made by Wang et al. [16]. An analytical expression was  
derived from a straight application of the many-body ex-  
pansion theory [17] of polyatomic interaction potentials.  
The parameters were taken from potential curves of the  
dissociation products  $C_2$  and  $C_2^+$  and from the harmonic  
force constants pertaining to the  ${}^2B_2$  global minimum.  
Since no reference was made to the peculiarities of the  
electronic structure of  $C_3^+$  this analytical surface can not  
be expected to be really useful.

For the sake of completeness we mention some recent  
theoretical results which have been generated in the con-  
text of experimental work. McAnoy et al. [18] performed  
single-point CCSD(T) calculations with an aug-cc-pVTZ  
basis, albeit for B3LYP optimized geometries. While the  
 $\Delta E^e$  of 5.7 kcal/mol does fit into the picture of previous  
work, a rather high barrier of 2.9 kcal/mol is suggested.  
Fura et al. [19] applied a quite similar but unrestricted  
scheme to a larger number of ion clusters and reported a  
rather small  $\Delta E^e$  of only 1 kcal/mol. Nicolas et al.  
[8] presented CASSCF/MR-CI potential curves along the  
bending coordinate while fixing the CC bond length to  
a value favouring the linear form. Thus, the suggested  
 $\Delta E^e$  of 2.2 kcal/mol as well as a flat, suspiciously looking  
hump may be artifactual.

All these variations in the results for  $\Delta E^e$  and the  
transition state barrier indicate that great care has to be  
taken if a potential surface is to be generated that can  
be trusted. It is the purpose of the present paper to es-  
tablish a global potential energy surface (PES) which is  
reliable up to some lower vibrational levels of linear  $C_3^+$   
, i.e. the conformation in which it is most likely formed.  
Both sheets of the adiabatic PES are obtained from MR-  
CI calculations in numerical form on a grid that covers all  
conformations which are accessible within the given en-  
ergy range (Section II). These data are used to generate  
an analytical PES suitable for integration with global vi-  
brational basis functions (Section III). A transformation  
based on a J-T parametrization of the potential in the  
cyclic region and on the dipole moment matrix in the  
linear region provides a PES in diabatic form for efficient  
account of vibronic coupling (Section IV). Vibrational  
states are finally obtained from variational calculations  
which use hyperspherical coordinates for proper treatment  
of symmetry requirements and boundary conditions at  
 $D_{3h}$  and  $D_{\infty h}$  geometries. They are characterized in  
terms of their distribution over the cyclic and linear do-  
mains and in terms of approximate vibrational quantum  
numbers (Section V).

With respect to analytical fits and vibrational states  
we follow the technical procedures previously applied to  
degenerate states of  $\text{Na}_3$  [20] and  $\text{Li}_3$  [21–23]. Measured  
vibrational band positions of excited  $E''$  states could be  
reproduced to within 2  $\text{cm}^{-1}$  and excellent agreement

with rotational resolved spectra was found. Of course, electronic structure calculations were facilitated by the small number of valence electrons and vibrational motion turned out to be confined to the lower sheets of the  $E''$  states. In the more recent analysis of vibrational structures in the  ${}^2E' \leftarrow {}^2A'_1$  absorption spectrum of  $B_3$  we arrived at a satisfactory simulation of a spectrum taken in a Ne matrix, but it was difficult to draw conclusions about the accuracy achieved for vibrational levels [24]. In the case of  $C_3^+$  the low-lying conical intersection and the presence of the three local linear minima will substantially complicate the vibrational analysis. Since there is not yet a convincing suggestion of how  $C_3^+$  will eventually be seen spectroscopically we restrict the present investigation to a vibrational analysis of the ground state. It is hoped that this might help understand the unexpected features of its reactivity.

## II. AB INITIO ELECTRONIC STRUCTURE CALCULATIONS

From the discussion of the previous investigations of  $C_3^+$  it seems clear that a global PES requires a computational procedure which is based on a multi-configuration reference wavefunction. Lacking a multi-state, multi-configuration coupled cluster program, we have decided to use the multi-reference configuration interaction method (MR-CI) as implemented in the MOLPRO program package [25]. Its particular strength is the efficiency derived from the concept of internally contracted configurations, i.e. singles and doubles substitution operators are applied to the reference function as a whole. This was first formulated by Meyer [26] but developed into an fully operative algorithm and efficiently implemented by Werner and Reinsch [27].

The reference function is obtained from CAS-SCF calculations. The definition of the active orbital space is not trivial in the present case and no perfect solution has been found in test calculations along the minimum energy path (MEP) between the  $D_{3h}$  and  $C_{2v}$  conformations. The reason is the C-C bond breaking upon opening the apex angle from  $60^\circ$  to  $180^\circ$  which is - as well known for multiple bonds in diatomic molecules, e.g.  $N_2$  [28] - connected with a series of avoided crossings of Rydberg-type with valence-type orbitals and, consequently, crossings of the corresponding configurations. This is seen in Fig. 1 which shows CAS-SCF potential curves of the lower excited states of  $C_3^+$ . The optimized orbitals for a preselected set of states therefore undergo rather sudden changes which may affect also the PES of lower-lying states. The resulting humps may be rather large at the CAS-SCF level (e.g.  $100\text{-}200\text{ cm}^{-1}$  for the lower component of the ground state). They are effectively reduced at the MR-CI level but may still be at a disturbing size of  $10\text{-}20\text{ cm}^{-1}$ . Since extending the set of states did not improve the situation significantly, we restricted this set to the two components of the ground state which is suf-

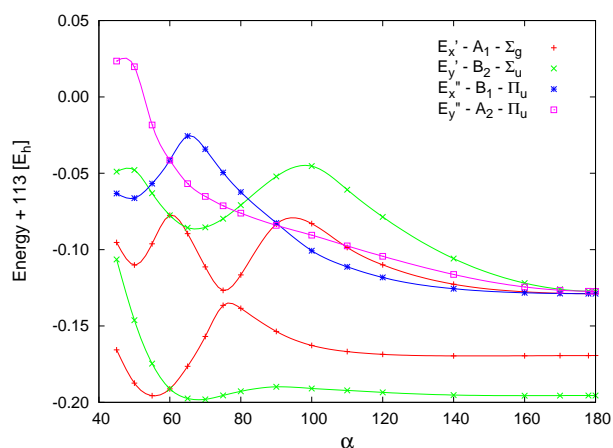


FIG. 1: Cuts of PESs of lower electronic states of  $C_3^+$  along the ground state MEP, from CAS-SCF with basis AVTZ

ficient to avoid the unphysical symmetry breaking. The avoided crossing of the upper component with the next higher state appears at about  $75^\circ$  of the apex angle, i.e. in a region where the upper component is not significant in itself for the low-energy vibrations. Thus, the effect of the crossing could be diminished by introducing an angle-dependent weighting of the two components which reaches a 1:1 ratio only above  $100^\circ$ . After some experimenting we found as best compromise a set of 13 active orbitals (more precisely  $11a'$ ,  $3a''$  orbitals in  $C_s$  symmetry or  $6a_1$ ,  $5b_2$ ,  $2b_1$ ,  $1a_2$  orbitals in  $C_{2v}$  symmetry), which is one orbital in excess of the full-valence set used previously. In order to reduce computing expense, an occupation restriction was imposed to the configurations of the reference function in the MR-CI: only two electrons were allowed to occupy simultaneously the four uppermost active orbitals. This restriction has an insignificant effect on the surface even though it may introduce an additional risk for small bumps.

Standard correlation consistent basis sets are used throughout [29]. For the full set of grid points, the cc-pVQZ basis was used [25]. For a significantly smaller subset of points, the VTZ and V5Z basis sets were applied to perform complete basis set extrapolation (CBS) [30]. Since dynamic electron correlation is significantly larger for the more compact cyclic structures than for linear structures, basis set deficiencies are also more serious there. Thus, CBS extrapolation increases  $\Delta E^e$  by 13%, up from  $2125\text{ cm}^{-1}$  to  $2401\text{ cm}^{-1}$ . The truncation of the configuration space inherent to a SD-MR-CI expansion has the opposite trend since configuration mixing is stronger for linear structures. Thus, approximately account for higher substitutions by Davidson's correction as implemented in the MR-CI of MOLPRO, lowers  $\Delta E^e$  by  $164\text{ cm}^{-1}$  to our final value of  $2238\text{ cm}^{-1}$ . Adding up both extrapolations leads to a bending potential curve which is quite similar to the uncorrected one. Still, the small barrier for bending changes from  $359\text{ cm}^{-1}$  to  $289$

$\text{cm}^{-1}$  upon the extrapolations. After corrections, the Jahn-Teller (JT) stabilization energy is predicted as  $1334 \text{ cm}^{-1}$  and the pseudorotation barrier has a height of only  $376 \text{ cm}^{-1}$ .

Potential curves along the MEP for apex angles between  $50^\circ$  and  $180^\circ$  are shown in Fig. II. It includes also results from standard CCSD(T) calculations. This

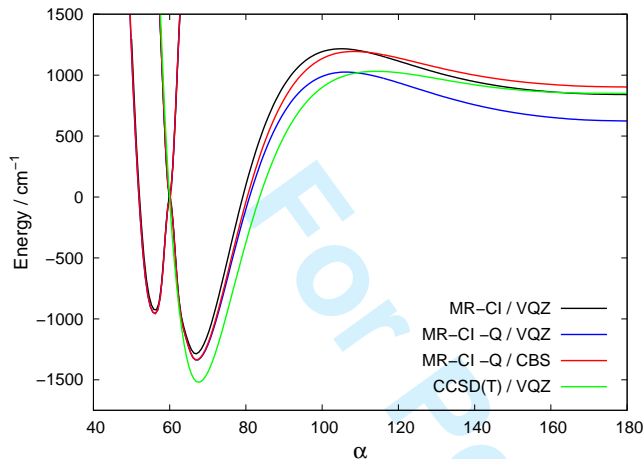


FIG. 2: Cut of ground state PES along the MEP of the lower sheet from fits of MR-CI energies without and with Davidson correction (-Q) or/and CBS extrapolation. Also included is the CCSD(T) PES (cut along an approximate MEP)

method accounts better for dynamical correlation than uncorrected MR-CI but underestimates the effects from near-degenerate configuration mixing. Both differences add up, yielding after CBS extrapolation an  $\Delta E^e$  isomerization energy of  $2660 \text{ cm}^{-1}$  and in particular a strongly reduced linear-to-cyclic barrier of only  $40 \text{ cm}^{-1}$ . We note again that the CCSD(T) treatment converges nicely for  $C_{2v}$  geometries (as those of Fig. II) provided the occupied orbitals obey  $C_{2v}$  symmetry. But it is difficult to devise a proper one-determinant wavefunction for general  $C_s$  geometries: convergence is not stable and may end up in unphysically distorted wavefunctions.

The much discussed question of the stability of the linear minima with respect to asymmetric stretch deformation is elucidated in Fig. 3 and seen to be somewhat academic. The harmonic force constant turns out to be very small indeed: It is weakly positive at equilibrium bond distances but soon changes sign with symmetric bond elongation, i.e. we find a valley-to-ridge transition. The effective force constant related to the fundamental frequency for asymmetric stretch is then largely determined from the positive quartic force constant.

The molecular constants derived from the MR-CI PES are collected in Tab. II. The structural parameters agree quite well with the better calculations of the literature. Our value for the  $\Delta E^e$  of  $2237 \text{ cm}^{-1} = 6.4 \text{ kcal/mol}$  is within the confidence range proposed by Taylor et al. [15] but close to its upper end. We are certain that the

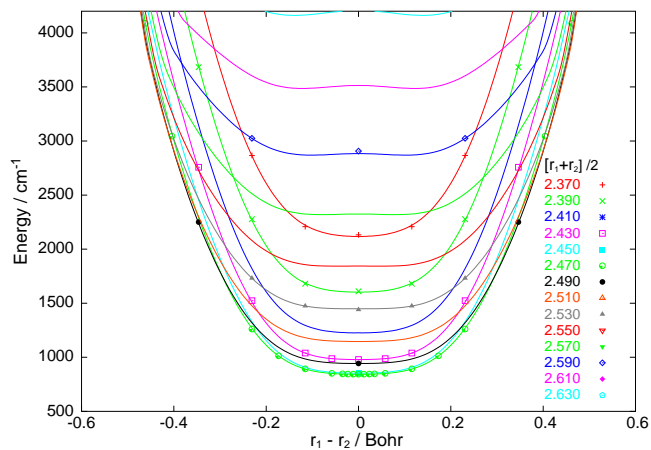


FIG. 3: Cuts of the  $X^-$  PES for asymmetric stretch deformation of linear  $C_3^+$ ; calculated energies (points) and fitted PES (solid lines)

TABLE I: Characteristic points of the  $X^2E'$  PES.

Methods and Basis sets: a: this work, MR-CI/cc-pVQZ; b: this work, MR-CI-Q/CBS; c: [10] QCISD(T)/6-31G\*; d: [12] CISD(TQ)/TZ2P; e: [13] CCSD(T)/5s4p3d2f1g; f: [15] MR-CI-Q/5s3p2d1f. Numbers in brackets: ZPE from harmonic frequencies.

		$E + 113$	$E_r$	$r_{1,2}$	$\alpha$	ZPE	$\omega_1(a_1)$	
		$E_h$	$\text{cm}^{-1}$	$\text{\AA}$	$^\circ$	$\text{cm}^{-1}$	$\text{cm}^{-1}$	
$C_{2v}$	a	-0.42535	0	1.322	66.9	1692	1785	
	min	-0.44094	0	1.321	67.1	1626	1780	
	$^2B_2$	c	-0.40392	0	1.333	67.2	(1811)	1684
		d	-0.29780	0	1.296	71.0	(1759)	1724
	e	-0.43724	0	1.324	67.8			
	f	-0.32590	0	1.324	68.0	1675	1612	
$C_{2v}$	a		357	1.391	55.9			
	sadd		376	1.391	55.8		1760	
$D_{3h}$	a		1278	1.358	60.0			
	min		1334	1.358			1792	
$C_{2v}$	a		2487	1.308	104.7			
	b		2526	1.308	110.1		1465	
			2432					
	f		2026					
$D_{\infty h}$	a		2120	1.303	180.0	1251	1190	
	min		2237	1.303		1195	1188	
$^2\Sigma_u^+$	c		1051	1.318				
			1400	1.283			1261	
	e		2383	1.307				
	f		1372	1.314			1154	
$^2\Sigma_g^+$	b	$T_v$	6035					

barrier between linear and bent structures is real, but it is difficult to qualify the accuracy of the  $289 \text{ cm}^{-1}$  obtained, given the large absolute amounts of CBS and Davidson extrapolations of together about  $4700 \text{ cm}^{-1}$ .

### III. ANALYTICAL REPRESENTATION OF POTENTIAL ENERGY SURFACES

MR-CI calculations as just described have been performed for a grid of non-redundant conformations defined by varying the bond lengths in steps of  $\Delta r = 0.05$  Bohr, starting from  $r = 2.56$  Bohr, which is close to  $r$  at the  $D_{3h}$  geometry with minimum energy, and stopping only after the relative energy exceeded  $4000 \text{ cm}^{-1}$  above the  $D_{3h}$  minimum. With some extra points for bending from linear conformations, this amounted to 250 grid points.

As outlined in section V, the variational calculation of vibrational states involves basis functions that are global in the sense that they span the complete space of the cyclic hyperspherical coordinates  $\theta$  and  $\phi$ , e.g. harmonic functions of  $\phi$  for pseudorotation. Numerical integration of the potential then requires substantial extension beyond the ranges covered by the grid of the electronic structure calculations. Therefore, we like to have the potentials in an analytical form which is well-behaved everywhere. Moreover, since finite basis sets are used which do not allow a vibrational wavefunction to be exactly zero over finite ranges in coordinate space, we have to make sure that the potential does nowhere assume very large values, including the ranges where two C atoms come very close to each other.

In previous applications to  $A_3$  molecules (with JT distorted potentials) we found it convenient to base an analytical expression on the three bond distances since they are most directly related to bonding forces. In order to further adapt the coordinates to the typical shape of bonding potentials, we introduced 'Morse coordinates' defined as  $\tilde{r} = (1 - \exp(-\beta(r - r_o)))/\beta$  [31]. This transforms a Morse potential into a parabola with a finite range for positive  $\tilde{r}$  and usually allows to represent a bonding potential by a low-order polynomial of  $\tilde{r}$ . As compared to functions of the apex angle, the steep  $C_3^+$  potentials around  $60^\circ$  are widened and the shallow potentials above  $130^\circ$  are compressed. Thus,  $\beta$  has to be chosen with care so that bending forces can be accounted for also in the linear domain. For the present case we found  $\beta = 0.90$  suitable.

The coordinates  $\tilde{r}_i$  are combined to form symmetry-adapted coordinates in the usual form:

$$\tilde{r}_a = (\tilde{r}_1 + \tilde{r}_2 + \tilde{r}_3)/\sqrt{3}, \quad (3.1)$$

$$\tilde{r}_x = (2\tilde{r}_3 - \tilde{r}_1 - \tilde{r}_2)/\sqrt{6}, \tilde{r}_y = (\tilde{r}_1 - \tilde{r}_2)/\sqrt{2} \quad (3.2)$$

$$\tilde{r}_e = (\tilde{r}_x^2 + \tilde{r}_y^2)/\sqrt{2}, \tilde{\phi} = \arctan(\tilde{r}_y/\tilde{r}_x) \quad (3.3)$$

Symmetry is solely carried by functions of  $\phi$ :  $\cos(k\tilde{\phi}), \sin(k\tilde{\phi})$  form a pair of  $E'$  species if  $(k \bmod 3) \neq 0$ , otherwise they are  $A'_1 + A'_2$  species. Since the adiabatic

PES belong to the  $A'_1$  representation, the following linear ansatz for the potential is appropriate:

$$V = \sum_{ijk} v_{ijk} \tilde{r}_a^i \tilde{r}_e^j \cos(3k\tilde{\phi}) \quad (3.4)$$

$$i, j, k = 0, 1, 2, \dots; i + j \leq n; k \leq k_{max}(j) \quad (3.5)$$

$\tilde{r}_e$  is by definition non-negative and is zero only for  $D_{3h}$  geometries. At these points, the two components of an  $E'$  state PES are smooth continuations of each other if taken as functions of  $\tilde{r}_x$  and  $\tilde{r}_y$  due to the fact that they are eigenfunctions of a regular Hamiltonian. Thus, the ansatz above can describe both components with a single set of  $v_{ijk}$  if the range of  $\tilde{r}_e$  is simply extended to negative values, which then correspond to the upper component PES. In the close neighborhood of  $\tilde{r}_e = 0$ ,  $v_{ijk}$  should vanish for  $3k > j$  but we only enforce this for  $j = 1$ . Admittedly, such a one-center polynomial expansion, which worked very well for our previous cases  $Li_3$  and  $Na_3$ , is somewhat under stress in the present case where linear geometries matter which have little to do with the conical intersection. In order to ensure sufficient flexibility, we adopted the pragmatic choice of  $k \leq 0, 1, 4$  for  $j \leq 1, j = 2, j \geq 3$ , respectively. The linear ansatz has the obvious advantage that it can easily handle a large number of parameters. It has been preferred over a non-linear fit in terms of the elements of a  $2 \times 2$  diabatic potential matrix because the upper potential has less quality than the lower. In the present case we have chosen  $n=8$  and  $i, j \leq 7$ . The total number of terms then amounts to 132. It should be noted that powers of up to 7 in a polynomial expansion are acceptable here because the relevant ranges of  $\tilde{r}_a$  and  $\tilde{r}_e$  are effectively limited to little more than 1 by the Morse transformation, and extended extrapolation is regularized as described below.

For a sensible least-squares fit the input energies should be weighted according to their relevance. We have chosen the weight function  $w(e) = (1 + e/e_0)^{-2}$ , where  $e = E - E_{min}$ , i.e. squares of errors of energy differences are minimized for  $e < e_0$  but squares of relative errors for  $e > e_0$ . Above a certain threshold,  $e > e_{max}$ , the weight is further reduced by the linear cut-off factor  $(2 - e/e_{max})$  and it is set to zero above  $2e_{max}$ . We have chosen  $e_0 = 3000 \text{ cm}^{-1}$  and  $e_{max} = 5000 \text{ cm}^{-1}$ , respectively. Around  $\alpha = 75^\circ$  a small ridge in the error surface was observed (see discussion in previous chapter) and the weights were reduced to half in that region. The final result comes with a rms error of  $4.5 \text{ cm}^{-1}$  for fitting the weighted MR-CI energies while the MR-CI-Q energies are fitted with an rms error of 2.9. (The Davidson correction reduces the ridge just mentioned.) Largest deviations amount to less than  $15 \text{ cm}^{-1}$ . The quality of the fit can also be judged by inspection of Fig. 3 for the most critical region around linear  $C_3^+$ .

We need to deal with the fact that there are large regions of conformation space for which the PES is not controlled by calculated energies and for which the polynomial fit may go wild. Extending the grid does not help since the range of the fit can not easily be extended



without loss of quality in the relevant region. Therefore, the analytical fit needs to be regularized in the regions which are beyond the grid but are accessed by (our) vibrational basis functions. The transformed coordinates  $\tilde{r}_a$  and  $\tilde{r}_e$  are quite convenient for simple regularizations [20–22, 24, 31]: It is usually possible to determine an ellipsoidal cut-off boundary with respect to a reference point  $\tilde{r}_{ao}, \tilde{r}_{eo}$  which encloses all regions where the PES is low and well represented by the fit. Beyond this boundary the PES may be defined with reference to its values at the boundary or close to it. E.g., if the boundary is defined by requiring

$$D^2(\tilde{r}_a, \tilde{r}_e) = d_a \tilde{r}_a^2 + d_e \tilde{r}_e^2 + 2d_{ae} \tilde{r}_a \tilde{r}_e = \tilde{r}_{cut}^2 \quad (3.6)$$

where  $\tilde{r}_a = \tilde{r}_a - \tilde{r}_{ao}$  and  $\tilde{r}_e = \tilde{r}_e - \tilde{r}_{eo}$ , the external potential may simply be derived by radial scaling of the coordinates  $\tilde{r}_a, \tilde{r}_e$  with  $t = \tilde{r}_{cut}/D(\tilde{r}_a, \tilde{r}_e)$ :

$$E(\tilde{r}_a, \tilde{r}_e, \phi) = E(\tilde{r}_a t + \tilde{r}_{ao}, \tilde{r}_e t + \tilde{r}_{eo}, \phi)(2 - t^2) \quad (3.7)$$

This scheme has been chosen here. In addition, the potential is not allowed to exceed an energy  $E_{max}$ , chosen here as 14000 and 19000  $\text{cm}^{-1}$  for lower and upper surface, respectively. It has been made sure that the calculated vibrational energies are not affected by reasonable changes of all of these parameters.

#### IV. VIBRONIC COUPLING

In the region of the Jahn-Teller well there is significant vibronic coupling. In adiabatic framework the electronic coupling factor  $\langle \Phi_2^a | \partial/r \partial \phi | \Phi_1^a \rangle$  has a pole at the conical intersection,  $r \rightarrow 0$ , which has to be removed by the vibrational factor  $\langle \chi_2^a | \partial/r \partial \phi | \chi_1^a \rangle$ . For practical reasons, among them program limitations, we prefer a diabatic framework in which the vibronic coupling turns into simple potential coupling. Diabatic electronic states are the specific combinations of the two adiabatic states which have minimal dependence on the angle  $\phi$ . Applying a 2x2 rotation matrix the two adiabatic PESs  $V_+^{ad}, V_-^{ad}$  from the MR-CI calculations turn into the diabatic 2x2 matrix PES

$$2V^d = V_+^{ad} + V_-^{ad} + (V_+^{ad} - V_-^{ad}) \begin{pmatrix} \cos \delta & \sin \delta \\ \sin \delta & -\cos \delta \end{pmatrix}$$

where  $\delta$  is twice the angle for rotating the wavefunctions. The diabatic wavefunctions may be chosen to be identical to the adiabatic ones,  $\delta = 0$ , at the  $C_{2v}$  geometries linked to  $\phi = 0$ . In the cyclic region around the conical intersection, the angle  $\delta$  can be determined from the familiar expansion of the Hamiltonian in a power series of the polar deformation coordinate  $r$  [32, 33]. Since the diabatic wavefunctions  $\Phi_x^d, \Phi_y^d$  form a pair of  $E'$  symmetry, their Hamiltonian matrix elements generate the representation  $A'_1 \oplus E'$  and may be approximated up to cubic

terms in  $r$  by

$$H_{xx} + H_{yy} = a_2 r^2 + a_3 r^3 \cos(3\phi) \quad (4.8)$$

$$H_{xx} - H_{yy} = e_{13} r \cos(\phi) + e_2 r^2 \cos(2\phi)$$

$$H_{xy} = e_{13} r \sin(\phi) - e_2 r^2 \cos(2\phi)$$

where  $H_{xy}$  abbreviates  $\langle \Phi_x^d | H | \Phi_y^d \rangle$  and  $e_{13} = e_1 + e_3 r^2$ . Diagonalization provides the rotation angle  $\delta$  as

$$\tan(\delta) = \frac{\sin(\phi) - \sin(2\phi) r e_2 / e_{13}}{\cos(\phi) + \cos(2\phi) r e_2 / e_{13}} \quad (4.9)$$

The expansion coefficients  $a_n, e_n$  obtained from a fit of the adiabatic potentials in the J-T region lead to the function  $\delta(\phi)$  shown in Fig. 4. Note that  $\delta = \phi$  not

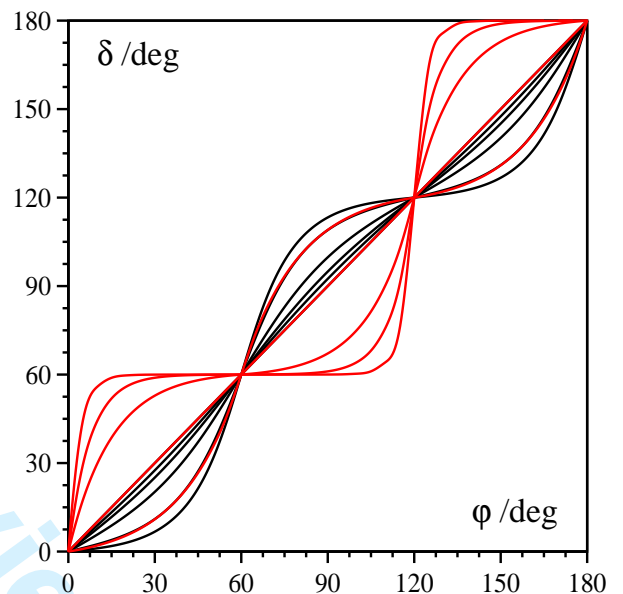


FIG. 4: Rotation angle  $\delta$  as function of  $\phi$ . Black curves: J-T region,  $\alpha$  60-15-105°; Red curves: linear domain,  $\alpha$  = 105-15-180°

only in the linear J-T region but generally at the angles  $\phi = n\pi/3$ ,  $n=0,1,\dots$ , i.e. for all  $C_{2v}$  geometries. This has also to be required for a consistent continuation of  $\delta(\phi)$  beyond the J-T well.

At the other ends of the bending potential valleys, for linear  $C_3^+$  the separation of the surfaces is 6032  $\text{cm}^{-1}$ , much larger than the vibrational energies considered there. Still, vibronic coupling is not negligible for asymmetric stretch motion due to a strong coupling between two diabatic states correlating to  ${}^2\Sigma_u^+$  and  ${}^2\Sigma_g^+$ , respectively, which is obvious from the shape of the asymmetric stretch potential, Fig. 3. As asymmetric stretch is always described by the angle  $\phi$ , diabatization is again a matter of defining  $\delta(\phi)$ . The pertinent quantity is the derivative  $(\partial\delta/\partial\phi)_0$  since only small ranges of  $\phi$  around the  $C_{2v}$  geometries are probed. (For linear structures the

turning points are below  $4^\circ$  for  $v_3 = 0$ ). The rotation angle has been determined from the matrix elements of the dipole moment operator under the assumption that their changes are solely due to changing mixtures of the diabatic states upon asymmetric stretch. We are led to the relation

$$\begin{aligned} \tan(\delta) &= \Delta\mu/\mu_{12} \\ \Delta\mu &= |\mu_{11}(\phi) - \mu_{22}(\phi) - (\mu_{11}(0) - \mu_{11}(0))|/2 \end{aligned} \quad (4.10)$$

This procedure seems validated by the facts that  $\mu_{12}^2 + \Delta\mu^2$  is nearly independent of  $\phi$  and that the resulting diabatic potential has nearly quadratic diagonal elements with a nearly linear coupling element. In Fig. 5 the di-

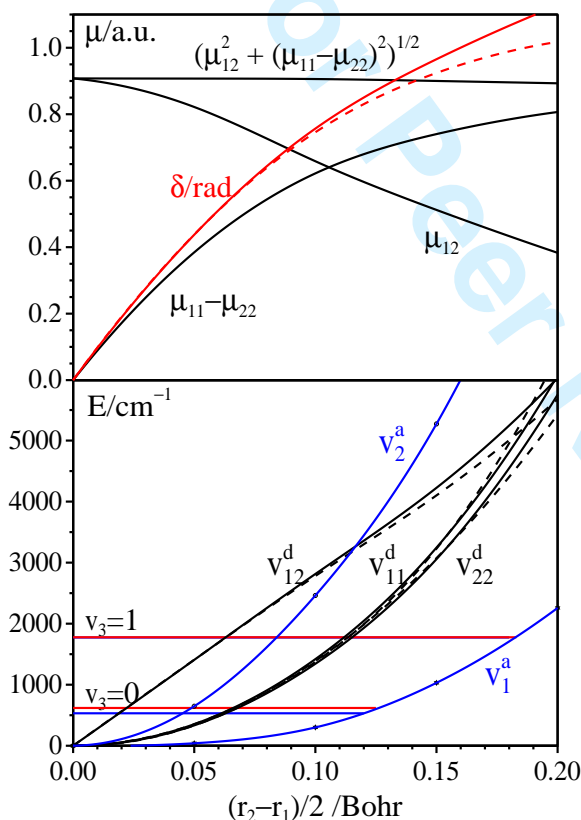


FIG. 5: Transformation to diabatic frame for asymmetric stretch deformation of linear  $C_3^+$ . Upper panel: Dipole moment matrix elements and rotation angle. Lower panel: Adiabatic and diabatic potentials; vibrational levels from uncoupled adiabatic and coupled diabatic treatment.

abatization procedure is illustrated for linear geometries but the picture changes little for apex angles down to about  $120^\circ$ . Remarkable large values of  $\delta/\phi \approx 17$  are obtained which are somewhat difficult to reconcile with the requirement that  $\delta \rightarrow \phi$  for  $\phi \rightarrow n\pi/3$ . After some experimenting we have settled at an exponential approximation for  $\pi/3 - \delta$ . This modeling, indicated by dashed

curves in Fig. 5, does not affect the vibrational levels, however. The function  $\delta(\phi)$  is also shown in Fig. 4. The effect of the vibronic coupling is quite substantial for the zero-point energy of the asymmetric stretch mode: In the linear 1-D model, see Fig. 5, this energy is increased by about 13% or  $85 \text{ cm}^{-1}$ . After averaging over symmetric stretch and bending motions, there remains a shift of  $65 \text{ cm}^{-1}$  for low vibrational states that populate linear structures, as compared to the states that populate cyclic structures. Note that the level  $v_3 = 1$  is hardly changed so that the fundamental transition is reduced by  $85 \text{ cm}^{-1}$ .

## V. CALCULATION OF VIBRATIONAL STATES

We employ hyperspherical coordinates for ease of taking into account symmetry and proper boundary conditions for cyclic motions. For three equal masses they may be defined by expressing the squares of interatomic distances as [34] ( $k=1,2,3$ )

$$r_k^2 = 3^{-1/2} \rho^2 (1 + \sin \theta \cos(\phi + \epsilon_k)); \quad \epsilon_k = k2\pi/3. \quad (5.11)$$

where the hyperspherical radius  $\rho$  determines the size of the molecule,  $\sin \theta$  gives the strength of deformation and  $\phi$  describes the pseudorotation motion around points of  $D_{3h}$  symmetry. With this definition  $\theta = 0, \pi$  correspond to equilateral geometries ( $D_{3h}$ ) and  $\theta = \pi/2$  to linear ones ( $D_{\infty h}, C_{\infty v}$ ),  $\phi = 0, \pm 2\pi/3$  gives isosceles triangles ( $C_{2v}$ ).

For convenient visualization of the actual motion with changing  $\phi$  it is useful to note the relations

$$R = 3^{-3/4} \rho \cos(\theta/2); \quad r = 3^{-3/4} \rho \sin(\theta/2) \quad (5.12)$$

where  $R$  is the distance of the atoms from the center of mass in the  $D_{3h}$  reference geometry and  $r$  is the polar radius for the motion around this reference point, see Fig. 6. We shall use  $(r, \phi)$  as coordinates for 2-D plots

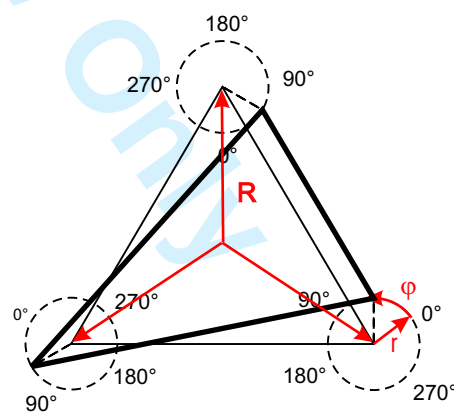


FIG. 6: Pseudorotation motion by varying  $\phi$  with constant  $\rho, \theta$  or  $R, r$ , respectively. Shown is the situation for  $\phi = \pi/3$

of the PES and vibrational wavefunctions. Of course, the coordinate surface of the plots should contain the MEP. The latter is close to a pure bend, i.e.  $r_1 = r_2 = \rho^2(1 - 0.5 \sin \theta)$  constant. Actually,  $\rho$  increases somewhat less than this along the MEP and is nearly constant close to  $\theta = 0$ . It is approximated rather well by defining

$$\rho'^2 = \rho^2[2 - (\sin^2 \theta + a^2)^{1/2}]/(2 - a); \quad a = 0.1885 \quad (5.13)$$

and requiring  $\rho' = \text{const.}$  Fig. 7 shows contour lines of the lower component PES in a  $(\rho', \theta; \phi = 0)$  2-D cut, verifying this relation. Fig. 8 displays contour lines for the 2-D surface  $(r, \phi; \rho' = \rho'_0)$ . One recognizes the typical structure of a JT potential with its central cone, three equivalent minima separated only by small barriers and three rather narrow valleys along  $C_{2v}$  deformations towards linear conformations which are reached at the circular boundary. Note the large coordinate space which is not accessible for its highly repulsive potential. With a strongly anharmonic potential as the one shown in Figs 7 and 8, a variational treatment of the vibrational motion is mandatory. Based on the hyperspherical kinetic energy operator introduced by Johnson [34], efficient procedures for the calculation of the Hamiltonian have been devised and implemented by Carter and Meyer [35, 36]. This implementation was originally designed for vibrational and rovibrational states on a single adiabatic PES with geometric phase boundary conditions 0 or  $\pi$ , respectively, but it has more recently been extended by Carter to handle also two electronic states in a diabatic frame-

work. This latter version is applied here since the upper component of the  $E'$  ground state has to be taken into account if vibrational states are investigated that reach out to linear conformations.

The particular strength of our scheme is the tailoring of the vibrational basis functions to fit the given potential. The full-dimensional basis set is generated from lower-dimensional sets by a sequence of contractions based on the eigenfunctions of appropriate parts of the kinetic energy operator and corresponding effective potentials. The effective potentials are defined as lower-dimensional minima of the full-dimensional potential surface and ensure that the basis functions cover adequately all energetically accessible regions of the coordinate space while effectively excluding the regions with high potentials. The primitive 1-D functions are taken to be Morse functions for the radius  $\rho$ , Jacobi polynomials for the angle  $\theta$  and harmonic functions  $\cos(m\phi), \sin(m\phi)$  with integer or half-integer modulus  $m$  for the angle  $\phi$ . Two-dimensional contracted functions may either be defined for the  $\theta - \phi$  subspace or the  $\rho - \theta$  subspace. Primitive functions as well as contracted functions and their derivatives are represented on a grid and Gauss integration is used for the Hamiltonian. In the present calculations we have used the following sets of primitive and contracted (numbers in parenthesis) functions:  $\rho$  : 28(16),  $\theta$  : 60(54),  $\phi$  : 132(78),  $\rho - \theta$  (69). The dimension of the Hamiltonian matrices were

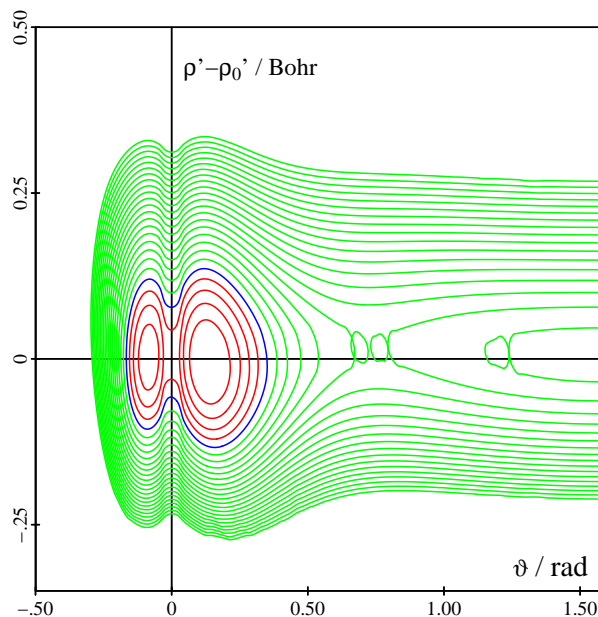


FIG. 7: Contour map of the  $X_-$  PES in the  $\rho'/\vartheta$  plane for  $\phi = 0$  with increments of  $250 \text{ cm}^{-1}$ . Blue line at lowest energy of conical intersection, red lines below, green lines above this energy, respectively.

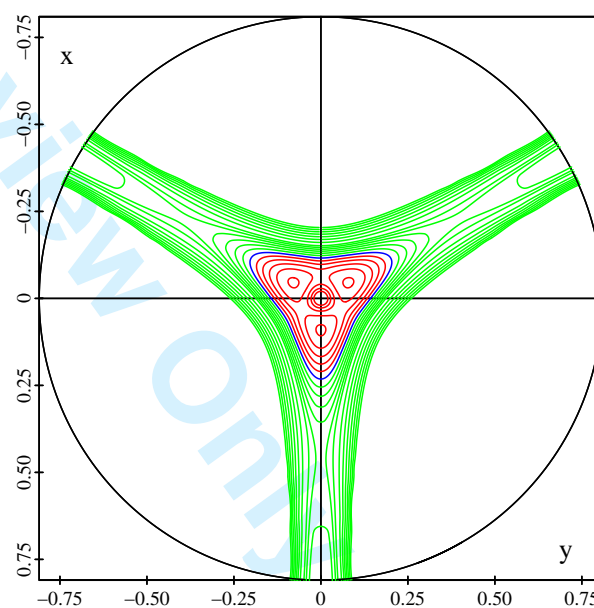


FIG. 8: Contour map of the  $X_-$  PES in the  $r/\phi$  polar plane for  $\rho' = \text{const.}$ , i.e. including the MEP of equ. 5.13. The polar radius  $r$  is given in equ.5.12 and shown in Fig. 6. Increments are  $250 \text{ cm}^{-1}$  up to  $1250 \text{ cm}^{-1}$  and  $500 \text{ cm}^{-1}$  beyond. Blue line at zero energy = lowest energy of conical intersection; red lines below, green lines above zero energy, respectively.

1850 for  $A'$  symmetries and 3700 for the  $E'$  symmetry, respectively.

The vibronic wavefunctions are symmetry-adapted products of electronic and vibrational wavefunctions. Since the coordinates  $\rho$  and  $\theta$  are not affected by any symmetry operation, vibrational symmetry is determined only by the factor-functions of the coordinate  $\phi$ : The pairs  $(\cos(m\phi), (-)^l \sin(m\phi))$  are  $e'$  symmetry species if  $l = (2m \text{ modulo } 3) \neq 3$ , otherwise they generate  $a'_1$  and  $a'_2$  species, respectively [35, 36]. (The phase factor  $(-)^l$  is required to ensure identical representation matrices of  $C_3$  rotations;  $l$  is defined to allow for half-integer values of  $m$  required in the adiabatic frame A.) As customary, small case letters are used for the symmetry species of vibrational functions. With the diabatic electronic wavefunctions  $\Phi_x, \Phi_y$  one finds the following four species of vibronic wavefunctions:

$$\begin{aligned}\Psi_{A_1} &= \Phi_{Ex}\chi_{e_x} + \Phi_{Ey}\chi_{e_y} \\ \Psi_{E_x} &= \Phi_{Ex}(\chi_{e_x} - \chi_{a_1}) + \Phi_{Ey}(\chi_{e_y} + \chi_{a_2}) \\ \Psi_{E_y} &= -\Phi_{Ex}(\chi_{e_y} - \chi_{a_2}) + \Phi_{Ey}(\chi_{e_x} + \chi_{a_1}) \\ \Psi_{A_2} &= \Phi_{Ex}\chi_{e_y} - \Phi_{Ey}\chi_{e_x}\end{aligned}\quad (5.14)$$

These symmetry species provide the only exact classification of vibronic states. The calculated vibronic energy levels are collected in Tab. V and are displayed in Fig. V. As the shape of the bending potential suggests, we find three groups of vibrational states. The first group, labeled by index b in Tab. V, starts lowest in energy and is confined to strongly bent regions around the conical intersection and the J-T minima. The second group, labeled by l, should appear above energies of  $1870 \text{ cm}^{-1}$  and is localized in the linear domain. The third group, labeled by g, is global in the sense that the barrier at about  $2200 \text{ cm}^{-1}$  is overcome and both regions are visited. A further characterization of the vibrational states may be attempted in terms of approximate quantum numbers. For group b states they may be derived by reference to the limiting case of a very high pseudorotation barrier, for which the states are superpositions of the three equivalent states of vibrators localized in the three deep wells. The states can then be labeled by the usual quantum numbers for small-amplitude vibration of a bent molecule. This situation is signaled by near degeneracy of pairs of  $E'$  and  $A'_1$  states. The other limiting case, a very low barrier, lends itself to quantum numbers of a free internal rotation and is signaled by near degeneracy of pairs of  $A'_1$  and  $A'_2$  states. Comparison of the low-energy  $A'_1$ - $E'$  splittings of  $201$  and  $285 \text{ cm}^{-1}$ , respectively, with the first  $A'_2$ - $A'_1$  splitting of  $605 \text{ cm}^{-1}$  indicates that the first option is more adequate. Although the pseudorotation barrier of only  $376 \text{ cm}^{-1}$  seems rather low, a quite effective separation of the three wells is caused by the fact that the saddle point region is rather narrow in bending direction (see Fig. II). Indeed, the lowest  $A'_2$  state, which heavily populates the saddle point region, lies as much as  $806 \text{ cm}^{-1}$  above the ground state. Thus, group b states are assigned with vibrational quantum numbers

TABLE II: Calculated vibrational energy levels (in  $\text{cm}^{-1}$ ) and local vibrator assignments.

$A'_1$		$E'$		$A'_2$	
E	$v_1 v_2 v_3$	E	$v_1 v_2 v_3$	E	$v_1 v_2 v_3$
201	000 b	0	000 b		
966	010 b	681	010 b	806	001 b
		1245	001 b		
1571	020 b	1377	020 b		
1821	100 b	1596	100 b		
1869	000 l	1820	030 b		
2011	030 b	1869	000 l		
		1962	011 b	1806	011 b
2015	020 l	2014	020 l		
2141	040 l	2130	040 l		
2227	070 g	2154	070 g		
2311	080 g	2252	080 g		
2428	090 g	2277	110 b		
2555	110 b	2329	002 b		
2565	0x0 g	2386	090 g		
2658	002 b	2499	0x0 g		
2735	0x0 g	2637	0x0 g		
		2741	021 b	2333	101 b
2894	0x0 g	2794	0x0 g		
		2885	101 b	2569	021 b
		2956	031 g	2935	111 b
		2971	111 b	3002	001 l
		3002	001 l	3110	003 b
3064	100 l	3064	100 l		
3075	0x0 g	3071	120 b		
		3161	021 l	3161	021 l
3173	120 b	3170	0x0 g		
3258	120 l	3194	200 b		
3281		3258	120 l		

$(v_1, v_2, v_3)$  for symmetric stretch, bend and asymmetric stretch modes of a  $C_{2v}$  vibrator, respectively. A particular set of such quantum numbers should appear twice, that is in  $E'$  and  $A'_1$  symmetry for  $v_3$  even but in  $A'_2$  and  $E'$  symmetry for  $v_3$  odd. Within such a pair, the energy order  $E' < A'_1$  and  $E' > A'_2$ , respectively, is usually observed. However, this assignment becomes less and less stringent at higher energies as different local vibrators interact across the pseudorotation barrier. From the energies given in Table II one derives frequencies for fundamental transitions of  $1596$ - $1620$ ,  $681$ - $765$  and  $1245 \text{ cm}^{-1}$  for symmetric stretch, bend and antisymmetric stretch modes, respectively.

In the case of group "l" states one finds  $E'+A'$  pairs which are exactly degenerate because the three components around  $\phi = \epsilon_k$ ,  $k=1-3$ , correspond to conformations with different center atoms which do not interact at all.

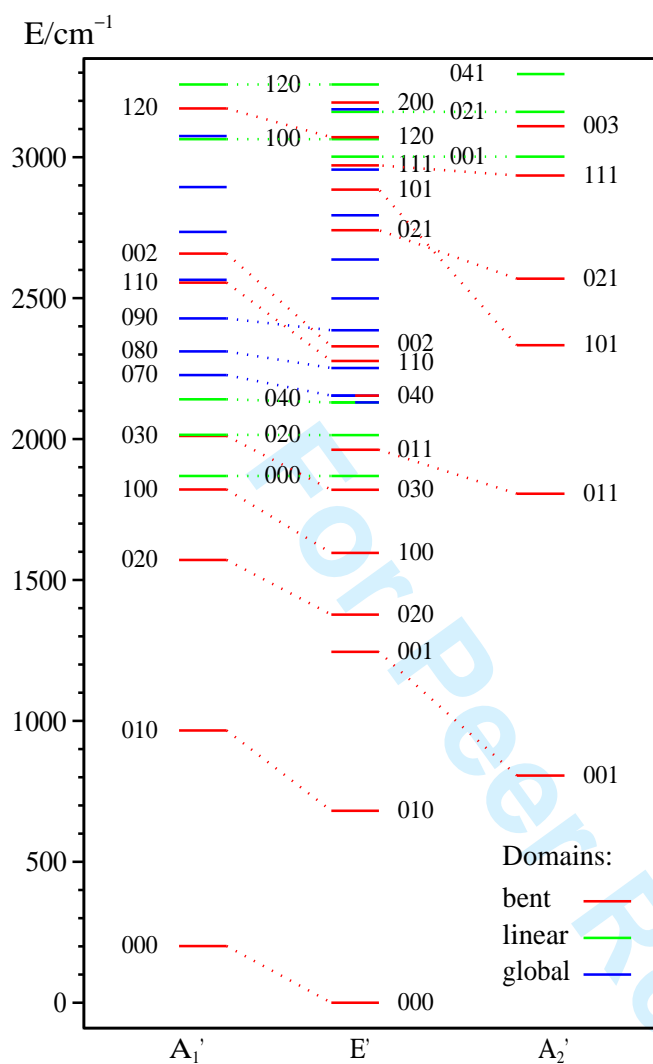


FIG. 9: Vibronic energy levels for  $\text{XC}_3^+$ . States with bent (b), linear (l) and global (g) domains indicated in red, green and blue, respectively. Tentative assignment by local vibrator quantum numbers  $(v_s, v_b, v_{as})$ ; related E-A pairs linked by dotted lines.

Vibrational quantum numbers as used above apply even better to these states but it should be noted that states with  $v_2$  odd do not appear in a rotation-free calculation. We observe only a short progression,  $v_2 = 0, 2, (4)$ , of low-energy bending vibrations with a fundamental frequency of only  $73 \text{ cm}^{-1}$  (level distance  $146 \text{ cm}^{-1}$ ), which is due to the broad barrier of  $289 \text{ cm}^{-1}$  at  $\alpha = 110^\circ$ . The levels  $v_2 = 4$  at  $272$  and  $261 \text{ cm}^{-1}$ , respectively, show already a small splitting from interactions with group b vibrations. The first excitations of symmetric and anti-symmetric stretch modes appear with energies  $1195$  and  $1133 \text{ cm}^{-1}$ , respectively, above the lowest linear level. (The latter value differs by  $-25 \text{ cm}^{-1}$  from the result of the linear 1-D model for vibronic coupling, which is due to vibrational averaging.) Short bending progres-

sions start with these excitations.

Finally, group g states spread rather evenly over cyclic and linear domains. The vibrational motions appear as 1-D bending vibrations within the three energy valleys, with rather regular interconnections in the cyclic region. (Quantum mechanics appears to effectively suppresses chaotic vibrational motion in the energy range under consideration here.) The states are simply denoted by the number of nodes along the minimum energy path (not double-counting the nodes as in the linear case). The average spacing between consecutive levels is around  $110 \text{ cm}^{-1}$ , but interactions with the pure cyclic and/or pure linear modes of similar energy cause significant variations. The assignment of quantum numbers becomes more and more questionable with increasing energy but the grouping according to domains turned out rather definite. The assignment has been carried out by applying small diagnostic step potentials and monitoring the resulting shifts of the energy levels. Of course, contour plots of the vibrational wavefunctions have been illuminating and very helpful in this regard.

Selected low-energy vibrational wavefunctions are displayed in figures 9 - 11. For an adequately representation of global motions the wavefunctions are shown for the  $(r, \phi; \rho'_0)$  surface as used for the potential in Fig. V. Since it appeared difficult to produce comprehensive pictures of the diabatic wavefunctions, we have transformed them to adiabatic ones. Only the component at the lower surface is shown since there is little population of the very narrow upper adiabatic surface. Note that the nonadiabatic wavefunctions are double-valued with a sign change for the second full period of  $\phi$ . The plots show only the period  $-\pi$  to  $\pi$  and indicate by a bold black line the cut where the wavefunction continues smoothly on the other sheet. As to the symmetry assignment please note that we do not invoke the double group notation. Thus, adiabatic electronic states are considered of  $A'$  symmetry and the symmetry species of adiabatic vibrational states are identical to that of the vibronic states. The lowest vibrational state then has  $E'$  symmetry and shows only a single node over the  $4\pi$  period ( $j_{psr} = 1/2$ ) while the lowest  $A_1'$  and  $A_2'$  states have three nodes ( $j_{psr} = 3/2$ ). The latter resides pretty much on top of the pseudorotation barrier. Plots for the  $\rho - \theta$  plane have been instructive for the assignment of vibrational quantum numbers but are not shown here.

The vibrational analysis provides a vibronic value for the isomerization energy of  $\Delta E^{ev} = 1668 \text{ cm}^{-1}$  ( $4.75 \text{ kcal/mol}$ ). The significant difference of  $570 \text{ cm}^{-1}$  to the purely electronic value of  $\Delta E^e = 2238 \text{ cm}^{-1}$  results from the reduction of the ZPE from  $1626 \text{ cm}^{-1}$  for the cyclic ground state to the vibrational energy of the lowest linear state of only  $1195 \text{ cm}^{-1}$ , which is due to much weaker bending and antisymmetric stretch potentials.

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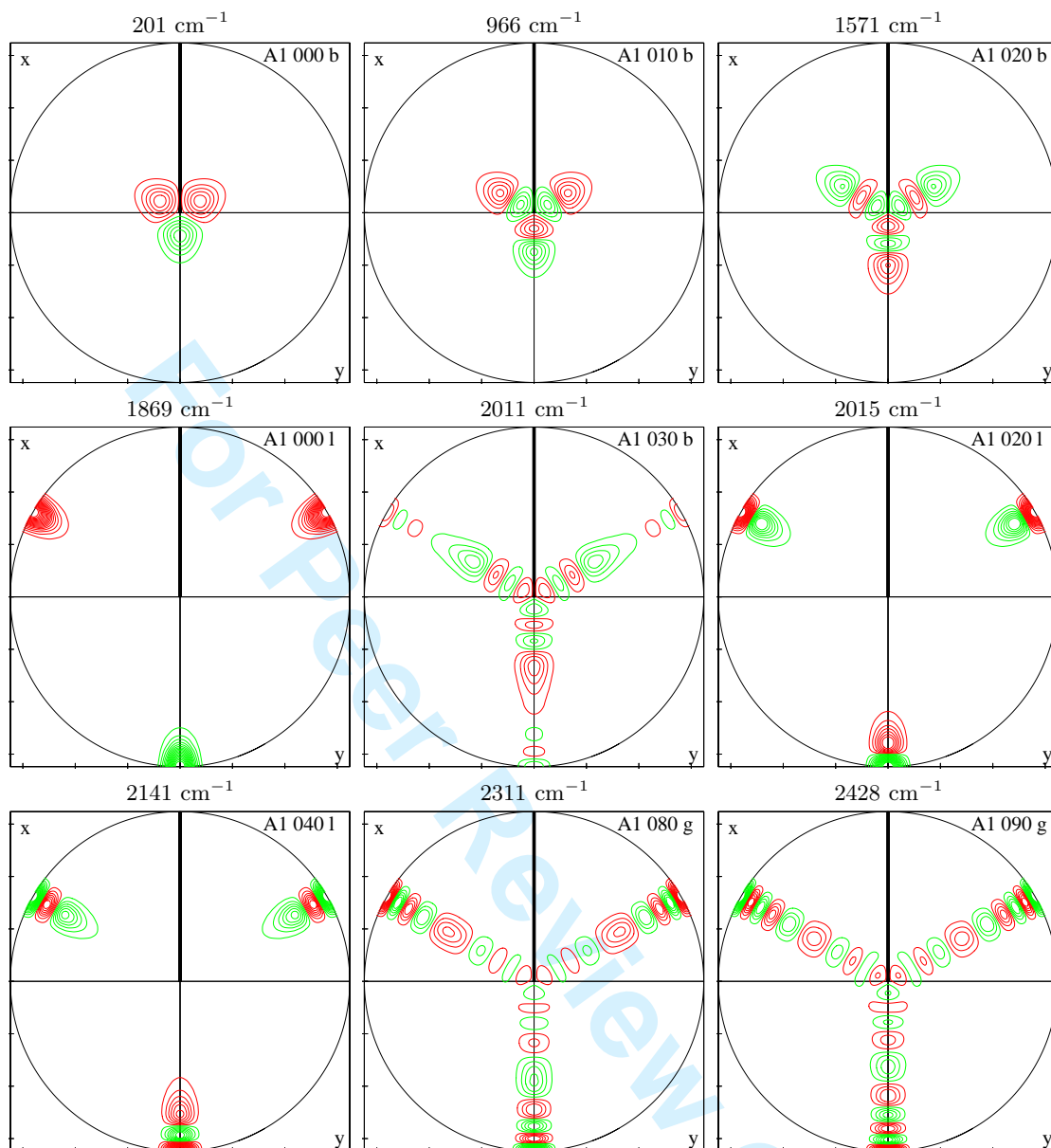
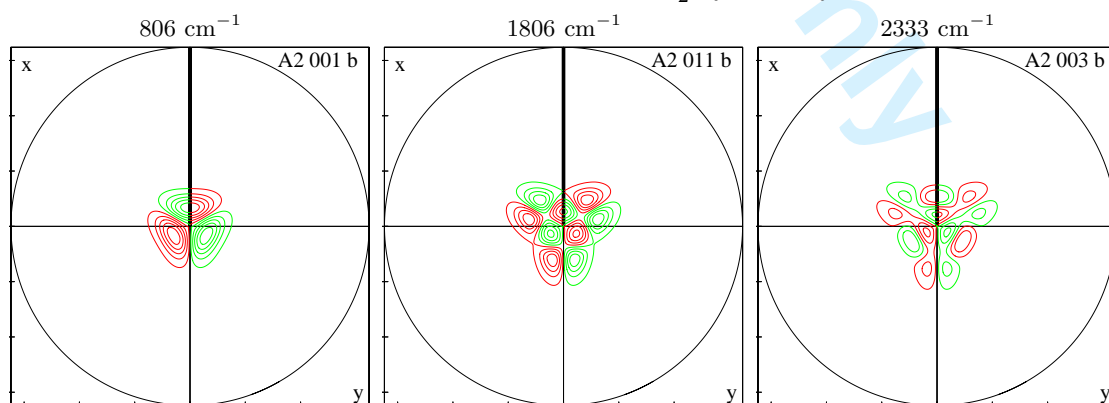
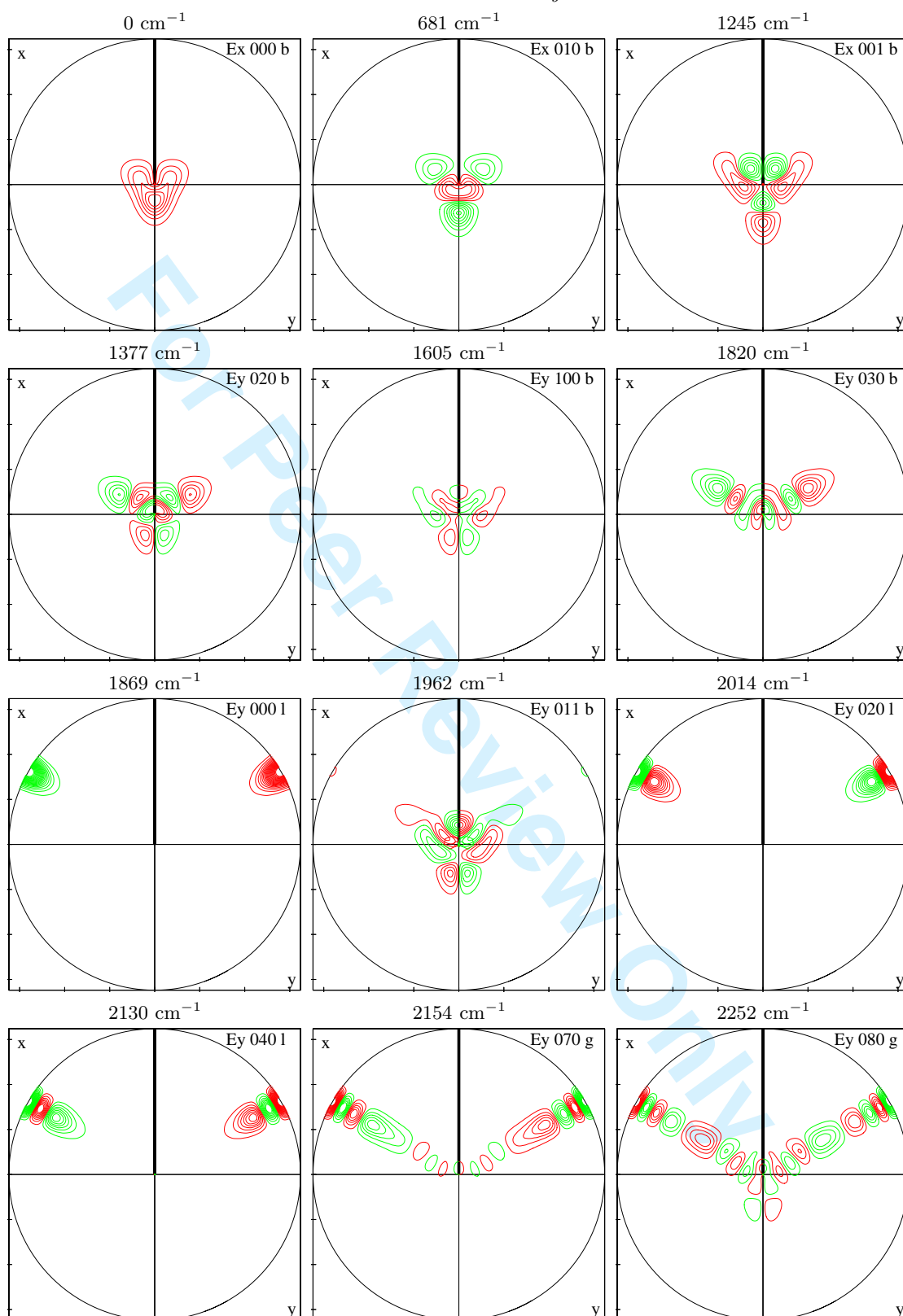
FIG. 10: Vibronic states of  $A_1'$  symmetry.FIG. 11: Vibronic states of  $A_2'$  symmetry

FIG. 12: Vibronic states of  $E'_y$  symmetry



## VI. CONCLUSIONS

We have presented a first attempt to generate a reliable global potential energy surface for the electronic ground state of  $C_3^+$ . The internally contracted MR-CI method proved to be adequate to deal with the strong multi-configurational character of the electronic structure. A large one-electron basis and beyond full-valence active orbital space as well as extrapolations to complete basis and complete configuration space are believed to support a rather uniform accuracy of about  $\pm 1$  kcal/mol for the lower sheet of the potential surface. The electronic isomerization energy  $\Delta E^e$  is calculated as  $2238 \text{ cm}^{-1}$  (6.4 kcal/mol), cyclic and linear structures appear separated by a transition state barrier of only  $290 \text{ cm}^{-1}$ .

The variational calculations of vibronic states take full account of strong vibronic coupling in the cyclic (J-T) and the linear domains of the vibrational motion and are converged to better than  $1 \text{ cm}^{-1}$ . The complex manifold of low-energy ( $< 3000 \text{ cm}^{-1}$ ) states could be classified according to local motions in the cyclic or the linear domains or global motions connecting all low-energy parts of the PES. A more detailed characterization has been attempted in terms of quantum numbers of local vibrators. The vibrational analysis shows that the isomerization energy is reduced by vibrational effects to the vibronic value of  $1626 \text{ cm}^{-1}$  (4.75 kcal/mol), due to a rather low ZPE in the linear domain.

The present work is focused on the electronic ground state because of the substantial problems we encountered with the accurate determination of the upper state

wavefunction beyond the J-T region. Work is in progress to derive reliable transition moments for simulation of spectra which may be helpful for identifying  $C_3^+$  spectroscopically.

In a final remark we like to address again the low reaction rate for  $C_3^+ + H_2 \rightarrow C_3H^+ + H$  and the puzzle of its inverse temperature dependence mentioned in the introduction. No new options seem to emerge from the knowledge of potential surface and vibrational manifold of  $C_3^+$ : Only one vibrational level, the  $A_1'$  level at  $202 \text{ cm}^{-1}$ , is subject to temperature dependent population but its structure hardly differs from the ground vibrational state. More information is required on the ionic collision complex. Some exploratory calculations have revealed the following situation: Multipole Coulomb and induction forces form a van der Waals complex with several local minima, the deepest of which is bound by about  $3500 \text{ cm}^{-1}$ . On the one hand, it may be stabilized with respect to the entrance channel by internal vibrational redistribution. On the other hand, it is separated from the reactive channel by a barrier which rises up to about the educt energy - our calculations are not yet conclusive for the sign of the small difference in energy. This barrier may well explain the low reaction rate at room temperature and maybe even its inverse temperature dependence because the decay channel to the educts is effectively closed at low energies by rotational energy redistribution. Work is in progress to map out the van der Waals potential well and accurately locate the barrier to the reaction channel.

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