SHALLOW WATER EQUATIONS FOR LARGE BATHYMETRY VARIATIONS

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Abstract. In this study, we propose an improved version of the nonlinear shallow water (or Saint-Venant) equations. This new model is designed to take into account the effects resulting from the large spatial and/or temporal variations of the seabed. The model is derived from a variational principle by choosing the appropriate shallow water ansatz and imposing suitable constraints. Thus, the derivation procedure does not explicitly involve any small parameter.

1. Introduction

The celebrated classical nonlinear shallow water (Saint-Venant) equations were derived for the first time in 1871 by A.J.C. de Saint-Venant [dSV71], an engineer working at Ecole Nationale des Ponts et Chaussées. Currently, these equations are widely used in practice and the literature counts many thousands of publications devoted to the applications, validations or numerical solutions of these equations [Syn87, ZCIM02, AGN05, DPD10].

Some important attempts have been also made to improve this model from physical point of view. The main attention was paid to various dispersive extensions of shallow water equations. The inclusion of dispersive effects resulted in a big family of the so-called Boussinesq-type equations [Per67, Nwo93, MBS03, DD07]. Many other families of dispersive wave equations have been proposed as well [Ser53, GN76, MS85, DP99].

However, there are a few studies which attempt to include the bottom curvature effect into the classical Saint-Venant or Savage-Hutter[1] [SH89] equations. One of the first study in this direction is perhaps due to Dressler [Dre78]. Much later, this research was pursued almost in the same time by Keller, Bouchut and their collaborators [Kel03, BMCPV03]. We note that all these authors used some variant of the asymptotic expansion method. The present study is a further attempt to improve the classical Saint-Venant equations by including a better representation of the bottom shape. Moreover, as a derivation procedure we choose a variational approach based on the relaxed Lagrangian principle [CD11].

In the next Section, we present the derivation and discussion of some properties of the improved Saint-Venant equations. Then we underline some main conclusions of this study in Section 5.

Key words and phrases. Shallow water; Saint-Venant equations; gravity waves; long waves.

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[1]The Savage-Hutter equations are usually posed on inclined planes and they are used to model various gravity driven currents such as snow avalanches.

2. Improved Saint-Venant equations derivation

Consider an ideal incompressible fluid of constant density ρ. The horizontal independent variables are denoted by $\boldsymbol{x} = (x_1, x_2)$ and the upward vertical one by y. The origin of the cartesian coordinate system is chosen such that the surface $y = 0$ corresponds to the still water level. The fluid is bounded below by a bottom at $y = -d(\boldsymbol{x}, t)$ and above by a free surface at $y = \eta(\boldsymbol{x}, t)$. Usually, we assume that the total depth $h(\boldsymbol{x}, t) = d(\boldsymbol{x}, t) + \eta(\boldsymbol{x}, t)$ remains positive $h(\boldsymbol{x}, t) \geqslant h_0 > 0$ for all times t. Traditionally in water wave modeling, the assumption of flow irrotationality is also adopted. The assumptions of fluid incompressibility and flow irrotationality lead to the Laplace equation for a velocity potential $\phi(\boldsymbol{x}, y, t)$.

It is well-known that the water wave problem possesses several variational structures [Pet64, Luk67, Zak68]. Recently we proposed a relaxed Lagrangian variational principle which allows much more freedom in constructing approximations in comparison with classical formulations. Namely, the water wave equations can be derived by minimizing the functional $\iiint \mathscr{L}\, \mathrm{d}^2\boldsymbol{x}\, \mathrm{d}t$ involving the Lagrangian density [CD11]:

$$\begin{aligned}\mathscr{L} \;&= (\eta_t + \tilde{\boldsymbol{\mu}} \boldsymbol{\cdot} \boldsymbol{\nabla}\eta - \tilde{\nu})\,\tilde{\phi} \;+\; (d_t + \check{\boldsymbol{\mu}} \boldsymbol{\cdot} \boldsymbol{\nabla} d + \check{\nu})\,\check{\phi} \;-\; \tfrac{1}{2}\, g\, \eta^2 \\ &+\; \int_{-d}^{\eta} \big[\, \boldsymbol{\mu} \boldsymbol{\cdot} \boldsymbol{u} - \tfrac{1}{2}\boldsymbol{u}^2 \;+\; \nu v - \tfrac{1}{2} v^2 \;+\; (\boldsymbol{\nabla} \boldsymbol{\cdot} \boldsymbol{\mu} + \nu_y)\,\phi \,\big]\, \mathrm{d}y, \end{aligned} \tag{2.1}$$

where g is the acceleration due to gravity force and $\boldsymbol{\nabla} = (\partial_{x_1}, \partial_{x_2})$ denotes the gradient operator in horizontal Cartesian coordinates. Other variables $\{\boldsymbol{u}, v, \boldsymbol{\mu}, \nu\}$ are the horizontal velocity, the vertical velocity and the associated Lagrange multipliers, respectively. The last two additional variables $\{\boldsymbol{\mu}, \nu\}$ are called the pseudo-velocities. They formally arise as Lagrange multipliers associated to the constraints $\boldsymbol{u} = \boldsymbol{\nabla}\phi$, $v = \phi_y$. However, once these variables are introduced, the ansatz can be chosen regardless to their initial definition. The over 'tildes' and 'wedges' denote, respectively, a quantity traces computed at the free surface $y = \eta(\boldsymbol{x}, t)$ and at the bottom $y = -d(\boldsymbol{x}, t)$. We shall also denote below with 'bars' the quantities averaged over the water depth. Note that the efficiency of the relaxed variational principle (2.1) relies on the extra freedom for constructing approximations.

In order to simplify the full water wave problem we choose some approximate, but physically relevant, representations of all variables. In this study, we consider very long waves in shallow water. This means that the flow is mainly columnar (Miles & Salmon 1985) and that the dispersive effects are negligible. In other words, a vertical slice of the fluid moves like a rigid body. Thus, we choose a simple shallow water ansatz, which is independent of the vertical coordinate y, and such that the vertical velocity v equals the one of the bottom, i.e.,

$$\phi \;\approx\; \bar{\phi}(\boldsymbol{x}, t), \qquad \boldsymbol{u} \;=\; \boldsymbol{\mu} \;\approx\; \bar{\boldsymbol{u}}(\boldsymbol{x}, t), \qquad v \;=\; \nu \;\approx\; \check{v}(\boldsymbol{x}, t), \tag{2.2}$$

where $\check{v}(\boldsymbol{x}, t)$ is the vertical velocity at the bottom. In the above ansatz, we take for simplicity the pseudo-velocity to be equal to the velocity field $\boldsymbol{\mu} = \boldsymbol{u}$, $\nu = v$. Note that in other situations they can differ (see [CD11] for more examples). With this ansatz, the

Lagrangian density (2.1) becomes

$$\mathscr{L} \;=\; (h_t + \bar{\boldsymbol{u}} \boldsymbol{\cdot} \boldsymbol{\nabla} h + h \boldsymbol{\nabla} \boldsymbol{\cdot} \bar{\boldsymbol{u}})\, \bar{\phi} \;-\; \tfrac{1}{2}\, g\, \eta^2 \;+\; \tfrac{1}{2}\, h\, (\bar{\boldsymbol{u}}^2 + \check{v}^2), \tag{2.3}$$

where we make appear the total water depth $h = \eta + d$.

Remark 1. *Note that for ansatz (2.2) the horizontal vorticity $\boldsymbol{\omega}$ and the vertical one ζ are given by:*

$$\boldsymbol{\omega} = \Big(\frac{\partial \check{v}}{\partial x_2}, -\frac{\partial \check{v}}{\partial x_1}\Big), \qquad \zeta = \frac{\partial \bar{u}_2}{\partial x_1} - \frac{\partial \bar{u}_1}{\partial x_2}.$$

Consequently the flow is not exactly irrotational in general. It will be confirmed below one more time when we establish the connection between $\bar{\boldsymbol{u}}$ and $\boldsymbol{\nabla}\bar{\phi}$.

Now we are going to impose one constraint by choosing a particular representation of the fluid vertical velocity $\check{v}(\boldsymbol{x}, t)$ at the bottom. Namely, we require fluid particles to follow the bottom profile:

$$\check{v} \;=\; -d_t \;-\; \bar{\boldsymbol{u}} \boldsymbol{\cdot} \boldsymbol{\nabla} d. \tag{2.4}$$

This last identity is nothing else but the bottom impermeability condition within ansatz (2.2). Substituting the relation (2.4) into Lagrangian density (2.3), the Euler–Lagrange equations yield

$$\delta\bar{\phi} : \quad 0 \;=\; h_t \;+\; \boldsymbol{\nabla} \boldsymbol{\cdot} [\, h\, \bar{\boldsymbol{u}}\,], \tag{2.5}$$

$$\delta\bar{\boldsymbol{u}} : \quad \boldsymbol{0} \;=\; \bar{\boldsymbol{u}} \;-\; \boldsymbol{\nabla}\bar{\phi} \;-\; \check{v}\, \boldsymbol{\nabla} d, \tag{2.6}$$

$$\delta\eta : \quad 0 \;=\; \bar{\phi}_t \;+\; g\, \eta \;+\; \bar{\boldsymbol{u}} \boldsymbol{\cdot} \boldsymbol{\nabla}\bar{\phi} \;-\; \tfrac{1}{2}\, (\bar{\boldsymbol{u}}^2 + \check{v}^2). \tag{2.7}$$

Taking the gradient of (2.7) and eliminating $\bar{\phi}$ from (2.6) gives us this system of governing equations

$$h_t \;+\; \boldsymbol{\nabla} \boldsymbol{\cdot} [\, h\, \bar{\boldsymbol{u}}\,] \;=\; 0, \tag{2.8}$$

$$\partial_t\, [\, \bar{\boldsymbol{u}} - \check{v}\, \boldsymbol{\nabla} d\,] \;+\; \boldsymbol{\nabla}\, [\, g\, \eta \;+\; \tfrac{1}{2}\, \bar{\boldsymbol{u}}^2 \;+\; \tfrac{1}{2}\, \check{v}^2 \;+\; \check{v}\, d_t\,] = \; 0, \tag{2.9}$$

together with the relations

$$\bar{\boldsymbol{u}} \;=\; \boldsymbol{\nabla}\bar{\phi} \;+\; \check{v}\, \boldsymbol{\nabla} d, \qquad \check{v} \;=\; -\, d_t \;-\; \bar{\boldsymbol{u}} \boldsymbol{\cdot} \boldsymbol{\nabla} d \;=\; -\frac{d_t \;+\; \boldsymbol{\nabla}\bar{\phi} \boldsymbol{\cdot} \boldsymbol{\nabla} d}{1 \;+\; |\boldsymbol{\nabla} d|^2}.$$

Remark 2. *The classical nonlinear shallow water or Saint-Venant equations can be recovered substituting $\check{v} = 0$ into the last system, yielding*

$$h_t \;+\; \boldsymbol{\nabla} \boldsymbol{\cdot} [\, h\, \bar{\boldsymbol{u}}\,] \;=\; 0,$$

$$\bar{\boldsymbol{u}}_t \;+\; \boldsymbol{\nabla}\, [\, g\, \eta \;+\; \tfrac{1}{2}\, \bar{\boldsymbol{u}}^2] = \; 0.$$

The last equation is most often written in the literature into the momentum flux form:

$$\partial_t\, [\, h\, \bar{\boldsymbol{u}}\,] \;+\; \boldsymbol{\nabla}\, [\, h\, \bar{\boldsymbol{u}}^2 \;+\; \tfrac{1}{2}\, g\, h^2\,] \;=\; g\, h\, \boldsymbol{\nabla} d.$$

3. Secondary equations

From the governing equations (2.8) and (2.9), one can derive an equation for the horizontal velocity $\bar{\boldsymbol{u}}$ and for the momentum density $h\bar{\boldsymbol{u}}$

$$\bar{\boldsymbol{u}}_t \;+\; \bar{\boldsymbol{u}}\boldsymbol{\cdot}\boldsymbol{\nabla}\bar{\boldsymbol{u}} \;+\; g\,\boldsymbol{\nabla}\eta \;=\; \gamma\,\boldsymbol{\nabla}d \;+\; \bar{\boldsymbol{u}}\wedge(\boldsymbol{\nabla}\check{v}\wedge\boldsymbol{\nabla}d), \tag{3.1}$$

$$\partial_t\,[\,h\,\bar{\boldsymbol{u}}\,] \;+\; \boldsymbol{\nabla}\,[\,h\,\bar{\boldsymbol{u}}^2 \;+\; \tfrac{1}{2}\,g\,h^2\,] \;=\; (g+\gamma)\,h\,\boldsymbol{\nabla}d \;+\; h\bar{\boldsymbol{u}}\wedge(\boldsymbol{\nabla}\check{v}\wedge\boldsymbol{\nabla}d), \tag{3.2}$$

where γ is the vertical acceleration at the bottom defined as

$$\gamma \;\equiv\; \frac{\mathrm{D}\,\check{v}}{\mathrm{D}t} \;=\; \check{v}_t \;+\; (\bar{\boldsymbol{u}}\boldsymbol{\cdot}\boldsymbol{\nabla})\check{v}. \tag{3.3}$$

Remark 3. *In the right hand sides of* (3.1) *and* (3.2)*, the last term cancel out in one horizontal dimension. It can be easily seen from the following analytical representation which degenerates to zero in one horizontal dimension*

$$\bar{\boldsymbol{u}}\wedge(\boldsymbol{\nabla}\check{v}\wedge\boldsymbol{\nabla}d) \;\equiv\; (\boldsymbol{\nabla}\check{v})\,(\bar{\boldsymbol{u}}\boldsymbol{\cdot}\boldsymbol{\nabla}d) \;-\; (\boldsymbol{\nabla}d)\,(\bar{\boldsymbol{u}}\boldsymbol{\cdot}\boldsymbol{\nabla}\check{v}).$$

This property has the geometrical interpretation that $\bar{\boldsymbol{u}}\wedge(\boldsymbol{\nabla}\check{v}\wedge\boldsymbol{\nabla}d)$ is a horizontal vector orthogonal to $\bar{\boldsymbol{u}}$, thus vanishing for two-dimensional flows.

One can also derive an equation for the energy flux

$$\partial_t\left[\,h\,\frac{\bar{\boldsymbol{u}}^2+\check{v}^2}{2} \;+\; g\,\frac{\eta^2-d^2}{2}\,\right] \;+\; \boldsymbol{\nabla}\boldsymbol{\cdot}\left[\left(\frac{\bar{\boldsymbol{u}}^2+\check{v}^2}{2}+g\eta\right)h\,\bar{\boldsymbol{u}}\right] \;=\; -(g+\gamma)\,h\,d_t. \tag{3.4}$$

Obviously, the source term on the right-hand side vanishes if the bottom is fixed $d = d(x)$, or equivalently if $d_t = 0$. This last conservation law is closely related to the Hamiltonian structure of the improved Saint-Venant equations (2.5) – (2.7). Namely, these equations possess a canonical Hamiltonian structure for the variables h and $\bar{\phi}$, i.e.,

$$\frac{\partial\,h}{\partial t} \;=\; \frac{\delta\,\mathcal{H}}{\delta\bar{\phi}}, \qquad \frac{\partial\,\bar{\phi}}{\partial t} \;=\; -\frac{\delta\,\mathcal{H}}{\delta h},$$

where the Hamiltonian is

$$\mathcal{H} \;=\; \tfrac{1}{2}\int\left\{g(h-d)^2 \;+\; h\,|\boldsymbol{\nabla}\bar{\phi}|^2 \;-\; \frac{h\,[\,d_t \;+\; \boldsymbol{\nabla}\bar{\phi}\boldsymbol{\cdot}\boldsymbol{\nabla}d\,]^2}{1\;+\;|\boldsymbol{\nabla}d|^2}\right\}\mathrm{d}^2\boldsymbol{x}. \tag{3.5}$$

One can easily check, after computing the variations, that the Hamiltonian (3.5) yields the equations

$$h_t \;=\; -\boldsymbol{\nabla}\cdot\left[\,h\boldsymbol{\nabla}\bar{\phi} \;-\; \frac{d_t+\boldsymbol{\nabla}\bar{\phi}\boldsymbol{\cdot}\boldsymbol{\nabla}d}{1\;+\;|\boldsymbol{\nabla}d|^2}h\boldsymbol{\nabla}d\right],$$

$$\bar{\phi}_t \;=\; -g(h-d) \;-\; \tfrac{1}{2}\,|\boldsymbol{\nabla}\bar{\phi}|^2 \;+\; \frac{[\,d_t \;+\; \boldsymbol{\nabla}\bar{\phi}\boldsymbol{\cdot}\boldsymbol{\nabla}d\,]^2}{1\;+\;|\boldsymbol{\nabla}d|^2},$$

which are equivalent to the system (2.5)–(2.7).

Remark 4. *Rewriting the Hamiltonian (3.5) in the equivalent form*

$$\mathcal{H} \;=\; \tfrac{1}{2} \int \left\{ g\eta^2 \,+\, h\,\bar{\boldsymbol{u}}^2 \,+\, h\,(\check{v} + d_t)^2 \,-\, h\,d_t^2 \right\} \mathrm{d}^2\boldsymbol{x}, \tag{3.6}$$

one can see that (3.5) is actually positive definite if the bottom is static, i.e., if $d = d(x)$ or $d_t = 0$. In other words, if there is no external input of energy into the system.

4. Hyperbolic structure

For simplicity we consider in this section two-dimensional flows only, i.e., we have one horizontal dimension, say the x_1 and $u = u_1$ for brevity. Introducing the potential velocity $U = \bar{\phi}_x$, we have

$$\bar{u} \;=\; \frac{U \,-\, d_x\,d_t}{1 \,+\, d_x^2}, \qquad \check{v} \;=\; -\,\frac{d_t \,+\, U\,d_x}{1 \,+\, d_x^2}, \qquad \bar{u}^2 \,+\, \check{v}^2 \;=\; \frac{U^2 \,+\, d_t^2}{1 \,+\, d_x^2},$$

and the equations of motion become

$$\partial_t\, h \;+\; \partial_x \left[h\,\frac{U \,-\, d_x\,d_t}{1 \,+\, d_x^2} \right] \;=\; 0, \tag{4.1}$$

$$\partial_t\, U \;+\; \partial_x \left[g\,(h - d) \,+\, \tfrac{1}{2}\,U^2 \,-\, \frac{(\,d_t \,+\, U\,d_x\,)^2}{2 \,+\, 2\,d_x^2} \right] \;=\; 0. \tag{4.2}$$

The Jacobian matrix of this quasilinear system along with its eigenvalues $\lambda_\pm$ can be easily computed:

$$\lambda_\pm \;=\; \bar{u} \;\pm\; c, \qquad c^2 \;=\; g\,h\,[\,1 \,+\, d_x^2\,]^{-1}, \tag{4.3}$$

where c represents the gravity wave celerity. Note that for the classical Saint-Venant equations $c^2 \;=\; g\,h$. Consequently, the gravity wave speed is slowed down in the new system by strong bottom variations.

5. Conclusions

In this study, we derived a novel non-dispersive shallow water model which takes into account larger bathymetric variations. Previously, some attempts were already made to derive such systems for arbitrary slopes and curvature [BMCPV03, Kel03]. However, our study contains a certain number of new elements with respect to the existing state of the art. Namely, our derivation procedure relies on a generalized variational structure of the water wave problem [CD11]. Moreover, we do not introduce any small parameter and our approximation is made through the choice of a suitable constrained ansatz which is partially equivalent to assume that the pressure is hydrostatic everywhere except at the bottom. Resulting governing equations have a simple form and hyperbolic structure. Another new element is the introduction of arbitrary bottom time variations. The Hamiltonian structure of the new model is provided as well. By comparing the vertical velocity profiles in our model and previous investigations [BMCPV03, Kel03], we conclude that a priori our model cannot be reduced to previous ones by choosing a particular asymptotic regime.

An interesting feature of the improved Saint-Venant equations is that this system is hyperbolic, like its classical counterpart. Thus, the same analytical and numerical methods can be used to study the new system. The wave propagation speed in the new system depends on the bottom gradient. This fact may have some important implications for practical problems and it will be investigated in the next study.

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