Control of a two-stage production/inventory system with products returns

Samuel VERCRAENE

G-Scop
Grenoble-INP

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Joint work with :

J.-P. GAYON (G-Scop Grenoble-INP)
Z. JEMAI (LGI Ecole Centrale Paris)

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Introduction

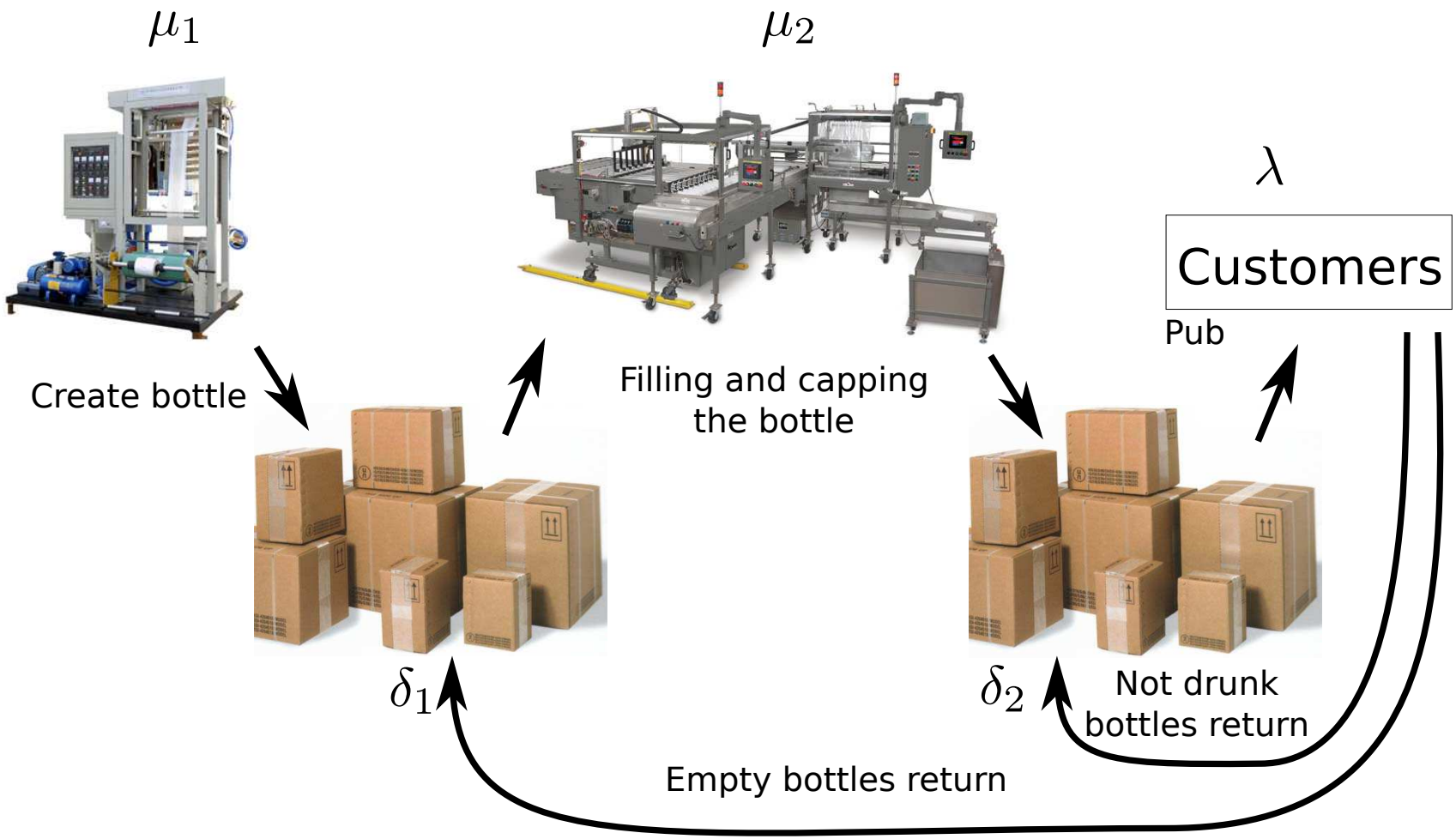
Context

- New regulations encouraging waste reduction, especially in Europe
- Some industries also make returns for economical and marketing reasons

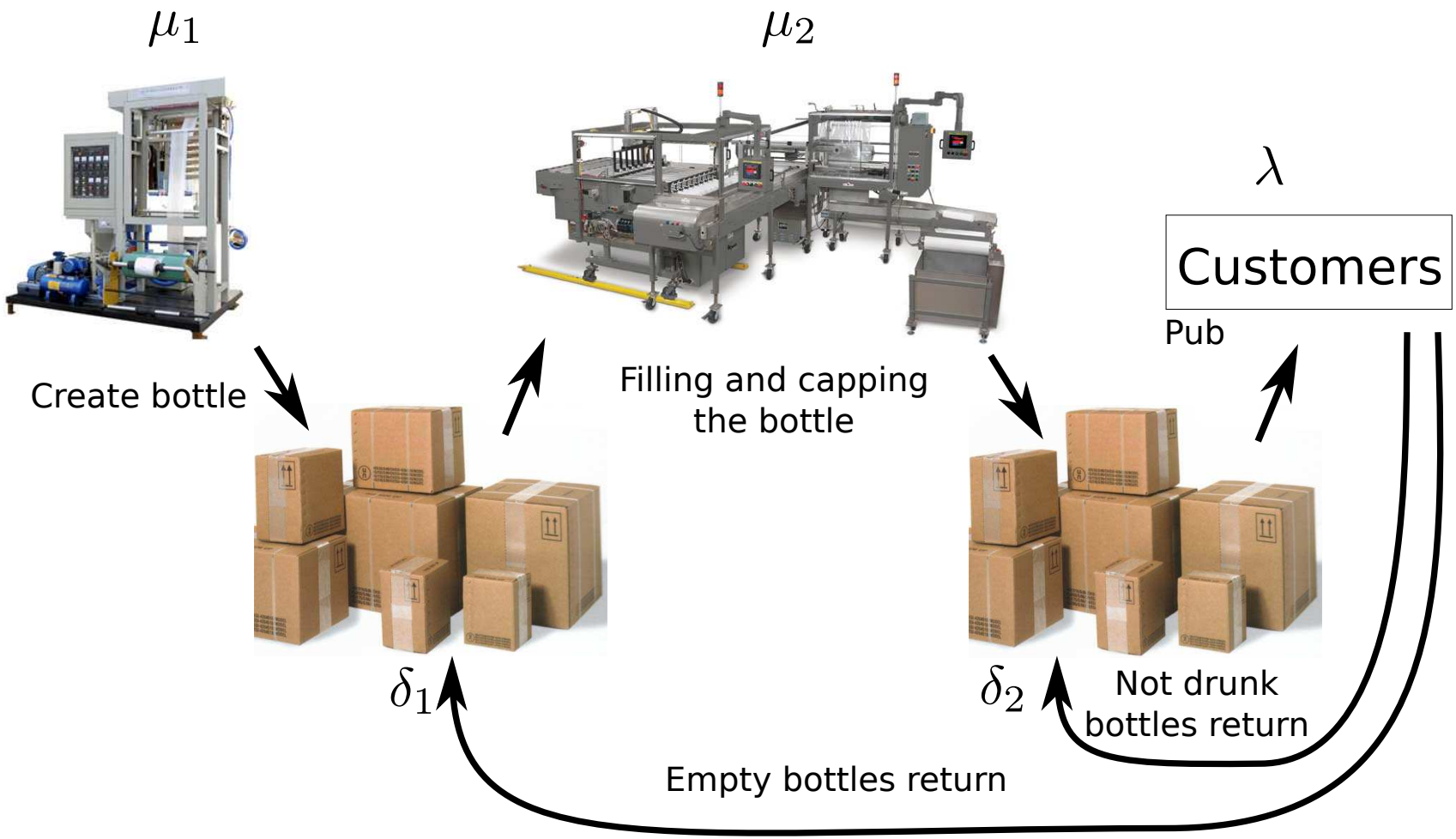
Why a multi stage and stochastic model ?

- Quality of a product returned
- Quantity of returned products
- Timing of returns

Exemple of Glass bottle



Exemple of Glass bottle

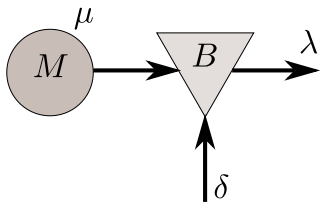


Objective :

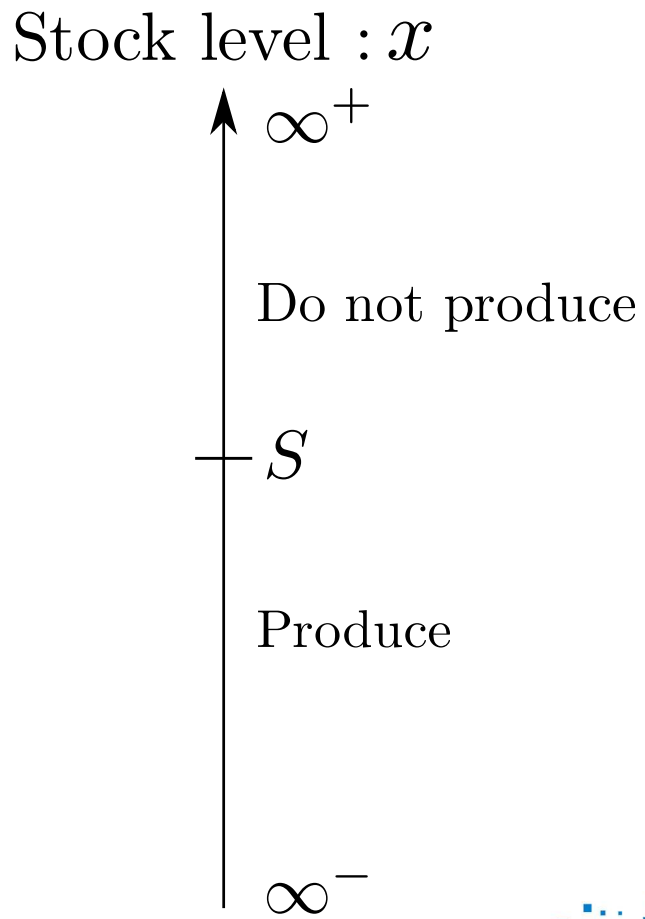
Compromise between holding cost and backlog (dissatisfaction) cost

Single Stage with products returns (Gayon 2005)

M/M/1 queue with forced return
Objective function : $\min_\pi C_\infty^\pi$

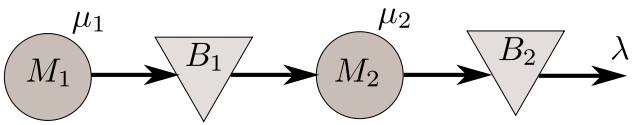


- The optimal policy is a Base Stock policy.
- The optimal Base Stock level ($S^\star$) is given by an analytical formula.



Two-stages system without product return (Veatch and Wein 1992 and 1994)

Two-stages model :



- Characterization of the optimal policy
- Comparison of heuristics

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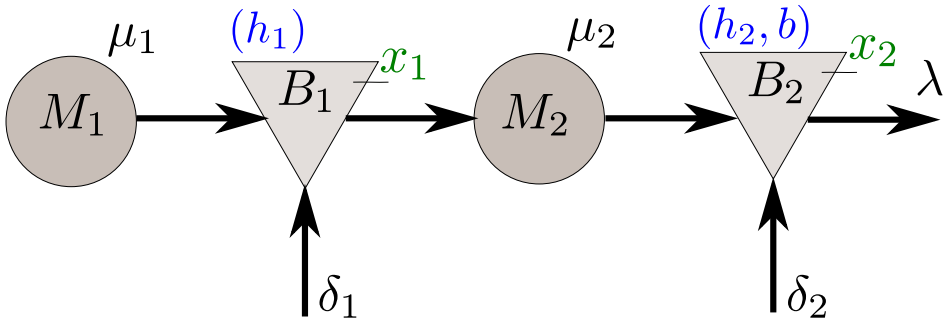
Model formulation



- Make-to-Stock
- Decision : start/stop servers M_i
- Exponential service time μ_i
- Poisson demand (λ) and returns (δ_i)
- Backorders are allowed with cost b
- Holding cost h_i
- No set-up cost
- Preemption is allowed
- Return independent of demand

Objective function

The objective is to find the production policy minimizing the average cost over an infinite horizon :

$$\pi^\star = \arg\min_{\pi} C_\infty^\pi$$

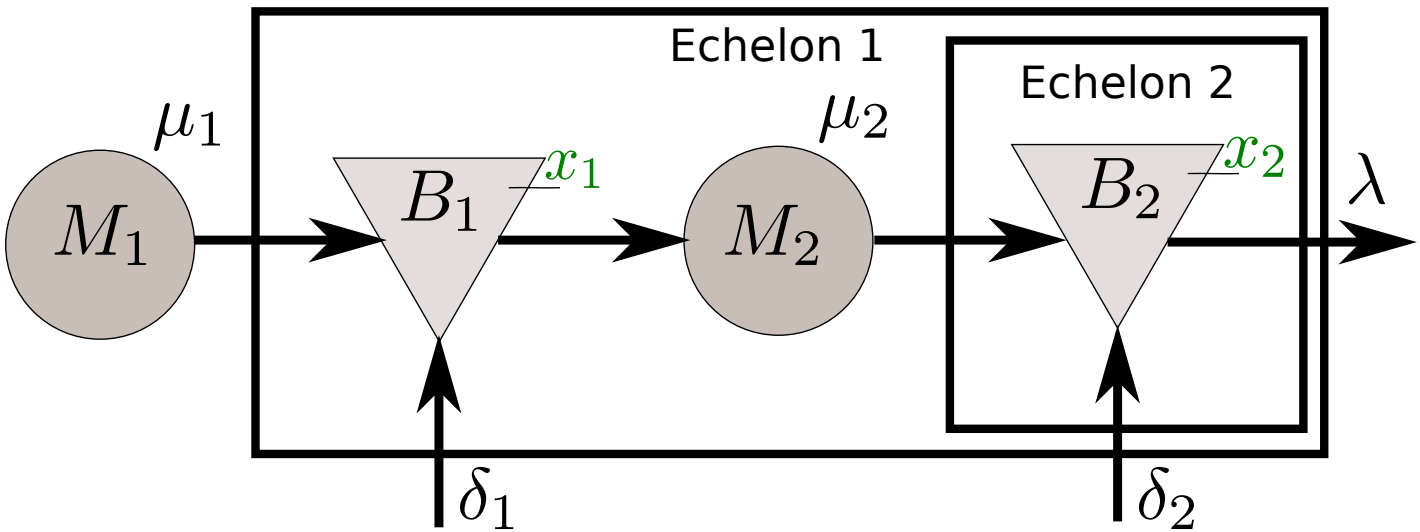
With :

$$C_\infty^\pi = \sum_{(x_1,x_2)} \left(h_1 x_1 + h_2 x_2^+ + b x_2^-\right) P^\infty(X_1 = x_1, X_2 = x_2 | \pi)$$

Stability

Every echelon should be capable to :

- Serve demand
- Consume returns



$$\delta_2 < \lambda < \mu_2 + \delta_2 \text{ (Echelon 2)}$$

$$\delta_1 + \delta_2 < \lambda < \mu_1 + \delta_1 + \delta_2 \text{ (Echelon 1)}$$

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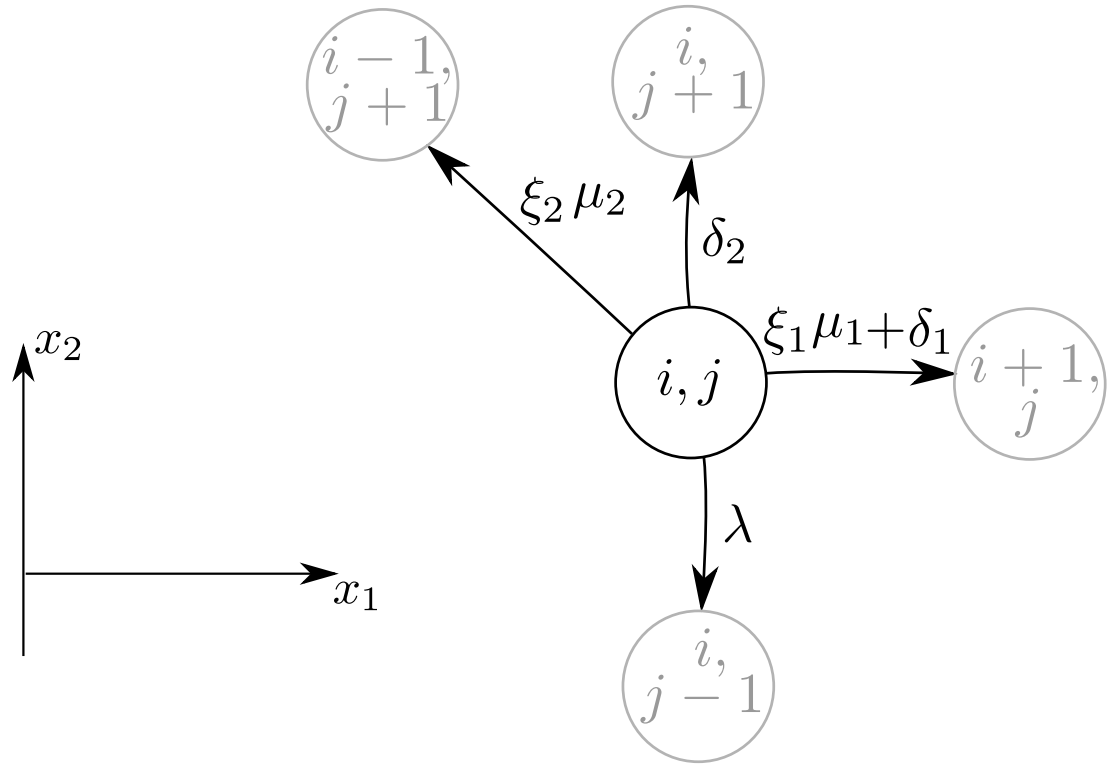
Continuous Markov chain



Decisions :

- $\xi_k(i,j) = 1$ the server k is producing
- $\xi_k(i,j) = 0$ the server k is stopped

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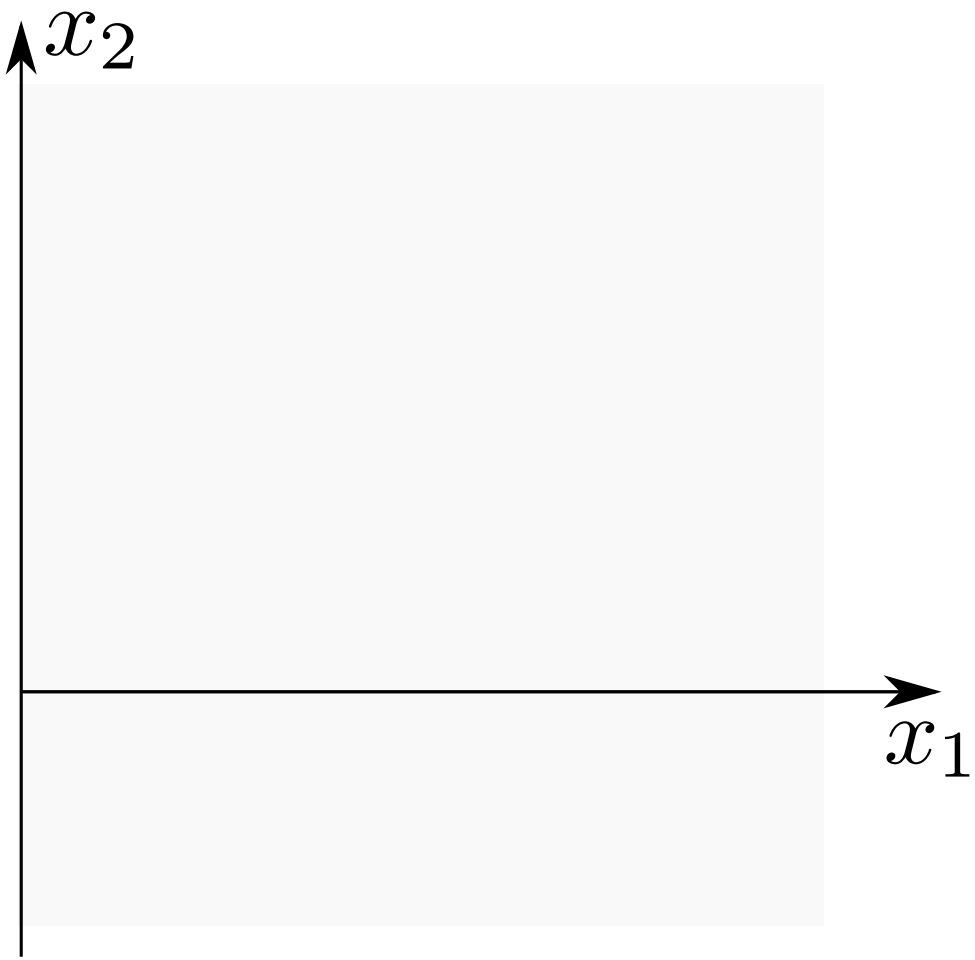
Characterization of the optimal policy : $\pi^\star$



Theorem

- $\pi^\star$ exists
- $\beta_1(x_2+1)-1 \leq \beta_1(x_2) \leq \beta_1(x_2+1)$
- $\beta_2(x_1-1)-1 \leq \beta_2(x_1) \leq \beta_2(x_1-1)$

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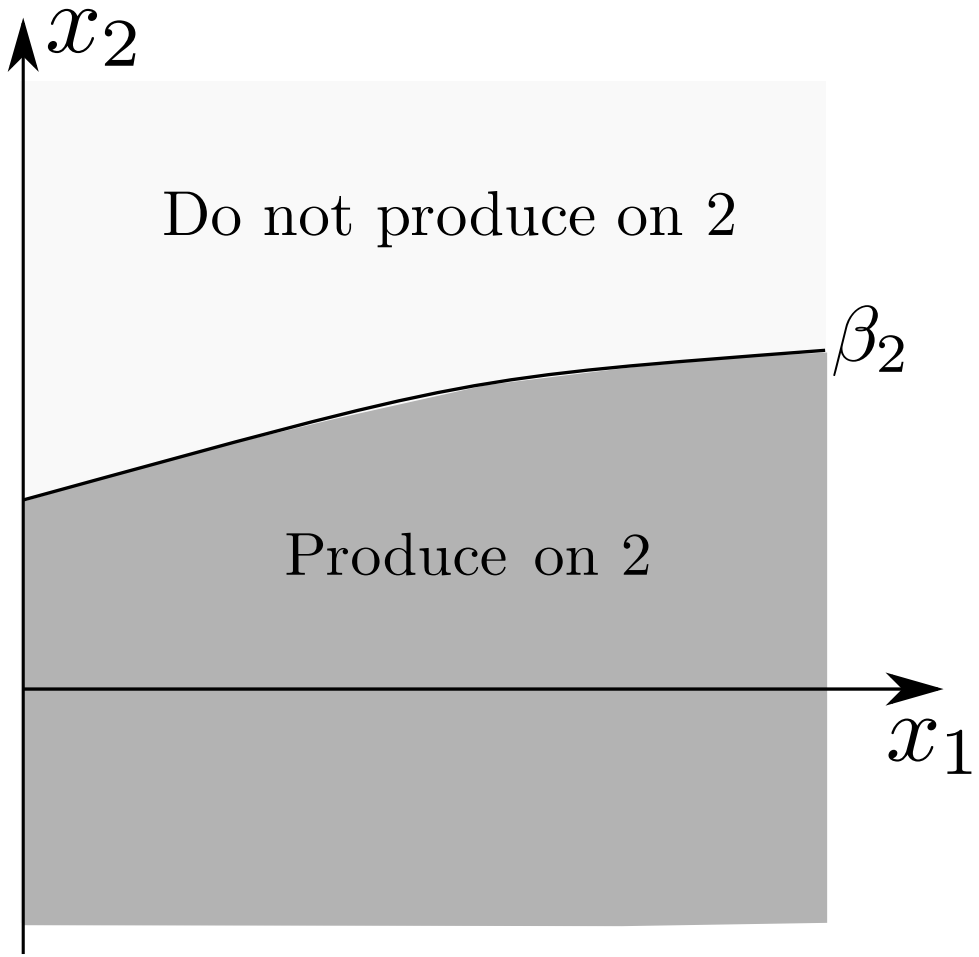

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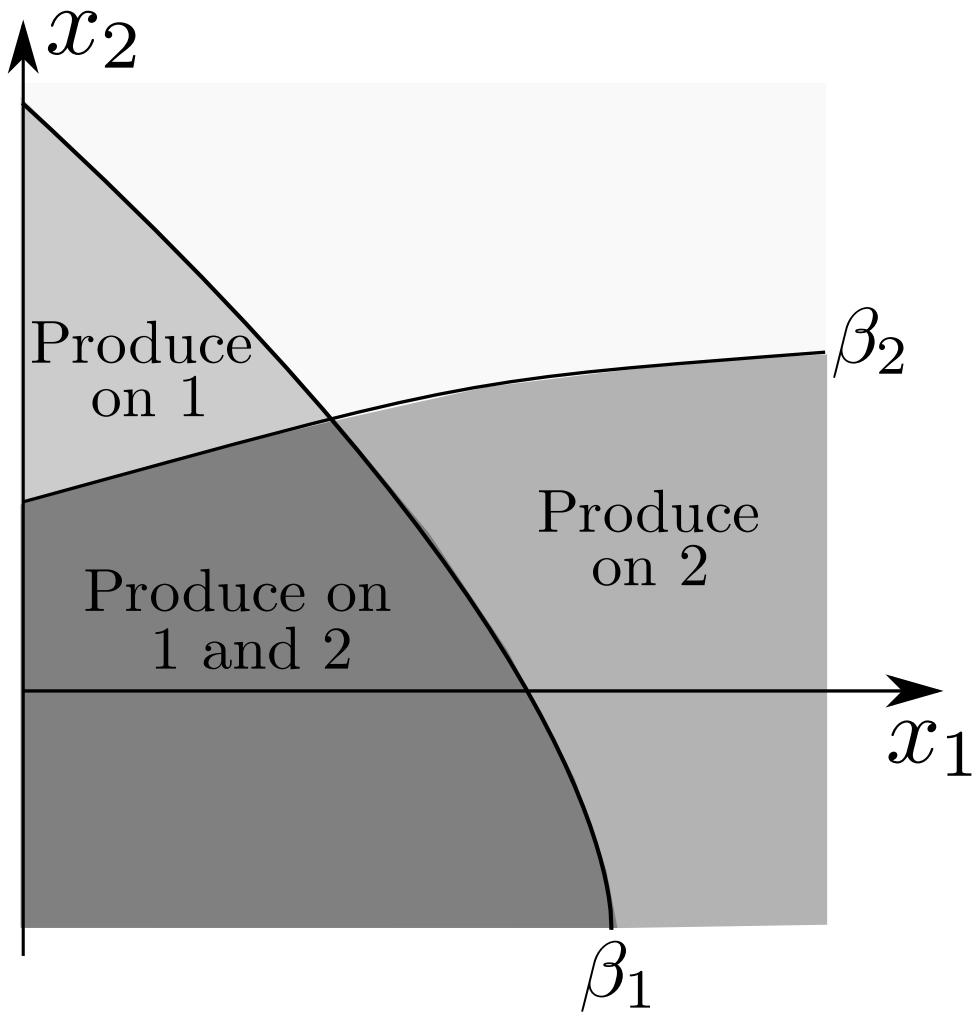

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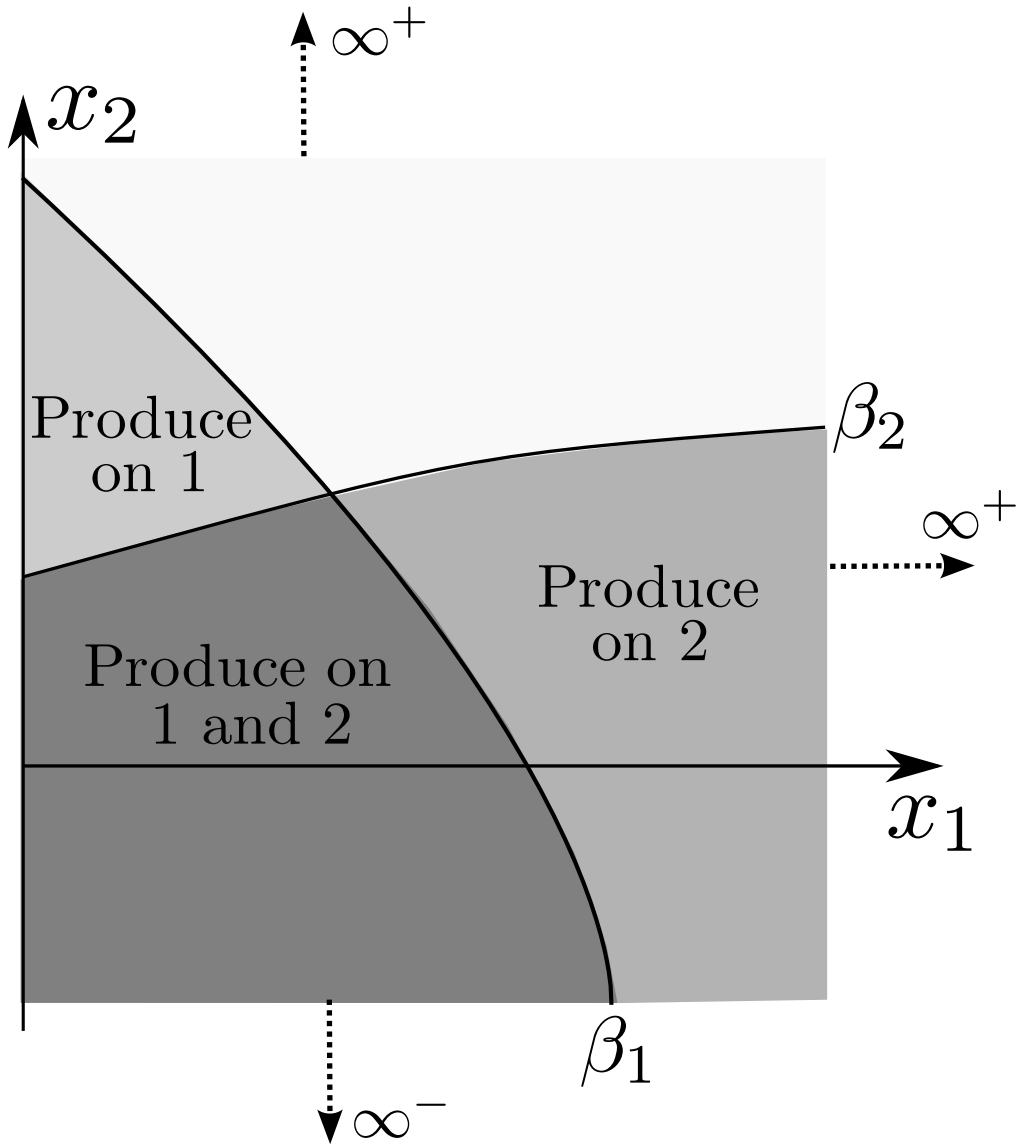

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Markov Decision Process (see Puterman 1994)

Optimality equation :

$$v^\star = Tv^\star$$

With :

$$\begin{aligned} Tv(x_1,x_2) = \tfrac{1}{\tau} \quad & \big(x_1 h_1 + x_2^+ h_2 + x_2^- b \\ & + T_1 v(x_1,x_2) + T_2 v(x_1,x_2) \\ & + T_3 v(x_1,x_2) + T_4 v(x_1,x_2) \\ & + T_5 v(x_1,x_2)\big) \end{aligned}$$

Production 1 : $T_1 v(x_1,x_2) = \mu_1 \min(v(x_1,x_2), v(x_1+1,x_2))$

Production 2 : $T_2 v(x_{1,2}) = \mu_2 \min(v(x_1,x_2), v(x_1-1,x_2+1))$

Return 1 : $T_3 v(x_1,x_2) = \delta_1 v(x_1+1,x_2))$

Return 2 : $T_4 v(x_1,x_2) = \delta_2 v(x_1,x_2+1))$

Demand : $T_5 v(x_1,x_2) = \lambda v(x_1,x_2-1)$

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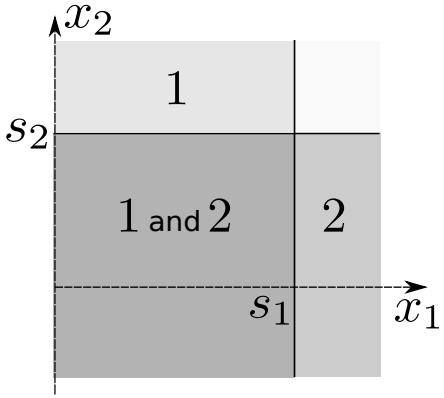
Sketch of the proof (see Koole 1998)

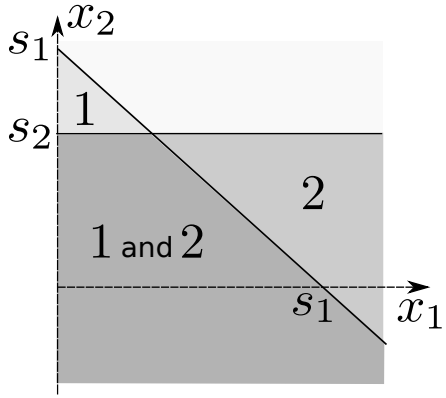
- T is a contracting mapping
- T propagates Supermodularity (Super) and SuperConvexity (SuperC)
- By induction $v^\star$ is Super and SuperC
- So $\pi^\star$ exists and β_i have specific slope

Heuristics

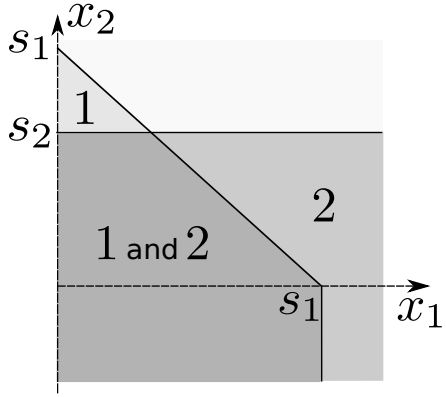
Fixed Buffer Policy : $FB(s_1, s_2)$



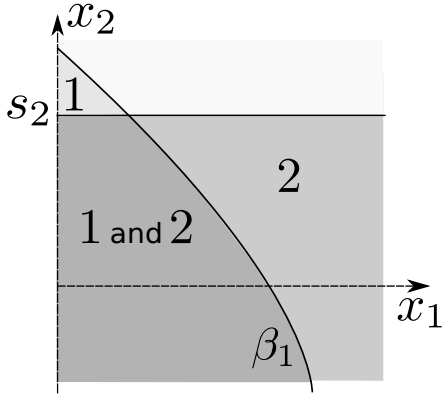
Base Stock Policy : $BS(s_1, s_2)$



Kanban Policy : $KB(s_1, s_2)$



Half Optimal Policy : $HO(s_2)$



Computation procedure

Infinite state space

We need to truncate state space (without impact on cost) in 3 directions :

- x_1^+ due to forced returns on B_1
- x_2^+ due to forced returns on B_2
- x_2^- due to backorders

Resolution for a state space

We use Fixed Point algorithm to compute the cost :

$$v_{n+1} = Tv_n$$

$$\lim_{n\to\infty} v_n = v^\star \ \forall v_0$$

The stop criteria is : $\mathrm{sp}\left|v^{n+1} - v^n\right| < \epsilon$

Procedure to compute heuristic policies

For instance, Base Stock policy :

$$T_1^{BS(s_1,s_2)} v(x_1,x_2) = \begin{cases} v(x_1,x_2) \text{ if } x_1 + x_2 \geq s_1 \\ v(x_1+1,x_2) \text{ else} \end{cases}$$

$$T_2^{BS(s_1,s_2)} v(x_1,x_2) = \begin{cases} v(x_1,x_2) \text{ if } x_2 \geq s_2 \\ v(x_1-1,x_2+1) \text{ else} \end{cases}$$

Optimisation of heuristics parameters

Objective

For a heuristic π with 2 parameters (s_1, s_2) we want to find :

$$(s_1^\star, s_2^\star) = \arg\min_{(s_1,s_2)} C(s_1, s_2)$$

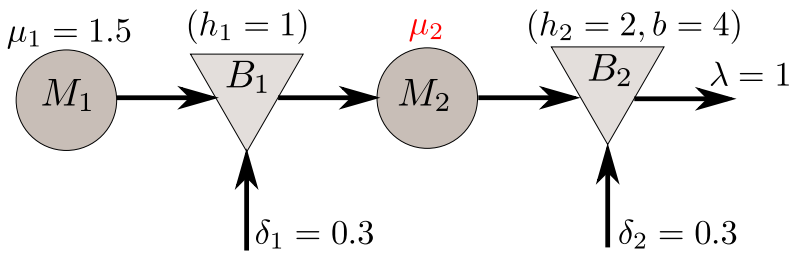
Conjecture

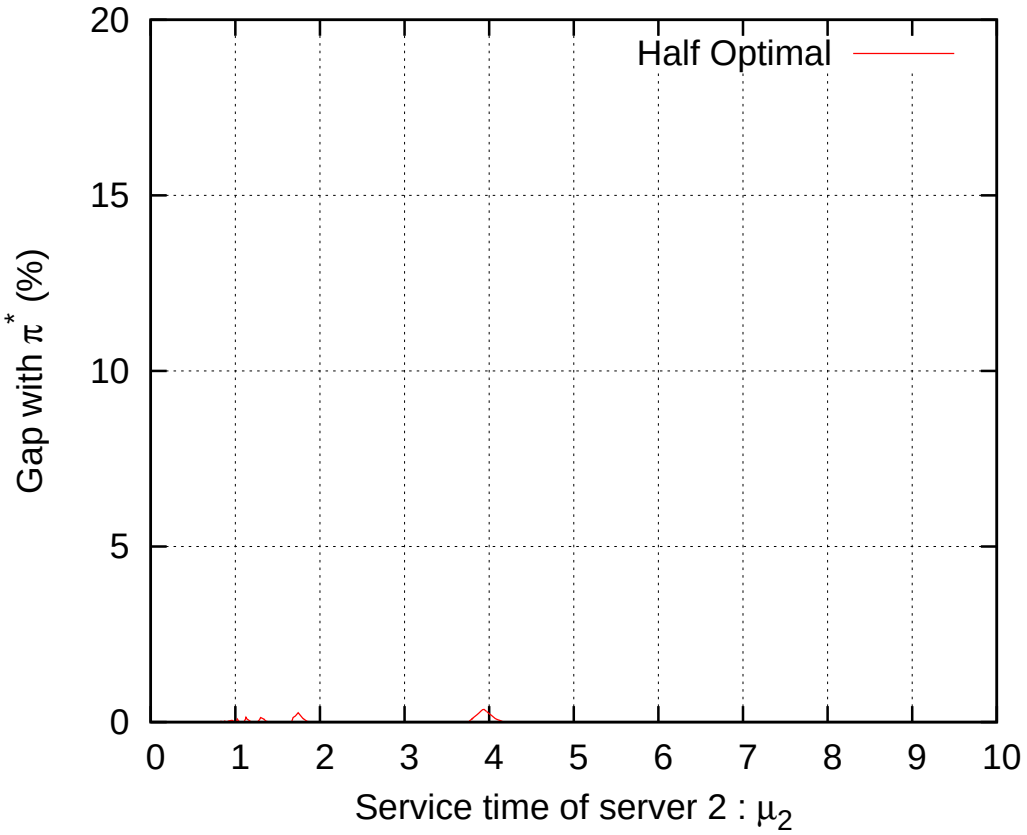
$C(s_1, s_2)$ is unimodal.

Procedure

Maximal gradient with step of 1

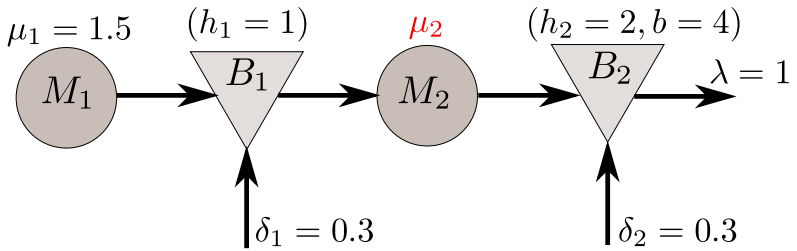
Comparison of heuristics

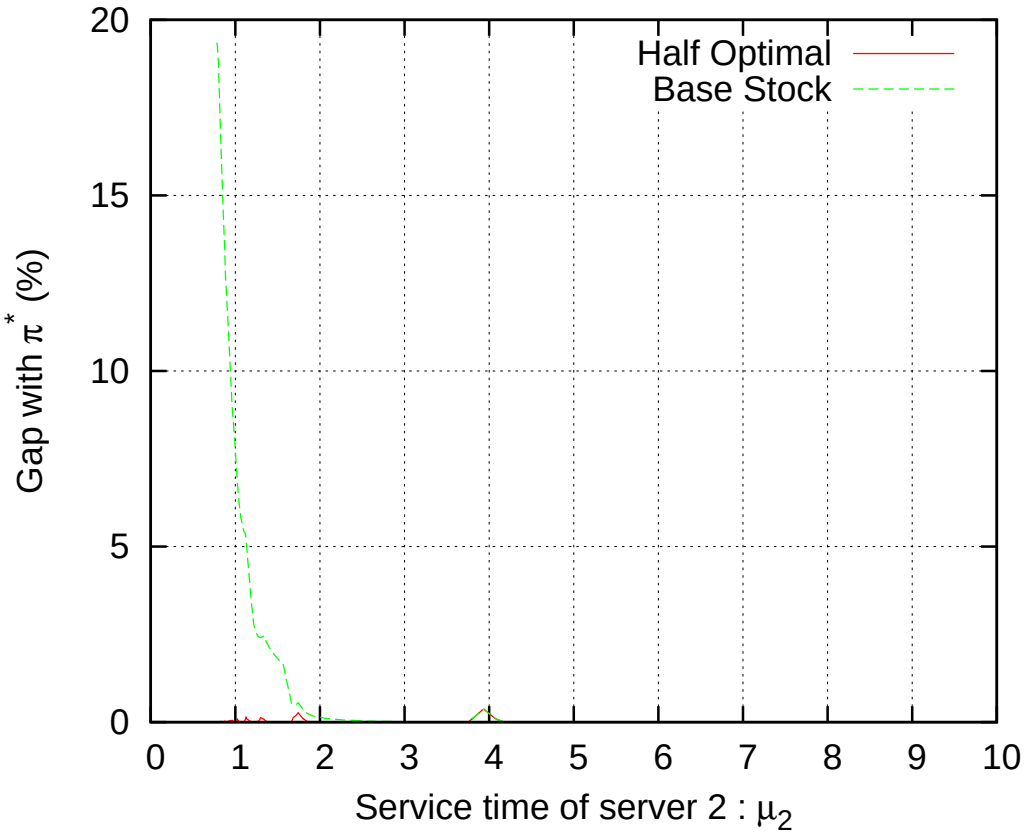




- HO is locally optimal

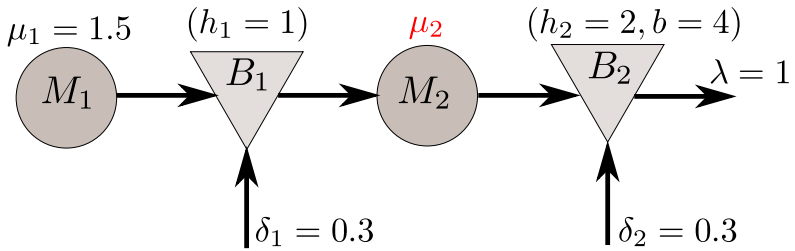
Comparison of heuristics

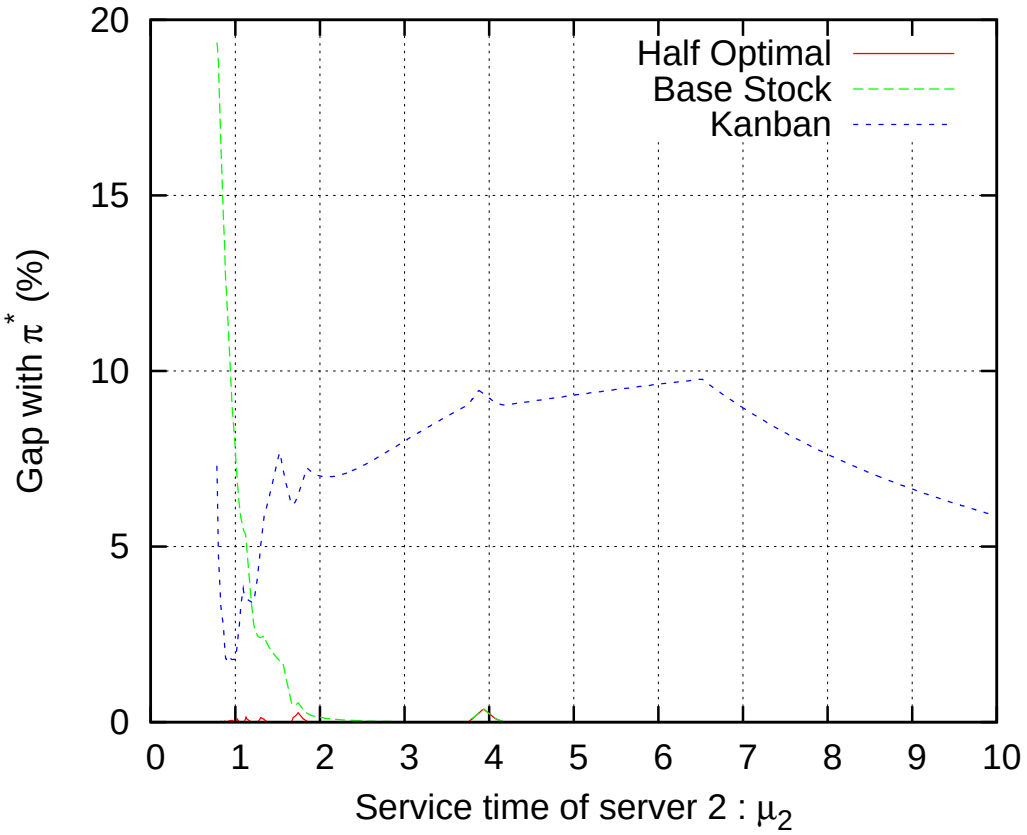




- HO is locally optimal
- BS is generally a good heuristic

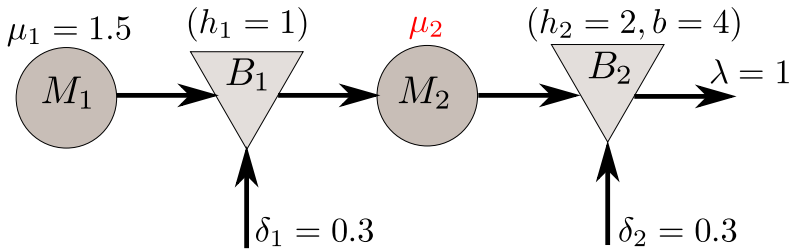
Comparison of heuristics

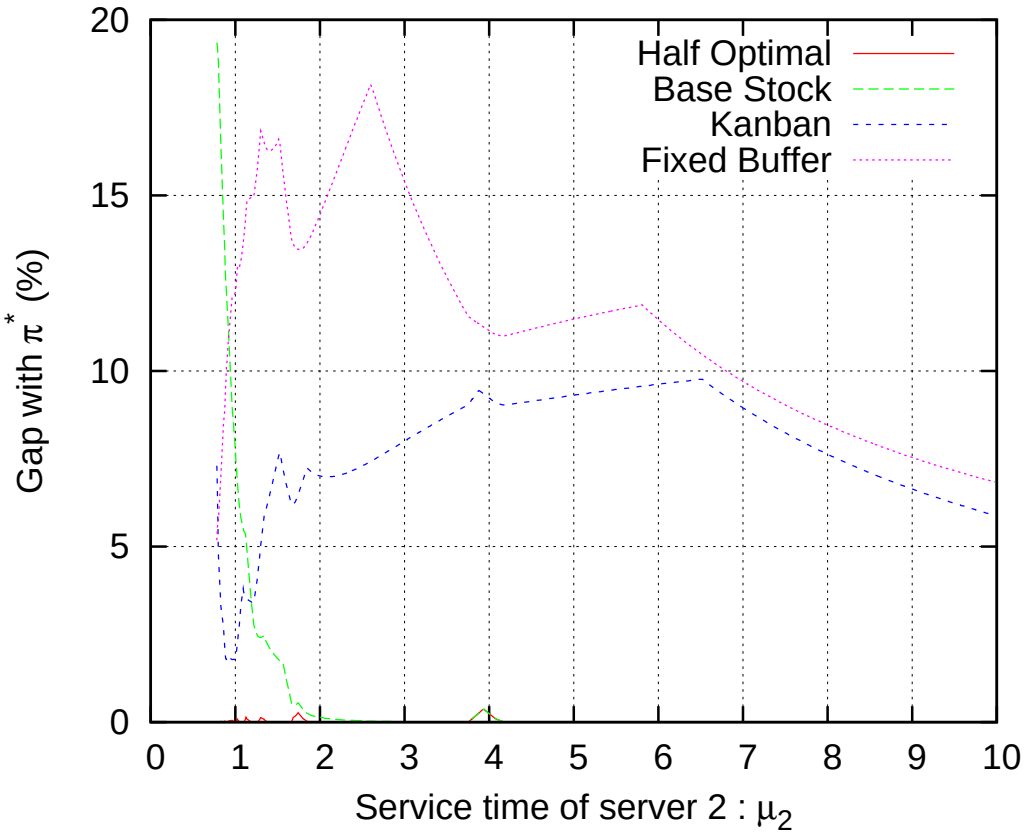




- HO is locally optimal
- BS is generally a good heuristic
- KB is good heuristic when server 2 is overloaded

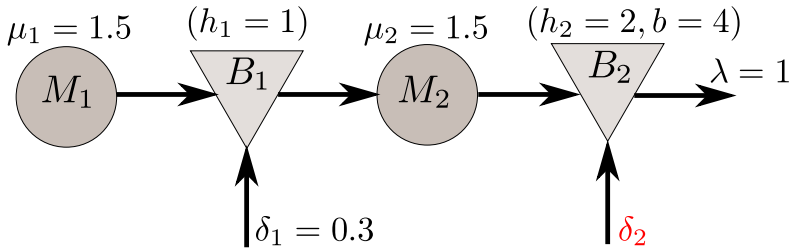
Comparison of heuristics

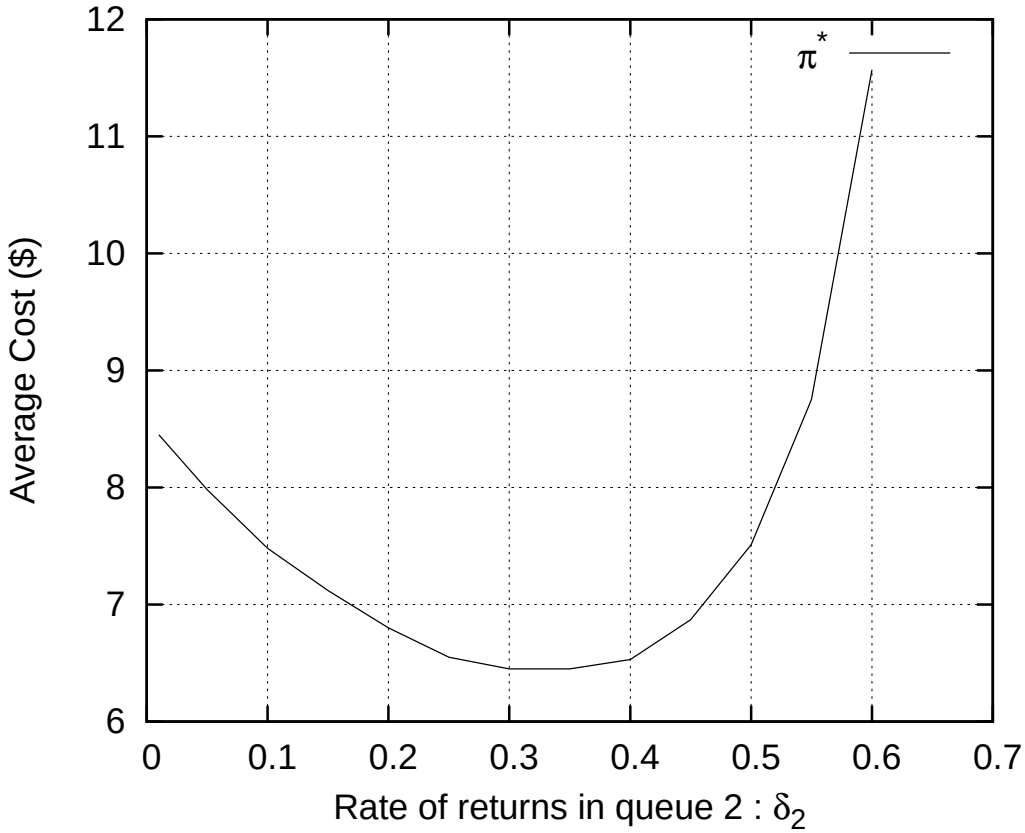




- HO is locally optimal
- BS is generally a good heuristic
- KB is good heuristic when server 2 is overloaded
- FB is good heuristic when server 2 is very overloaded

Impact of returns

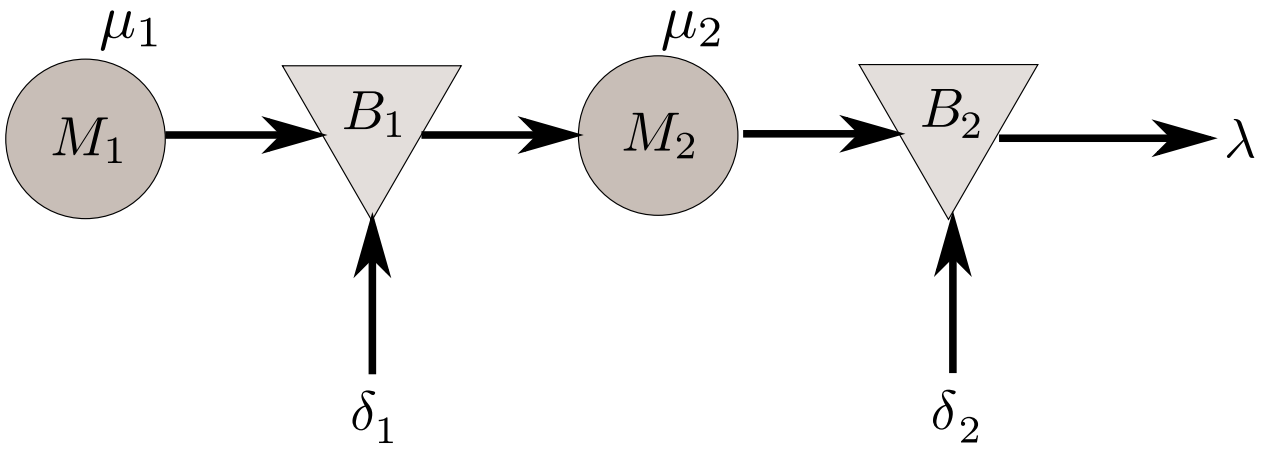




- Few returns help system to produce, so the cost decreases.
- Lot of returns saturate system, so the cost increases.

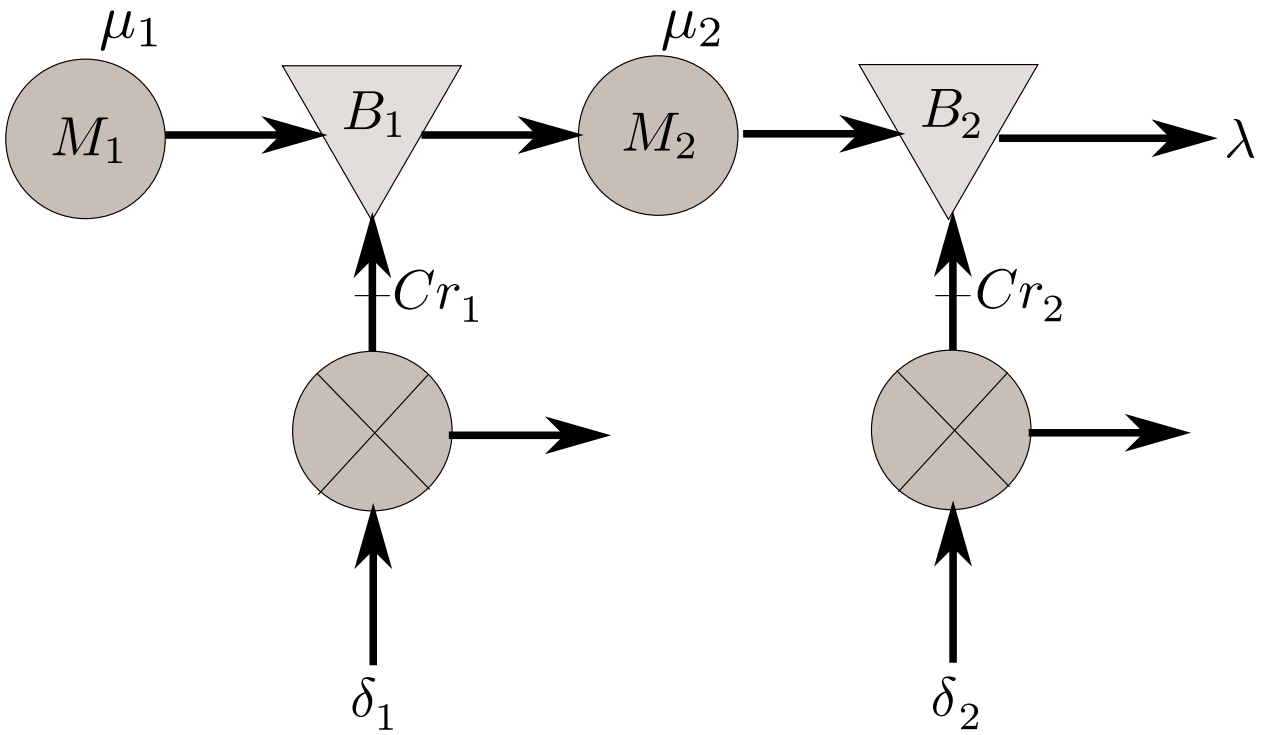
2-stages system with disposable returns

Before : we can not dispose returns



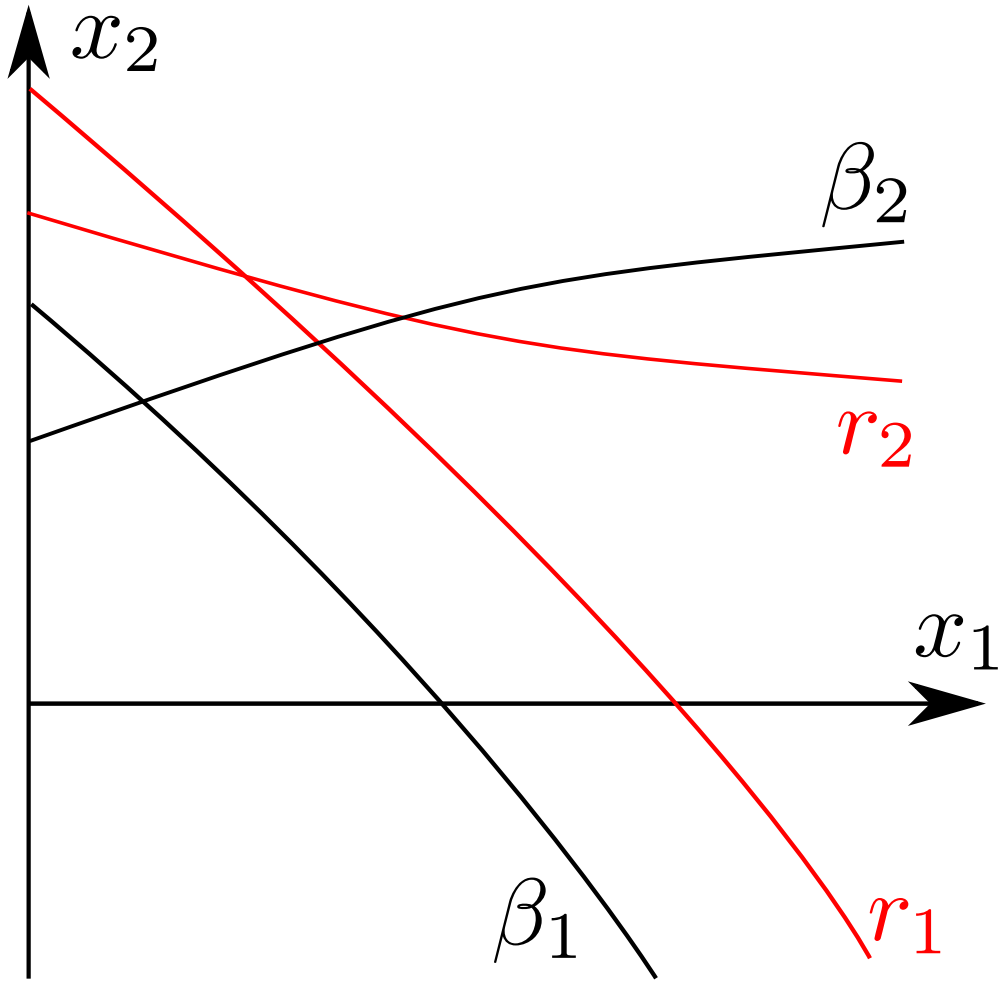
2-stages system with disposable returns

Now : we relax this assumption, so we can dispose returns

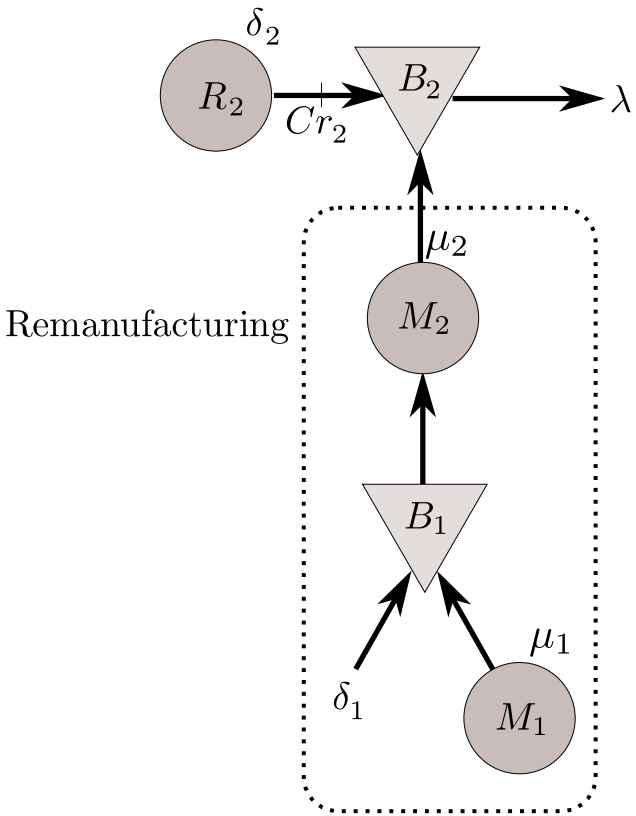


Optimal Policy for disposable case

Form of the Optimal Policy :



Equivalence (same Optimal Policy)



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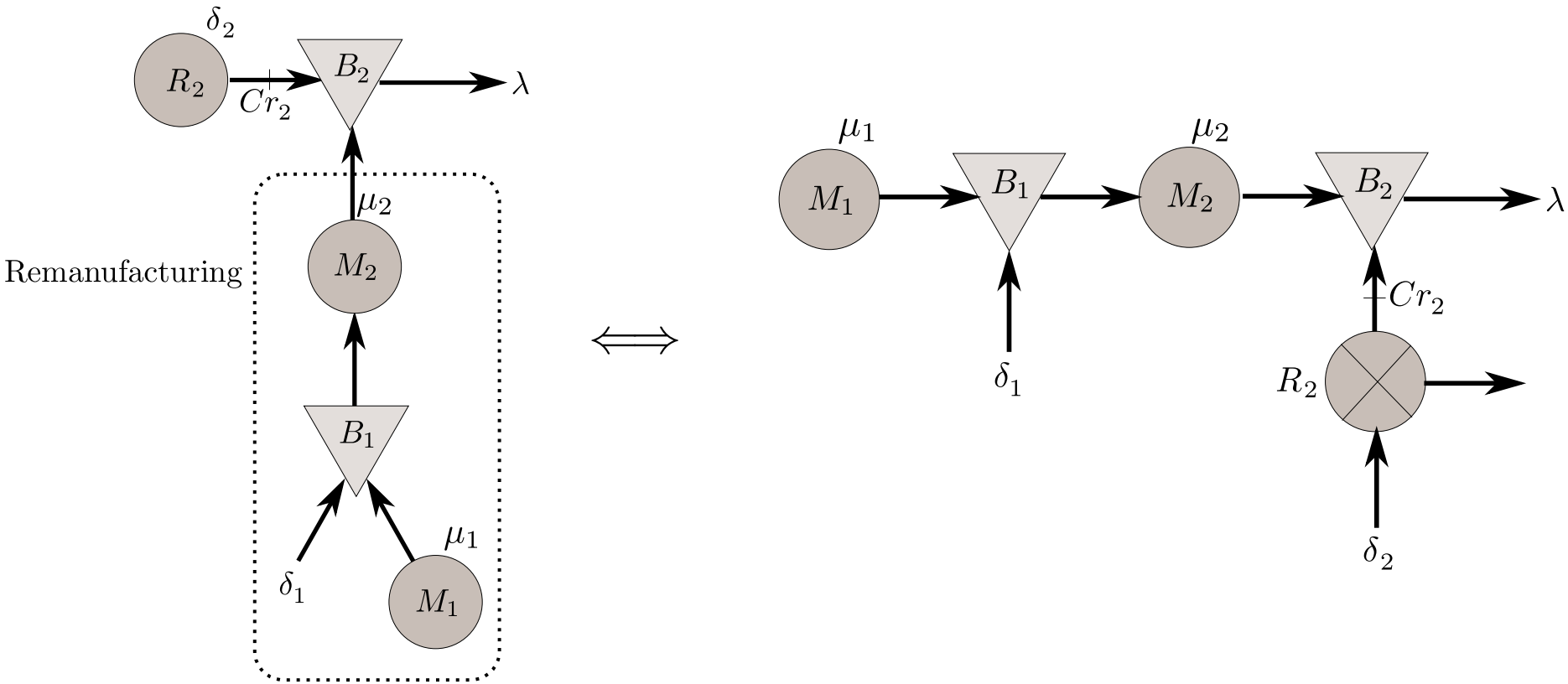

Contribution

- Proof of the form of $\pi^\star$ for forced and disposable cases.
- Proof of the form of $\pi^\star$ for remanufacturing case.
- Comparisons of heuristics and $\pi^\star$.
- New heuristics : Half Optimal near optimal.

Future research

- Investigate more complex systems with more than 2 stages
- Make-to-order system
- Explain why β_2 increases.